Deleveraging, deflation and depreciation in the euro area

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Abstract

During the post-crisis period, economic performance has been highly heterogenous across the euro area. While some economies rebounded quickly after the 2009 output collapse, others are undergoing a protracted further decline as part of an extensive deleveraging process. At the same time, inflation has been subdued throughout the whole of the euro area and intra-euro-area exchange rates have hardly moved. We interpret these facts through the lens of a two-country model of a currency union. We find that deleveraging in one country generates deflationary spillovers which cannot be contained by monetary policy, as it becomes constrained by the zero lower bound. As a result, the real exchange rate response becomes muted, and the output collapse—concentrated in the deleveraging economies.

Keywords: Deleveraging, currency union, real exchange rate, zero lower bound, downward wage rigidity, deflationary spillovers, paradox of flexiblity
JEL-Codes: F41, E42

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1 Introduction

Following the onset of the global financial crisis, the euro area has experienced a protracted economic slump, with aggregate output in 2015 still below the 2008 level. The dynamics of the slump among individual countries, however, have been highly heterogenous. Whilst all countries underwent a deep recession in 2009, some rebounded quickly whereas others have experienced a persistent and protracted further decline. These most adversely affected economies are also undergoing a severe deleveraging process (Martin and Philippon, 2014; Reinhart and Rogoff, 2014).[1] Meanwhile, inflation in the euro area has been subdued, both during the initial downturn and the subsequent years. Moreover this subdued inflation has taken hold throughout the union, both in countries experiencing a deep slump and those doing relatively well. As a consequence, intra-euro-area real exchange rates have hardly moved during the post-crisis period.

In this paper we ask why—despite the heterogenous deleveraging and output performance across the euro area—there has been no significant adjustment of intra-euro-area real exchange rates. To provide an answer, we put forward a stylised two-country model of a currency union which accounts for the key features of the crisis in the euro area. The two countries represent the two heterogenous groups in the euro area, namely the "stressed" economies undergoing a severe slump and the "non-stressed" economies which have performed relatively well. Each country specialises in production of differentiated goods. The real exchange rate fluctuates with the relative price of these goods because goods markets are imperfectly integrated. Whilst goods prices are flexible in both economies, wages are downwardly rigid as in Schmitt-Grohé and Uribe (2015). Monetary policy aims to stabilise union-wide inflation, but may be constrained by the zero lower bound on nominal interest rates.

Drawing on Eggertsson and Krugman (2012), we assume that a group of households in the stressed economy are forced to reduce their debt because their borrowing limit tightens exogenously. We study how the repercussions of this deleveraging shock transmit through the entire currency union. The model is able to account for key features of the data. First, the output performance is heterogenous. Output collapses in the stressed economy, but is hardly affected in the rest of the union. Second, there is—at the same time—union-wide deflationary pressure such that, third, the real exchange rate hardly moves. Our analysis reveals that size matters for these results: both the size of the country which is subject to the deleveraging shock and the size of the shock itself.

To establish this analytically, we first consider the case when the stressed economy is gener-

[1] For an early account of how deleveraging in the global banking sector helped the financial crisis in the United States to morph into a global crisis, see Kollmann et al. (2011).

ically small. In this case, deleveraging does not impact the rest of the union at all and depreciates the stressed economy's real exchange rate. In line with conventional wisdom, we find that the extent of depreciation is limited by the extent of downward wage rigidity in the stressed economy (Friedman, 1953).[2] By the same token, the recession turns out to be less severe, the more flexible wages are.

Once we turn to the other polar case and assume that the stressed economy is large, the effects of the shock turn out to be fundamentally different. This holds in particular if the shock, in addition to the economy, is large as well. In this case, monetary policy finds itself pushed to the zero lower bound, and hence unable to contain the deflationary effects of the shock—not only in the stressed economy, but in the entire union. Whilst relative wage rigidities in the two countries still play a role in the adjustment process, deflationary spillovers from the stressed to the non-stressed economy will generally dampen the extent of real depreciation. Under specific conditions the real exchange rate may even appreciate, echoing earlier findings of a "perverse" response of real exchange rates at the zero lower bound when exchange rates are flexible (Cook and Devereux, 2013).

Furthermore in this scenario—somewhat paradoxically—the real exchange rate may depreciate less, the more flexible wages in the stressed economy are. The reason for this can be traced back to the root cause of the crisis itself—debt, or more precisely debt deflation à la Fisher (1933). As prices in the domestic economy decline, the real value of debt increases, thwarting the initial efforts to reduce the debt. Increased wage flexibility gives rise to more debt deflation, amplifying the recession and dampening the real exchange rate response further—an instance of the "paradox of flexibility", as established by Eggertsson and Krugman (2012) for the closed economy. We offer an important qualification to this paradox in the currency union setting, however: it only applies if both the shock and the size of the domestic economy are large. In particular, if the domestic economy is small, the drop in domestic prices depreciates the real exchange rate, stabilising the economy. Moreover in this case, long-run purchasing power parity implies that the decline in domestic prices is met by future inflation, which is also stabilising (Corsetti et al., 2013).

What remains to be determined is whether the size of stressed economies in the euro area, and the magnitude of deleveraging are sufficiently large to push the union to the zero lower bound and generate enough deflationary spillovers for the real exchange rate response to be muted. In our quantitative analysis, we find this to be the case for plausible parametrisations

[2] Kollmann (2001) and Monacelli (2004) perform analyses of how nominal rigidities impact real exchange rate volatility under flexible and fixed exchange rates in small open economies. Broda (2004) provides evidence for developing countries in support of the received wisdom.

of the model.[3] As monetary policy becomes constrained by the zero lower bound, domestic output and prices decline, as do prices in the entire monetary union. Importantly, the decline in prices in the foreign economy implies that foreign output remains close to full employment. The model thus generates a heterogenous output response across the two countries. At the same time, deflationary spillovers ensure that the real exchange rate remains basically flat. Counterfactual simulations suggest that greater wage flexibility in the stressed economies is unlikely to be stabilising, and may instead deepen the recession.

The effects of debt deleveraging in an open economy context have been analysed in a number of other studies. Benigno and Romei (2014) examine the implications of deleveraging by one country within the world economy and study how monetary policy should be optimally set at a global level. However they do not consider the case of a monetary union. Fornaro (2015) studies the implications of deleveraging within a part of a monetary union under the zero lower bound constraint, as in our paper. But he abstracts from internal debt and debt deflation within countries, and does not focus his attention on relative price movements across stressed and non-stressed economies, which is the main focus of our paper. Gilchrist et al. (2015) also study the lack of real exchange rate adjustment across the euro area, but focus on financial constraints of firms rather than households. In their account, adverse financing conditions induce firms in stressed economies to keep prices high relative to what would be optimal under benign conditions. Firms in non-stressed economies, in turn, find it optimal to reduce prices in order to cannibalise the market share of stressed firms.

The remainder of the paper is organised as follows. The next section provides a number of basic facts regarding the post-2008 dynamics in the euro area. Section 3 introduces the model. We discuss analytical results for the limiting cases in Section 4. Section 5 presents results obtained from model simulations. A final section concludes.

2 Some facts

In this section we present time series evidence that highlights important aspects of the post-crisis slump in the euro area and provides some background for our model-based analysis. Our focus is on two regions, each consisting of a group of countries. The "stressed" region comprises countries where the crisis was particularly severe: Greece, Italy, Ireland, Portugal and Spain. The "non-stressed" region consists of Austria, Belgium, Finland, France, Germany and the Netherlands. For each group we aggregate time series using 2007 GDP weights. The stressed economies make up roughly 37% of GDP of all the countries in our sample.[4] Our

[3] Stressed economies make up 37% of the union and deleveraging is 34% of annual GDP per borrower, based on eurozone data.

[4] The 37% figure corresponds to pre-crisis levels. During the crisis, the share falls to around 34%.

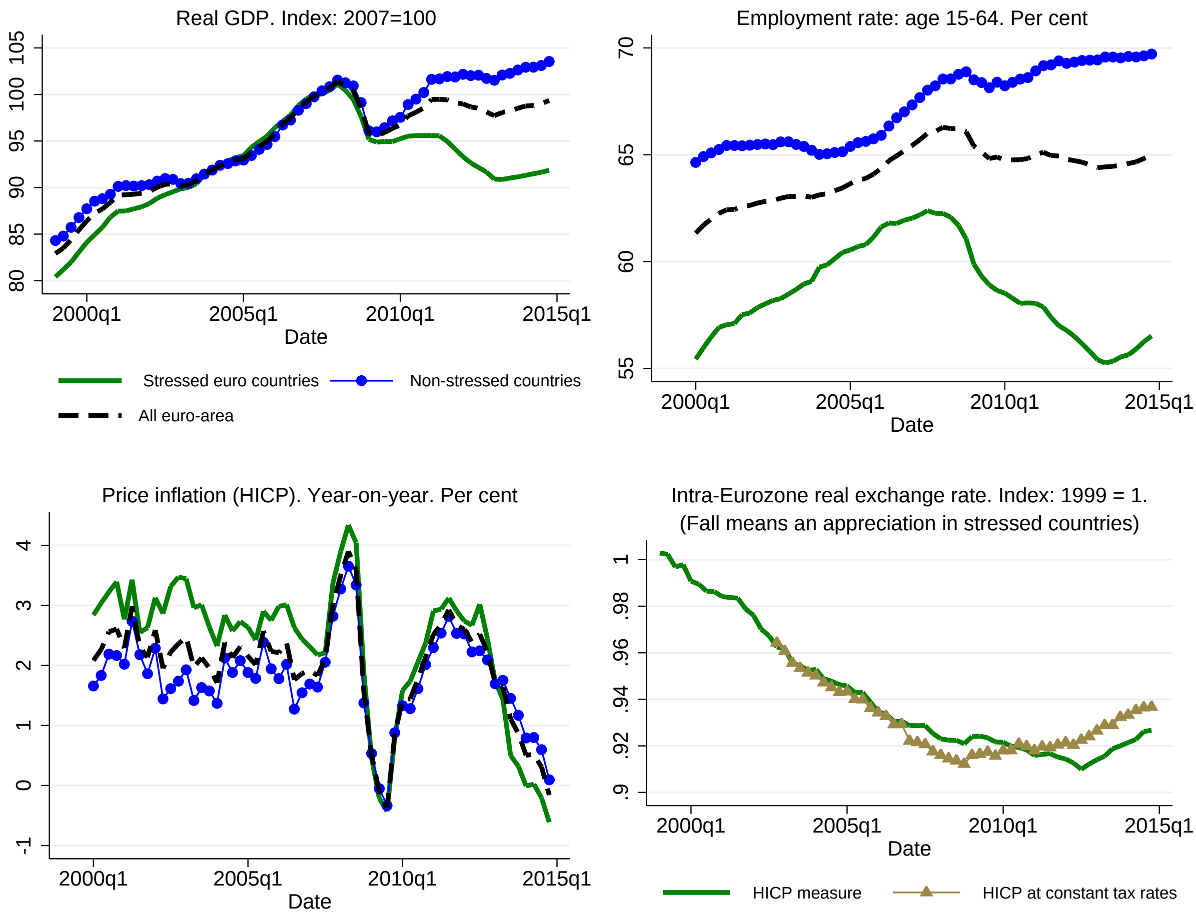

Figure 1: Development of macroeconomic aggregates. Source: Eurostat. Averages are weighted by 2007 nominal GDP. Irish and Finnish tax-adjusted exchange rates series start in 2013 and 2005 respectively; unadjusted data used beforehand.

sample, in turn, covers about 97% of euro-area GDP.

Figure 1 displays quarterly time series data for macroeconomic aggregates, covering the period 1999Q1–2014Q4. Dashed lines correspond to data for the euro area as a whole, solid lines correspond to the stressed economies and dotted lines—to the non-stressed economies. The upper-left panel shows real GDP normalised to 100 in the pre-crisis year 2007. In the run-up to the crisis output growth was quite synchronised across the two regions. Even during the early stages of the crisis, the GDP collapse in stressed and non-stressed countries was roughly the same. But since 2010 the growth performance has been quite distinct. The decline in GDP in the stressed economies seemed to bottom out in 2011–2012, only to decline further afterwards. While there is a small recovery at the end of our sample, stressed-economy GDP is still almost 10 percent below its pre-crisis level. In contrast, GDP recovered relatively quickly in the non-stressed countries, surpassing the pre-crisis level in early 2011. A similar picture emerges for employment data (top-right panel).

The intra-euro-area real exchange rate is displayed in the bottom-right panel of Figure 1. The solid line indicates the exchange rate measure based on the harmonised index of consumer prices. The line with markers (triangles) corresponds to a series which controls for tax changes. In both instances, we normalise the exchange rate to unity in 1999 and define it such that a decline corresponds to an appreciation for the stressed economies. In the years prior to the crisis the real exchange rate appreciated by about 8 percent, but has moved little after 2008. The HICP-based measure indicates that there was a further, if very mild, appreciation in the early stage of the crisis and an equally mild depreciation after 2013. The relative price adjustment is somewhat more pronounced but still muted for the series which is purged of the effect of tax changes. Within the euro area, changes in the real exchange rate are the result of differential inflation developments across the two regions, which are shown in the lower left panel. Prior to the crisis inflation in the stressed economies always exceeded inflation in the non-stressed region. Since the start of the crisis, however, inflation dynamics across the two regions have been markedly similar: inflation in both regions briefly turned negative in 2009, recovered afterwards but declined again after 2012.

In the model-based analysis that follows we explore the adjustment of the real exchange rate to a deleveraging process which takes place in one region of the currency union. That such a process contributed to the post-crisis slump has been suggested by many observers (see, for instance, Martin and Philippon, 2014; Reinhart and Rogoff, 2014) and is hardly controversial in light of the facts. The upper panels of Figure 2 display real credit growth (top row) and credit volumes relative to GDP (middle row) in the euro area, again distinguishing between area-wide developments and those in the stressed and non-stressed economies. In the left and right panels we show private and household debt respectively.

We observe that prior to the crisis the stressed economies experienced a particularly rapid expansion of credit. In fact their real credit growth averaged close to 13% per year, roughly three times the corresponding rate in the non-stressed economies. The advent of the crisis in 2008–09 coincided with a collapse in lending growth. Credit growth in both regions was close to zero in 2009 and then turned negative in the stressed economies whilst recovering somewhat in the non-stressed economies. In the former, it is still negative at the end of our sample. Overall, credit volumes thus declined considerably during the crisis—as far as the stressed economies are concerned. Expressed relative to GDP, overall private credit (household debt) peaked at some 170 (65) percent of GDP in the stressed economies in 2009. Since then the decline in credit relative to GDP has been muted by the sizeable decline in GDP.

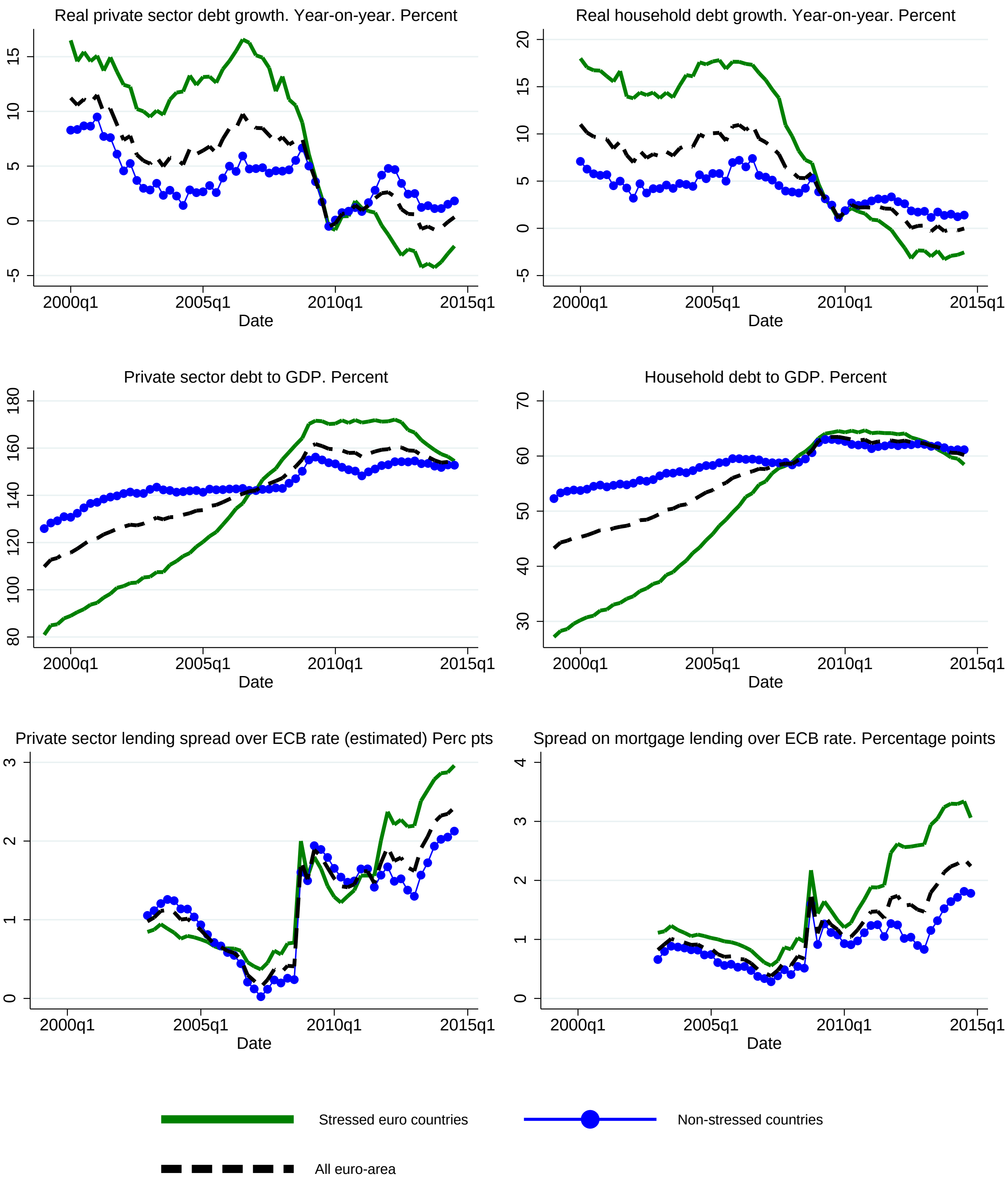

Figure 2: Development of debt and spreads. Sources: Eurostat, ECB and BIS. Averages are weighted by 2007 nominal GDP. Irish household debt data estimated before 2001Q4.

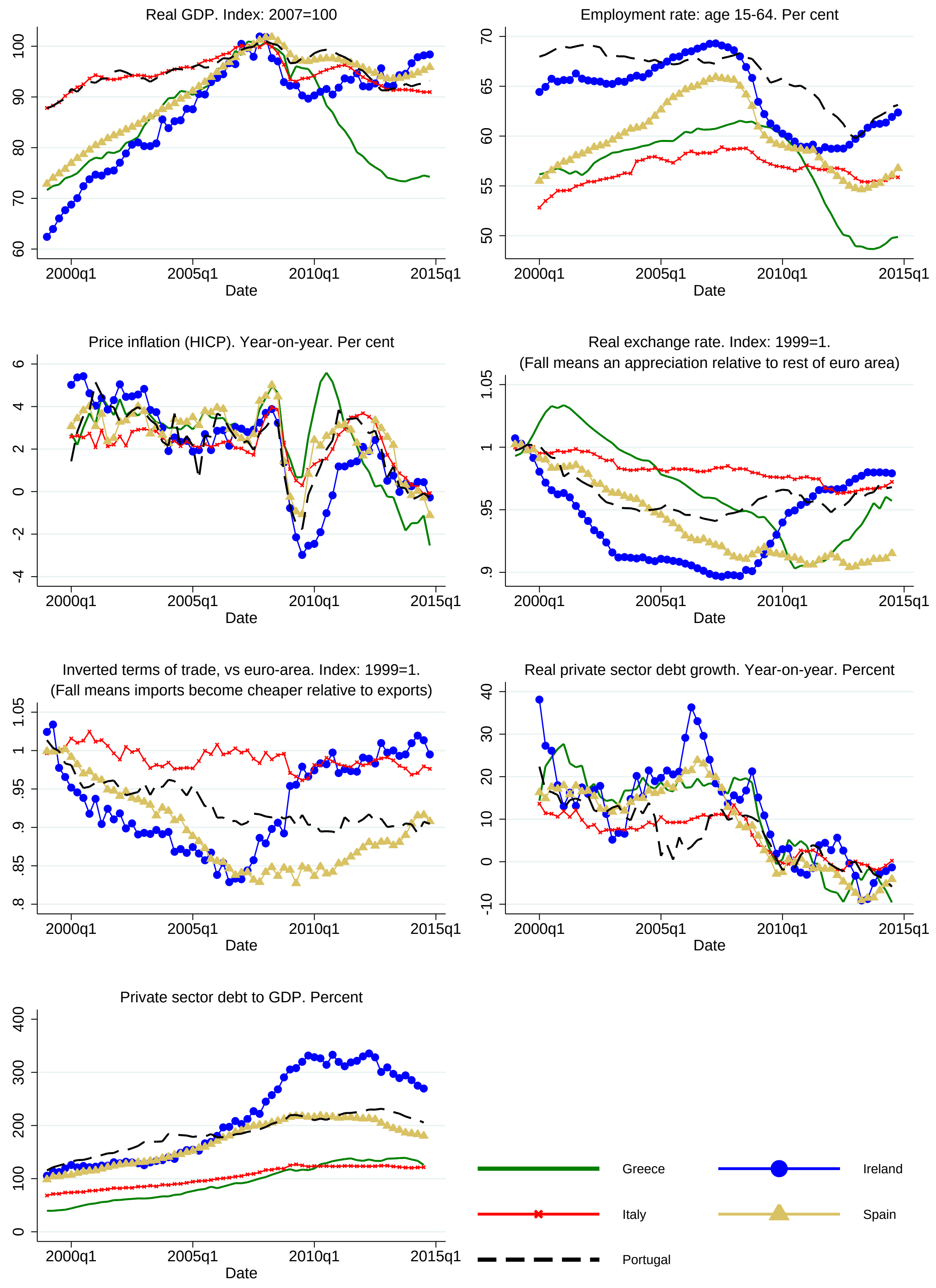

Figure 3: Developments in individual countries of the stressed region. Sources: Eurostat, BIS and OECD Economic Outlook.

In the bottom panels of Figure 2 we show the dynamics of interest rates. Specifically, we compute the difference of private-sector lending (left panel) and mortgage (right panel) rates relative to the ECB main refinancing rate. The spread on mortgages is higher in the stressed economies throughout the sample period. There is, however, a marked widening in the gap between stressed and non-stressed economies after 2010, reaching some 2 percentage points. The picture is less clear-cut for the spread on private sector lending rates. However, private sector spreads also tightened by considerably more in the stressed economies after 2009.

Figure 3 zooms in on the developments of individual countries of the stressed-economies aggregate. As we might expect, the disaggregated picture is more nuanced, with somewhat different dynamics from country to country. Ireland stands out with the largest lending boom, and also the most significant real exchange rate adjustment. Italy, on the contrary, did not experience much of a boom in lending or a significant real exchange rate appreciation before the crisis. Greece, in turn, shows the most dismal performance in terms of post-crisis output and employment.

Despite these differences, we find sufficiently strong similarities across the stressed euro area countries for our stressed/non-stressed country classification to be justified. First, the top panels of Figure 3 (real GDP and employment) show that all countries are experiencing a prolonged slump and are yet to fully recover. Second, inflation dynamics are quite synchronised (second row, left panel). Third, in all countries real exchange rates are still appreciated relative to the beginning of the sample (second row, right panel). For our sample, we also observe that the movements in real exchange rates reflect to a large extent movements of the terms of trade (displayed in the left panel of the third row).[5] This observation squares well with formal results of earlier studies which decompose real exchange rate movements (e.g., Engel, 1999).[6] Fourth and finally, note that all countries experienced a lending surge before the crisis and a lending slowdown after 2009, albeit to different degrees.

In sum, a simple inspection of the facts supports the view that the stressed economies of the euro area are experiencing a fully-fledged balance sheet recession. A sizeable build-up of debt was followed by a lengthy period of deleveraging, with very adverse consequences for economic activity. On the contrary, there is a recovery in the non-stressed region. Against this background it may be surprising that inflation is subdued not only in the stressed economies, but in the non-stressed region, too. Equivalently, the lack of real deprecation in the stressed

[5] For ease of comparison, the graph depicts “inverted” terms of trade: the price of imports relative to exports. That way it is directly comparable to our real exchange rate series. No observations are available for Greece.

[6] However, in a recent study Berka et al. (2015) find that for a pre-crisis sample of euro area countries, cross-country productivity differentials in the traded and non-traded good sector matter a great deal for real exchange rate movements, even in the short run. According to the authors this result is likely due to the fact that the countries in their sample maintain a common currency.

economies relative to the non-stressed economies may appear puzzling. We investigate this issue further by means of a model-based analysis.

3 The model

Our analysis is based on a simple two-country model of a currency union. Countries specialise in the production of specific goods which are traded across countries. Good market integration is incomplete, however, as countries' consumption is biased towards domestically produced goods. The real exchange rate may therefore deviate from unity. Given that the real exchange rate and the terms of trade co-move strongly in our sample (see Section 2), we abstract from the production of non-traded goods. Countries may differ in size and we assume "Home" makes up a mass $[0, n)$ of the total union population, where $n \in [0, 1]$. Each country is populated by a unit mass of agents, who supply labour inelastically to domestic firms. Within Home we distinguish between households with high and low discount factors as in Eggertsson and Krugman (2012). We refer to these households as "savers" and "borrowers" respectively. The rest of the union ("Foreign") is populated by savers only. Savers in one country can trade a nominally non-contingent bond with the savers in the other country. Savers in Home can, in addition, lend funds to domestic borrowers. Prices are fully flexible, but downward adjustment of nominal wages is restricted as in Schmitt-Grohé and Uribe (2015).

3.1 Households

In Home, borrowers account for a fraction $\chi \in (0, 1)$ of the population. They are less patient than savers, which account for the rest. A typical saver in Home maximises

$$\max_{\{C_t^s\}_{t=0}^{\infty}} \sum_{t=0}^{\infty} (\beta^s)^t \ln(C_t^s)$$

subject to

$$P_t C_t^s + R_t^{-1} B_t^s + R_t^{*-1} D_t = W_t L_t + B_{t-1}^s + D_{t-1} \tag{3.1}$$

and a constraint which rules out Ponzi games. Here $\beta^s < 1$ is the discount factor which exceeds the discount factor of the borrower: $\beta^s > \beta^b$. C_t^s denotes savers' per capita consumption and P_t is the consumer price level in Home. Savers earn rate R_t by lending to borrowers at home (B_t^s), and rate R_t^* by saving abroad (D_t). All debt and interest rates are denominated in nominal terms. W_t is the nominal wage rate in Home, and L_t are hours worked. Optimality requires the following Euler equation to hold

$$(C_t^s)^{-1} = \beta^s R_t \left(C_{t+1}^s\right)^{-1} \frac{P_t}{P_{t+1}}, \tag{3.2}$$

as well as a transversality condition. The absence of arbitrage possibilities between domestic and foreign assets requires that

$$R_t = R_t^*. \tag{3.3}$$

In turn, a typical borrower maximises

$$\max_{\{C_t^b\}_{t=0}^{\infty}} \sum_{t=0}^{\infty} (\beta^b)^t \ln(C_t^b)$$

subject to

$$P_t C_t^b + B_{t-1}^b = R_t^{-1} B_t^b + W_t L_t \tag{3.4}$$

$$B_t^b \leq \bar{B}_t. \tag{3.5}$$

Here, B_t^b denotes nominal debt vis-à-vis the savers which may not exceed an exogenous, potentially time-varying, debt limit $\bar{B}_t$. First order conditions imply that (3.5) holds with equality at all times.[7] Aggregate consumption in Home is given by

$$C_t = (1-\chi)C_t^s + \chi C_t^b. \tag{3.6}$$

In the rest of the union, all households are savers and we suppress the superscript s for simplicity. The objective is

$$\max_{\{C_t^*\}_{t=0}^{\infty}} \sum_{t=0}^{\infty} (\beta^s)^t \ln(C_t^*)$$

subject to

$$P_t^* C_t^* + R_t^{*-1} D_t^* = W_t^* L_t^* + D_{t-1}^*, \tag{3.7}$$

where stars denote variables in the rest of the union. First order conditions imply

$$(C_t^*)^{-1} = \beta^s R_t^* \left(C_{t+1}^*\right)^{-1} \frac{P_t^*}{P_{t+1}^*}. \tag{3.8}$$

We allow for home bias in consumption in both countries, and the elasticity of substitution between domestic and imported goods is unity. Specifically, aggregate consumption is a composite of two goods in both countries:

$$C_t = \frac{C_{H,t}^{\lambda} C_{F,t}^{1-\lambda}}{\lambda^{\lambda}(1-\lambda)^{1-\lambda}}, \qquad C_t^* = \frac{C_{H,t}^{\lambda^*} C_{F,t}^{1-\lambda^*}}{\lambda^{*\lambda^*}(1-\lambda^*)^{1-\lambda^*}},$$

where $\lambda = 1-(1-n)\omega$ and $\lambda^* = n\omega$. Here, $C_{H,t}$ is the locally produced good, $C_{F,t}$ is the good produced in the rest of the union, and $\omega \in [0,1]$ determines the degree of home bias in

[7] More precisely, optimality requires that $\left(C_t^b\right)^{-1} \geq \beta^b R_t \left(C_{t+1}^b\right)^{-1} \frac{P_t}{P_{t+1}}$, holding with equality whenever $B_t^b < \bar{B}_t$, and requiring $B_t^b = \bar{B}_t$ whenever holding with strict inequality. In the steady state of the model as well as during the deleveraging phase, the latter case always obtains, such that we omit this case distinction from the main text.

consumption, which we assume is symmetric across countries. If $\omega = 1$ there is no home bias. This formulation of relative consumption weights follows Sutherland (2005) and De Paoli (2009). Note that both country size n and home bias $1 - \omega$ affect the consumption shares of Home- and Foreign-produced goods; however they do so in different ways. As the size of the domestic country grows (as n increases), both domestic and foreign households consume a bigger share of the good produced in Home. By contrast, an increase in $1 - \omega$ implies that both domestic and foreign households consume a bigger share of goods produced in their own country (Home and Foreign respectively)—the classic notion of "home bias".[8]

The local good sells at price $P_{H,t}$, while the foreign good sells at price $P_{F,t}$. Expenditure minimisation implies

$$P_t = P_{H,t}^{\lambda} P_{F,t}^{1-\lambda}, \qquad P_t^* = P_{H,t}^{\lambda^*} P_{F,t}^{1-\lambda^*}, \tag{3.9}$$

that is, the consumer price indices domestically and abroad are a weighted average of the producer prices of the two goods.

Furthermore, we define the real exchange rate Q_t as the price of foreign consumption in terms of domestic consumption,

$$Q_t = \frac{P_t^*}{P_t}, \tag{3.10}$$

such that an increase in Q_t indicates a depreciation of Home's real exchange rate.

3.2 Firms

Firms operate in competitive goods and labour markets. They maximise profits $P_{H,t}Y_t - W_t L_t$ in Home, $P_{F,t}Y_t^* - W_t^* L_t^*$ in Foreign, subject to

$$Y_t = L_t, \qquad Y_t^* = L_t^* \tag{3.11}$$

respectively, and their first order conditions imply

$$P_{H,t} = W_t, \qquad P_{F,t} = W_t^*. \tag{3.12}$$

As in Schmitt-Grohé and Uribe (2015), the labour market is characterised by downward nominal wage rigidity. In each period, a maximum of $\bar{L}$ hours can be sold to firms

$$L_t \leq \bar{L}, \qquad L_t^* \leq \bar{L} \tag{3.13}$$

while wages may fall by at most $(1 - \gamma)$ in Home, $(1 - \gamma^*)$ in Foreign, in proportion to their previous level

$$W_t \geq \gamma W_{t-1}, \qquad W_t^* \geq \gamma^* W_{t-1}^*. \tag{3.14}$$

[8] This in turn implies that the real exchange rate is independent of the distribution of wealth *only if* home bias is zero ($\omega = 1$), regardless of the value of n. A formal analysis of this issue is available on request. Note that furthermore, the size of the economy n impinges on the supply side of the model, as more goods are being produced in a country that is larger (formally, this is implied from equation (3.21)).

We require that $1 \geq \gamma > 0$ and $1 \geq \gamma^* > 0$, where $\gamma, \gamma^* \to 0$ characterises flexible wages, and $\gamma, \gamma^* = 1$—full rigidity. The labour markets are closed by complementary slackness conditions of the form

$$(L_t - \bar{L})(W_t - \gamma W_{t-1}) = 0, \qquad (L_t^* - \bar{L})(W_t^* - \gamma^* W_{t-1}^*) = 0, \tag{3.15}$$

which imply that, as long as wages are free to adjust, the economy must operate at potential. Conversely, involuntary unemployment is possible as (3.14) becomes a binding constraint.

3.3 Monetary policy

We assume that monetary policy is characterised by a strict inflation targeting rule, adjusting the nominal interest rate such that area-wide inflation is zero, subject to a zero lower bound constraint. It targets

$$\Pi_t^u = 1 \text{ subject to } R_t \geq 1, \tag{3.16}$$

where $\Pi_t^u = (P_t)^n (P_t^*)^{1-n} / (P_{t-1})^n (P_{t-1}^*)^{1-n}$ is area-wide inflation, and sets

$$R_t = 1 \tag{3.17}$$

if due to deflationary pressure, the inflation target cannot be reached.

3.4 Market clearing

Goods market clearing requires that the supply of domestically produced goods equals domestic as well as export demand

$$Y_t = \left(\frac{P_{H,t}}{P_t} \right)^{-1} \left(\lambda C_t + \frac{\lambda^*(1-n)}{n} Q_t C_t^* \right). \tag{3.18}$$

Equivalently, we require for the Foreign-produced good[9]

$$Y_t^* = \left(\frac{P_{F,t}}{P_t^*} \right)^{-1} \left(\frac{(1-\lambda)n}{1-n} Q_t^{-1} C_t + (1-\lambda^*) C_t^* \right). \tag{3.19}$$

Moreover, asset market clearing requires

$$(1-\chi) B_t^s = \chi B_t^b \tag{3.20}$$

within Home and

$$(1-\chi) n D_t + (1-n) D_t^* = 0 \tag{3.21}$$

across the two countries.

An equilibrium is a sequence of endogenous variables $\{Y_t, Y_t^*, L_t, L_t^*, C_t, C_t^*, C_t^s, C_t^b, B_t^s, B_t^b, \ldots$ $D_t, D_t^*, R_t, R_t^*, P_t, P_t^*, P_{H,t}, P_{F,t}, W_t, W_t^*, Q_t, \Pi_t^u\}$ solving equations (3.1)—(3.21), for given parameters and initial conditions, and exogenous $\{\bar{B}_t\}$.

[9] Note that for the limiting cases we discuss in Section 4, $\frac{\lambda^*(1-n)}{n} \to \omega$ as $n \to 0$ and $\frac{(1-\lambda)n}{1-n} \to \omega$ as $n \to 1$.

3.5 Steady state

We assume that initially the economy is in a symmetric steady state: the real exchange rate, consumer and producer price indices are equal to unity, $P_H = P = P^* = P_F = 1$, which from (3.12) implies that $W = W^* = 1$. Moreover, we let $Y = Y^* = \bar{L}$ and $C^* = Y^*$. This implies $C = Y$ from equations (3.18) and (3.19). We obtain $R = R^* = 1/\beta^s$ from equations (3.2) and (3.8). Borrowers are up against the borrowing constraint, hence $C^b = Y - (1 - \beta^s)\bar{B}$. Savers in Home consume $C^s = Y + (1 - \beta^s)(B^s + D)$, and savers in Foreign $C^* = Y^* + (1 - \beta^s)D^*$. This combined with the fact that $C^* = Y^*$ yields $D^* = 0$, and from (3.21) $D = 0$. That is, net foreign assets must equal zero in the initial steady state. Note that the economy is characterised by non-stationary dynamics, that is, it will generally not revert back to its initial steady state, once it departs from it.[10]

4 Relative prices in a crisis

We now investigate how the economy adjusts to a deleveraging shock. More specifically, we consider a one-off tightening of the debt limit in Home from $\bar{B}_t = \bar{B}^H$ to a permanently lower level $\bar{B}^L$ in time period t. Our setup mimics Eggertsson and Krugman (2012), except that we consider a two-country model and restrict the tightening of the debt limit to take place in one country only. The adjustment will then depend on the size of this country. We illustrate this by first focusing on two limiting cases: $n \to 0$ and $n \to 1$. For these cases we obtain closed-form results, and develop an intuition of the underlying mechanisms. We discuss numerical results for intermediate n in Section 5. Throughout, our main interest is how the real exchange rate in period t responds to the shock.

4.1 Deleveraging in a small union member

If $n \to 0$, Home is effectively a small open economy (see De Paoli, 2009; Galí and Monacelli, 2005). The Home-good consumption weights are $\lambda \to 1-\omega$ and $\lambda^* \to 0$ in Home and Foreign, respectively. In this case, $P^*_t = P_{F,t}$ from equation (3.9), $Y^*_t = C^*_t$ from (3.19) and $D^*_t = 0$ from (3.21). In other words, the rest of the union resembles a closed economy and is not affected by the deleveraging shock in Home. It therefore remains in the initial steady state during the whole deleveraging process. This has important implications for monetary policy: from (3.16) we obtain $\Pi^u_t = P^*_t/P^*_{t-1} = 1$, such that average inflation in the union is zero, and as a result the nominal interest rate remains unchanged as well: $R_t^{-1} = \beta^s$. We summarise

[10] The economy is non-stationary for two reasons. First, international financial markets are incomplete, and second, households are heterogenous. The distribution of wealth, both across agents and across countries is a state of the economy which induces unit-root behaviour in some variables.

our main result in what follows.

Proposition 1. *Consider the economy defined in Section 3 and let* $n \to 0$*. Suppose that in period* t*, the debt limit in Home is unexpectedly and permanently reduced from* $\bar{B}^H$ *to* $\bar{B}^L$*. The real exchange rate at time* t *is then given by*

$$Q_t = P_t^*/P_t = \min\left(\gamma^{\omega-1}, [1 - \eta(\bar{B}^H - \bar{B}^L)/Y]^{\omega-1}\right) \geq 1, \tag{4.1}$$

where $\eta = (\beta^s(1-\omega)\chi)/(1-(1-\omega)\chi) > 0$*. Therefore, there is no depreciation* ($Q_t = 1$) *if wages are completely rigid* ($\gamma = 1$)*, and a greater depreciation, the more flexible wages are, where the upper threshold* $[1 - \eta(\bar{B}^H - \bar{B}^L)/Y]^{\omega-1} > 1$ *is reached once* (3.14) *ceases to bind.*

Proof. See Appendix. □

Intuitively, the deleveraging shock forces borrowers to cut consumption in order to repay their debts. This reduces aggregate demand and puts downward pressure on prices, resulting in a real exchange rate depreciation, as long as wages are allowed to adjust sufficiently. We now establish a second result.

Proposition 2. *Consider again the economy defined in Section 3 with* $n \to 0$*. In the period of deleveraging, output, saver consumption, borrower consumption and real wage income all decline strictly less if wages are more flexible (that is, if* γ *is reduced) up until* (3.14) *ceases to bind, point beyond which they do not vary further. The recession is deepest if wages are completely downwardly rigid* ($\gamma = 1$)*.*

Proof. See Appendix. □

Propositions 1 and 2 establish that wage flexibility and the associated movements in the real exchange rate dampen the response to country-specific shocks, as the received wisdom—going back to at least Friedman (1953)—suggests. Still, it is interesting to analyse this case for the following reasons. First, it will serve as a useful benchmark once we consider $n > 0$. Second, the fact that a real depreciation plays a stabilising role is actually not obvious during a deleveraging recession. The fall in Home prices required to bring about the depreciation increases the real value of debt, giving rise to debt deflation à la Fisher (1933).

To see this, consider how the borrowers respond to the deleveraging shock. When the shock hits, nominal debt of $\bar{B}^H - \beta^s \bar{B}^L$ has to be repaid to satisfy the new, lower, borrowing limit. By rearranging the budget constraint (3.4) as follows

$$C_t^b = -\frac{\bar{B}^H - \beta^s \bar{B}^L}{P_t} + \frac{W_t L_t}{P_t}, \tag{4.2}$$

we see that borrowers' consumption depends on debt repayment as well as on real wage incomes $W_t L_t / P_t$. For a given real income, a lower price level increases real debt and reduces consumption. Still, recall that borrower consumption declines less in general equilibrium with greater wage flexibility and real depreciation (Proposition 2). The reason is that in this case, overall economic activity is higher, which helps sustain the real wage income of the borrowers. First, a weaker real exchange rate crowds in foreign demand for the domestic good. Second, since long-run prices are pinned down by purchasing power parity, a temporary drop in the price level generates expected inflation, which reduces the real interest rate and increases spending by savers.[11] It turns out that these two mitigating factors necessarily outweigh the adverse impact of debt deflation. In other words, while rigidity in wages may rule out debt deflation altogether, the resulting drop in real wage income (which operates via a drop in working hours L_t) depresses borrower consumption all the same—in fact, depresses it by more, the more rigid the wages.

In sum, in the case of a small open economy, a lack of relative price adjustment reflects the presence of nominal rigidities, in our case downwardly sticky wages. We show the adjustment dynamics in Figure 4, which contrasts results for the cases of fully rigid ($\gamma = 1$, solid lines) and flexible ($\gamma = 0.75$, dashed lines) wages. We discuss the parameter choices which underlie the model simulations in Section 5 below. Importantly, we assume that a deleveraging of 17% GDP is undertaken over one year, in period 5.[12] Because the share of borrowers is 0.5, this amounts to a deleveraging of 34% GDP per borrower. Figure 4 echoes our discussion above: if wages are sticky, the exchange rate response is flat, while output collapses. If wages are fully flexible, the exchange rate depreciates strongly, while output remains constant.[13]

The response of consumption also differs across the two scenarios: both saver and borrower consumption are higher under flexible wages, relative to the rigid-wage scenario. This implies, in the context of our model, that welfare is higher under more flexible wages. Galí and Monacelli (2013), in contrast, find that higher wage flexibility may reduce welfare whenever monetary policy seeks to stabilise the exchange rate. This is because Galí and Monacelli (2013) assume a monopolistically competitive labour market and staggered wage setting. As a result, higher wage inflation induces wage dispersion which is detrimental to welfare. We do not consider this possibility. Moreover, recall that in our set-up labour effort has no direct bearing on household utility.

[11] Thus, implicit in fixed exchange rate regimes is an element of price level targeting, which has been emphasised in previous work (Corsetti et al., 2013). A more detailed discussion is provided in Section 4.2.

[12] Here and in the rest of the paper, we refer to deleveraging in terms of nominal GDP before the crisis, which equals 1 in our parametrisation.

[13] In our calibration, $\gamma = 0.75$ provides sufficient flexibility for (3.14) not to bind in the period of deleveraging. A further increase in wage flexibility would then leave results unaltered.

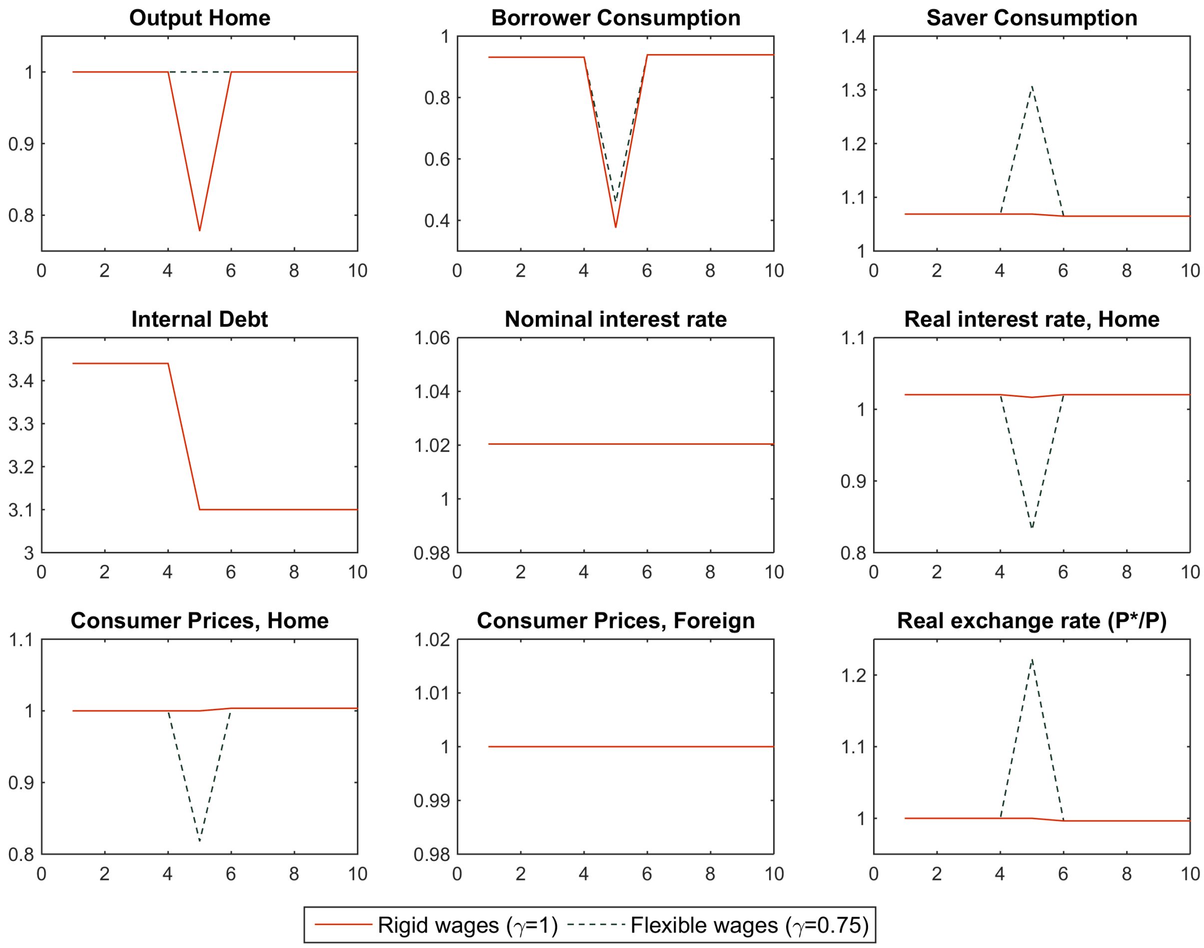

Figure 4: Deleveraging in a small member of a currency union (n → 0). Parameter values: $\bar{L} = 1, \bar{B}^H = 3.44, \bar{B}^L = 3.1, \chi = 0.5, \omega = 0.2, \beta^s = 0.98, \beta^b = 0.97$.

Martin and Philippon (2014) also study a deleveraging shock within a small member country of a currency union. They find their model to perform well in accounting for the dynamics of nominal GDP, employment and net exports in a number of euro area countries during the period 2000–2012. Through counterfactual simulations they explore the role of macroprudential and fiscal policy in influencing macroeconomic outcomes. Their analysis does not consider movements in relative prices, and in particular real exchange rates, which is the main focus of our paper. Furthermore, focussing on shocks to a small union member—as we do in this section—implies that a deleveraging shock does not generate spillovers on the rest of the union. As we show next, allowing for such spillovers has important implications for the area-wide response to the shock and, hence, for the adjustment of the real exchange rate.

4.2 Deleveraging in a (very) large union member

If $n \to 1$, the Home-good consumption weights are $\lambda \to 1$ and $\lambda^* \to \omega$ in Home and Foreign respectively. We obtain $Y_t = C_t$ from equation (3.18), $P_t = P_{H,t}$ from (3.9) and $D_t = 0$ from (3.21). Thus, Home accounts for almost the entire currency union and behaves like a closed economy. Foreign, in turn, effectively becomes a small open economy. Again, this has important implications for monetary policy: it is entirely geared towards developments in Home, as Foreign has a negligible effect on average union-wide inflation: $\Pi_t^u = P_t/P_{t-1}$ from (3.16).

As we now show, in this case the real exchange rate response hinges critically on the size of the shock, and the related monetary policy response. We establish that for a large shock, monetary policy becomes constrained by the zero lower bound (3.17) and, as a result, the real exchange rate response becomes muted, provided wages in Home are not much more flexible than in Foreign. Moreover, under certain conditions the real exchange rate may in fact appreciate rather than depreciate, reversing the usual dynamics.

Proposition 3. *Consider the economy defined in Section 3 and let $n \to 1$. Suppose that in period t, the debt limit in Home is unexpectedly and permanently reduced from $\bar{B}^H$ to $\bar{B}^L$. The real exchange rate response at time t depends on whether this shock is large enough to push the union to the zero lower bound.*

(a) If the deleveraging shock is small, $\beta^s \bar{B}^H - \bar{B}^L < \underline{\zeta}$, monetary policy is unconstrained by the zero lower bound and the real exchange rate depreciates. Formally, we have

$$Q_t \ (:= Q_t^{NoZLB}) = \left[(1-\omega)\left(1-\beta^s\left(1-\frac{(1-\chi)Y+\chi\bar{B}^H}{(1-\chi)Y+\chi\bar{B}^L}\right)\right)+\omega\right]^{1-\omega} > 1.$$

(b) If the deleveraging shock is large, $\beta^s \bar{B}^H - \bar{B}^L > \underline{\zeta}$, the zero lower bound binds in the period of deleveraging. If wages in Home are not much more flexible than in Foreign, $\gamma^*/\gamma < 1+\kappa$, where $\kappa > 0$, then the following inequality holds

$$Q_t \ (:= Q_t^{ZLB}) = \max\left(\left[\frac{\gamma^*}{\gamma}\right]^{1-\omega}, \left[1-\omega+(1-(1-\omega)\beta^s)\,\frac{Y_t}{Y}\right]^{1-\omega}\right) < Q_t^{NoZLB}.$$

That is, the zero lower bound generally dampens the real depreciation. Moreover, for a sufficiently large shock, $\beta^s \bar{B}^H - \bar{B}^L > \bar{\zeta} \geq \underline{\zeta}$, the second part in $\max(\cdot,\cdot)$ above is below one such that the real exchange appreciates ($Q_t^{ZLB} < 1$), provided wages in Home are less flexible than in Foreign, $\gamma^*/\gamma < 1$.

Proof. See Appendix. □

The critical values $\underline{\zeta}$, $\bar{\zeta}$ and κ are provided in the appendix along with the proof of Proposition 3. The solution for Y_t is given in equation (4.4) below; note that $Y_t < Y$, the full-employment output level. The first part of the proposition establishes that country size *per se* does not alter the sign of the real exchange rate response. Intuitively, if monetary policy is not constrained in pursuing the inflation target, it reduces the nominal interest rate sufficiently in response to the deleveraging shock. Average inflation in the currency union remains at zero, because Home inflation is zero ($n \to 1$). Lower interest rates, in turn, raise consumption in Foreign. In the presence of home bias, this pushes up the price level in Foreign. Thus the Home real exchange rate depreciates.

The second part of the proposition shows that if Home is large, the real exchange rate response is generally hampered whenever monetary policy becomes constrained by the zero lower bound. In this case, monetary policy is unable to stabilise Home prices, as this would require pushing nominal interest rates into negative territory. As Home demand collapses, nominal wages (and as a result: prices from (3.12)) decline up to the floor set by downward rigidity, parametrised by γ. This has implications for Foreign, too. As before, Foreign consumption will tend to increase to the extent that monetary policy reduces interest rates. However, there is now a second effect: the demand for Foreign-produced goods falls with Home consumption. This exerts downward pressure on the Foreign price level: there are deflationary spillovers which dampen the real exchange rate depreciation. In fact, if the shock is large enough ($\beta^s \bar{B}^H - \bar{B}^L > \bar{\zeta}$) and if Foreign wages are more flexible than Home wages, the Foreign price level may decline more strongly than the Home price level: the Home real exchange rate appreciates.[14]

This finding qualifies a result obtained by Cook and Devereux (2014). They use a two-country model to contrast the effect of a negative demand shock under flexible exchange rates with that under a common currency. In case the zero lower bound binds, they find the Home real exchange rate to appreciate, but only if the nominal exchange rate is flexible. Under a common currency, instead, there is no such "perverse adjustment" of the real exchange rate. As we allow for differential degrees of stickiness in the two countries, we find that, for a large enough shock, the real exchange rate response may in fact also reverse its usual pattern under a common currency.

Having established that real exchange rate movements are dampened at the zero lower bound, we now turn to the role of wage rigidity in the adjustment process. It turns out that at the zero lower bound, increasing wage flexibility is actually destabilising, in line with the findings

[14] Thus a sufficient condition for the real exchange rate depreciation to be dampened for an intermediate-sized shock, and even to be reversed for a larger shock, is given by $\gamma^* < \gamma$—that is, Foreign wages are more flexible than Home wages.

of Eggertsson and Krugman (2012) for the closed economy. We summarise this result in the following Proposition.

Proposition 4. *Paradox of flexibility. Consider the economy defined in Section 3 and let* $n \to 1$*. Assume the zero lower bound is binding in the period of deleveraging* ($\beta^s \bar{B}^H - \bar{B}^L > \underline{\zeta}$)*. In this case, if wages become more flexible domestically (that is, as* γ *is reduced)*

(a) Output, borrower consumption and real wage income all decline strictly more.

(b) The real exchange rate depreciates strictly less (or, if $\beta^s \bar{B}^H - \bar{B}^L > \bar{\zeta} \geq \underline{\zeta}$*, appreciates strictly more), provided the wage rigidity condition* (3.14) *is not binding in Foreign.*

Proof. See Appendix. □

The reason why more flexibility is harmful is simple: debt deflation. To see this, we again rearrange the borrower budget constraint (3.4) to obtain

$$C_t^b = -\frac{\bar{B}^H - \bar{B}^L}{P_t} + \frac{W_t L_t}{P_t}, \tag{4.3}$$

where we have used that $R_t = 1$ in the period of deleveraging. As before, the shock exerts downward pressure on domestic prices, making real debts harder to repay. And again, the general equilibrium response of real wage income is crucial for how the borrower responds to the shock.

As $n \to 1$, real wage income corresponds to domestic output, $W_t L_t / P_t = Y_t$, which follows from combining (3.11) with (3.12), and using that $P_t = P_{H,t}$ (see above). In turn, domestic output in the period of deleveraging solves[15]

$$Y_t = (\beta^s)^{-1} \left[Y - \frac{\chi}{1-\chi} \left(\frac{\beta^s \bar{B}^H - \bar{B}^L}{\gamma} \right) \right] < Y. \tag{4.4}$$

The second part of this expression is negative, and more so, the more flexible the domestic wages (the lower the γ). That is, in contrast to the case of $n \to 0$ analysed earlier, economic activity (and therefore real wage incomes) now decline with more flexible wages.

The intuition here is that while lower prices still increase the real value of debts, they no longer stimulate economic activity by crowding in foreign demand or by lowering the real interest rate. For foreign demand, at the zero lower bound deflationary spillovers imply that the real exchange rate response is dampened (Proposition 3).[16] Real interest rates could fall because of either a cut in nominal rates, or expected inflation. But the zero lower bound

[15] See the appendix, the proof of Proposition 4.

[16] Incidentally, in the case of $n = 1$, foreign demand would not be crowded in even if the real exchange rate depreciated substantially, given Foreign's negligible size. However, as we turn to $n < 1$ next, we believe the deflationary spillovers are the more relevant intuition.

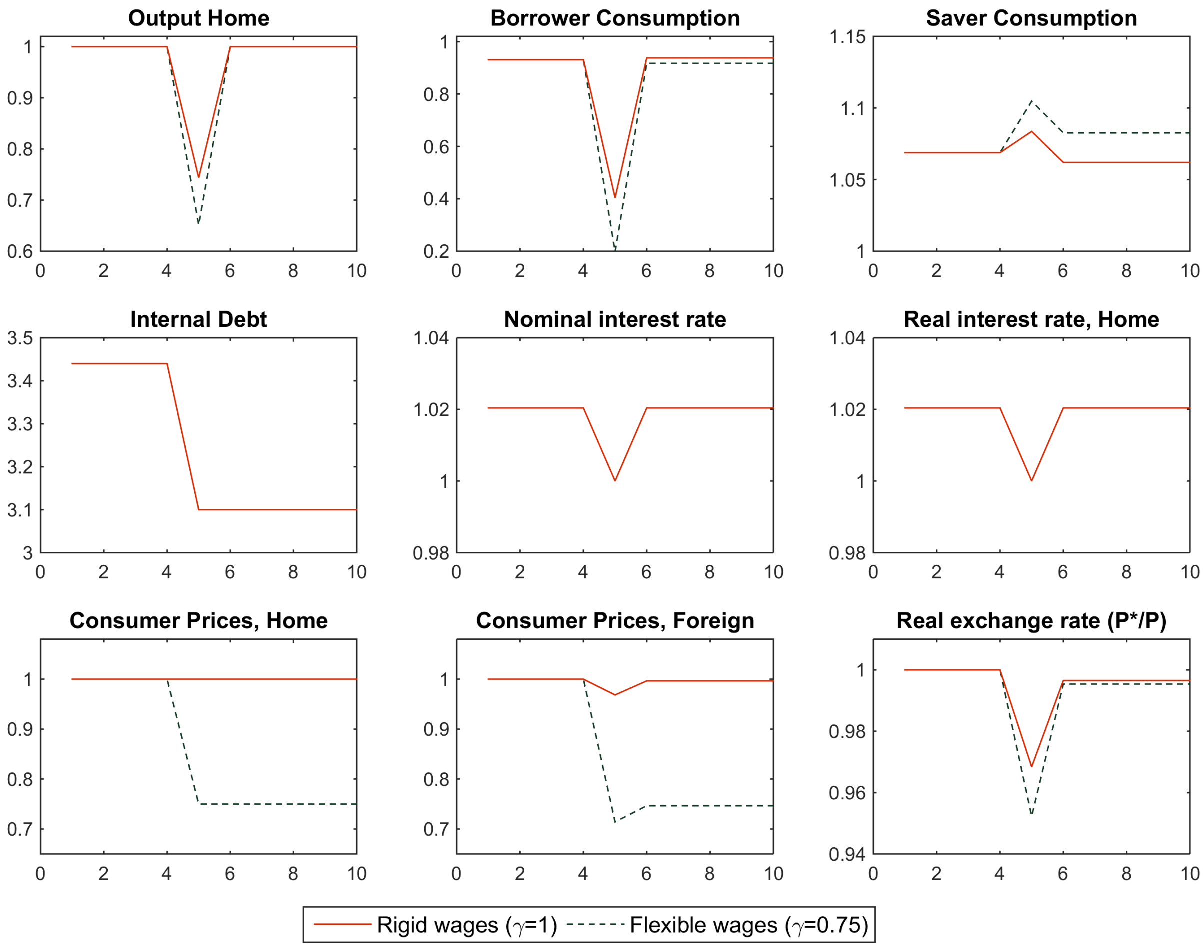

Figure 5: Deleveraging in a large union member (n $\to$ 1). Parameter values: $\bar{L} = 1, \bar{B}^H = 3.44, \bar{B}^L = 3.1, \chi = 0.5, \omega = 0.2, \beta^s = 0.98, \beta^b = 0.97,\ \gamma^* = 0.7$.

implies that nominal interest rates are stuck at zero, and under inflation targeting a drop in prices does not generate expected inflation.

As pointed out in Proposition 3, for a large enough shock the fall in Home consumption creates large enough spillovers such that Foreign prices fall by more than Home prices, to the extent that the former are more flexible than the latter. Part a) of Proposition 4, in turn, states that these spillovers can only *increase* as domestic rigidities are reduced. Thus, somewhat paradoxically, more flexible wages in Home may lead to an even larger appreciation of its real exchange rate (part b) of Proposition 4).[17] Figure 5 shows the full adjustment dynamics for precisely such a case, by comparing fully rigid ($\gamma = 1$, solid lines) and more flexible wages

[17] Note that $\gamma^* < \gamma$—the condition for the real exchange to appreciate if the shock is large (part b) of Proposition 3)—is implied by the wage rigidity condition (3.14) not binding in Foreign, as assumed in part b) of Proposition 4. See the appendix, the proofs of the two propositions for further details.

($\gamma = 0.75$, dashed lines).

From a conceptual point of view, we also note that there is a critical level of wage flexibility beyond which the model has no solution—or equilibrium—because the real value of debt repayments exceeds the level of output under full employment, which would imply $Y_t < 0$ in equation (4.4). Moreover, there is no solution even as wages become fully flexible. Intuitively, for a large deleveraging shock, the flexible-wage equilibrium requires a real interest rate below unity to induce savers to consume all of the repayment made by the borrowers. At the zero lower bound, this requires expected inflation—which is inconsistent with the central bank's inflation targeting mandate.[18] A flexible-wage equilibrium would exist, however, under an alternative monetary policy rule which generates the required expected inflation, such as a price level targeting rule. Moreover, such an alternative monetary policy rule could maintain full employment even under partially rigid wages, avoiding both the paradox of flexibility and the deleveraging recession itself.

The above insight relates to a point made by Cochrane (2015), who maintains that in New Keynesian models equilibria involving deep recessions and paradoxes of flexibility at the ZLB are the result of monetary policy rules containing an element of equilibrium selection. In our model, we account for alternative monetary policy rules as we vary the country size n. When $n \to 0$, monetary policy in Home corresponds to a simple exchange rate peg. As discussed in Corsetti et al. (2013), such a peg provides an implicit price level commitment—in our case, to prices in Foreign—which is stabilising because it generates expected inflation. As n increases, a Foreign country with more rigid wages can still provide such a commitment mechanism, albeit to a lesser extent. Instead, for $n \to 1$ the price level commitment disappears and we are left with a simple inflation targeting rule.

In the next section we turn to analyse the numerical solution of our model for an empirically relevant, intermediate value of n. Generally speaking though, the insights gained from looking at the limiting cases $n \to 0$ and $n \to 1$ carry over to our quantitative analysis. Most importantly, as long as the deleveraging shock is large enough to push the union to the zero lower bound, deflationary spillovers will generally mute the adjustment of relative prices, and may sometimes even reverse them. This lack of relative price adjustment, and the lack of movement in the real interest rate, imply that Home could enter a deep recession even if wages are quite flexible.

[18] We show this formally in note which is available on request.

5 Quantitative analysis

The analytical results established in the previous section show that country size matters for the adjustment dynamics to a region-specific deleveraging process. In 2009, credit growth stalled in most countries of the euro area. However only the stressed economies of the euro area experienced a full-fledged deleveraging process in the years thereafter (see Section 2). In the following we perform a quantitative assessment of our model and explore to what extent it can account for key features of the post-crisis slump in the euro area. Precisely, we examine the non-linear impulse response to a deleveraging shock in two settings. First, we consider the baseline model introduced in Section 3, where deleveraging takes place over one period. Second, we use a modified version of the model, where borrowers are allowed to deleverage gradually and choose the optimal path of deleveraging over a number of periods.

5.1 Baseline model

We assign parameter values in order to solve the model numerically. The specific values are summarised in Table 1. The parameters which govern the size of the stressed economy and the size of the shock are determined by the following observations. First, we set $n = 0.37$ to match the share of stressed countries in euro-area GDP on the eve of the crisis. Second, we observe that the level of total private sector debt in the stressed economy declines from 172% of GDP to 155% of GDP, that is, by 17 percentage points of GDP. The debt limit in our model, $\bar{B}_t$, is debt *per borrower*, which means that we have to scale up the economy-wide debt-to-GDP values by a factor $1/\chi$, in our case 2. This gives us deleveraging from 344% GDP per borrower ($\bar{B}^H = 3.44$) to 310% GDP per borrower ($\bar{B}^L = 3.1$), a total of 34% GDP.[19] We use the borrower share parameter χ from Martin and Philippon (2014), which captures the share of liquidity-constrained households in the data from Eurosystem Household Finance and Consumption Survey (HFCS).

The other parameters are standard. Assuming a discount factor β^s of 0.98 for the patient households implies an annual real interest rate of 2% in steady state. We set the home-bias parameter $\omega = 0.2$. Given $n = 0.37$ this implies an import share of 0.12, the average GDP weight of imports from the rest of the euro area in the stressed economies.[20] Finally, we vary the parameter γ which captures downward wage rigidities. Specifically, we consider a range of values between 0.93 and 0.99, which is equivalent to maximum wage deflation of between 7% and 1% per year respectively. The high downward wage rigidity implied by $\gamma = 0.99$ is

[19] The upper limit is the peak value, observed in 2012Q2, whilst the trough value is the latest observation in our sample, 2014Q3. As before, in our model nominal GDP equals 1 in the pre-crisis steady state, and a deleveraging shock of 34% is equivalent to 34% of pre-crisis nominal GDP.

[20] Source: OECD, Monthly Foreign Trade Statistics, period: 1999–2006.

Parameter:	n	$\bar{B}^H$	$\bar{B}^L$	β^s	ω	χ	γ
Value:	0.37	3.44	3.1	0.98	0.2	0.5	0.93–0.99

Table 1: Parameters for quantitative analysis: baseline model

broadly in line with estimates for the euro area (Schmitt-Grohé and Uribe, 2015).

Figure 6 shows the response of this economy to the deleveraging shock, contrasting three scenarios. Solid lines represent our baseline, characterised by high wage rigidities ($\gamma = 0.99$) in Home. In addition we consider two counterfactuals. The lines with crosses correspond to a scenario where Home rigidities are reduced, but remain above Foreign ($\gamma = 0.96$ vs $\gamma^* = 0.95$). The dashed lines correspond to a case where rigidity in Home is significantly lower, to the extent that Home becomes more flexible than Foreign ($\gamma = 0.93$ vs $\gamma^* = 0.95$).

The deleveraging shock is displayed in the bottom-left panel. Under our baseline scenario, it pushes the economy into a deep but asymmetric recession: while Home output collapses, Foreign output remains unaffected, even though monetary policy becomes constrained by the zero lower bound (first row of Figure 6). The dynamics of consumption also differ. While borrowers reduce consumption to repay their debts, Home savers' and Foreign households' consumption increases (second row of Figure 6), reflecting a reduced real interest rate (bottom-right panel for the case of Home).

The third row shows the responses of variables of particular interest. For the baseline case we find a mild decline of the price level in Home, reflecting the presence of downward wage rigidities. At the same time, Foreign prices decline—if only by little—due to deflationary spillovers. As a result, the real exchange rate remains broadly unchanged in the baseline scenario.

Turning to the counterfactuals, consider first the case of somewhat increased wage flexibility ($\gamma = 0.95$, crossed lines) in Home. Despite the sharper drop in prices in Home, the real exchange rate movement is virtually identical to that under the baseline scenario of higher wage rigidity (third row). This is because more flexible wages increase debt deflation and deepen the recession in Home—an instance of the paradox of flexibility (see Proposition 4 for the $n \to 1$ case). Deflationary spillovers increase, such that prices in Foreign decline by more to stabilise output in the face of an export collapse.

Under our second counterfactual, wage rigidities in Home are relaxed yet further, to the point where wages are more downwardly flexible than in the rest of the union ($\gamma = 0.93$, $\gamma^* = 0.95$, dashed lines). Following the shock, both countries find themselves against the

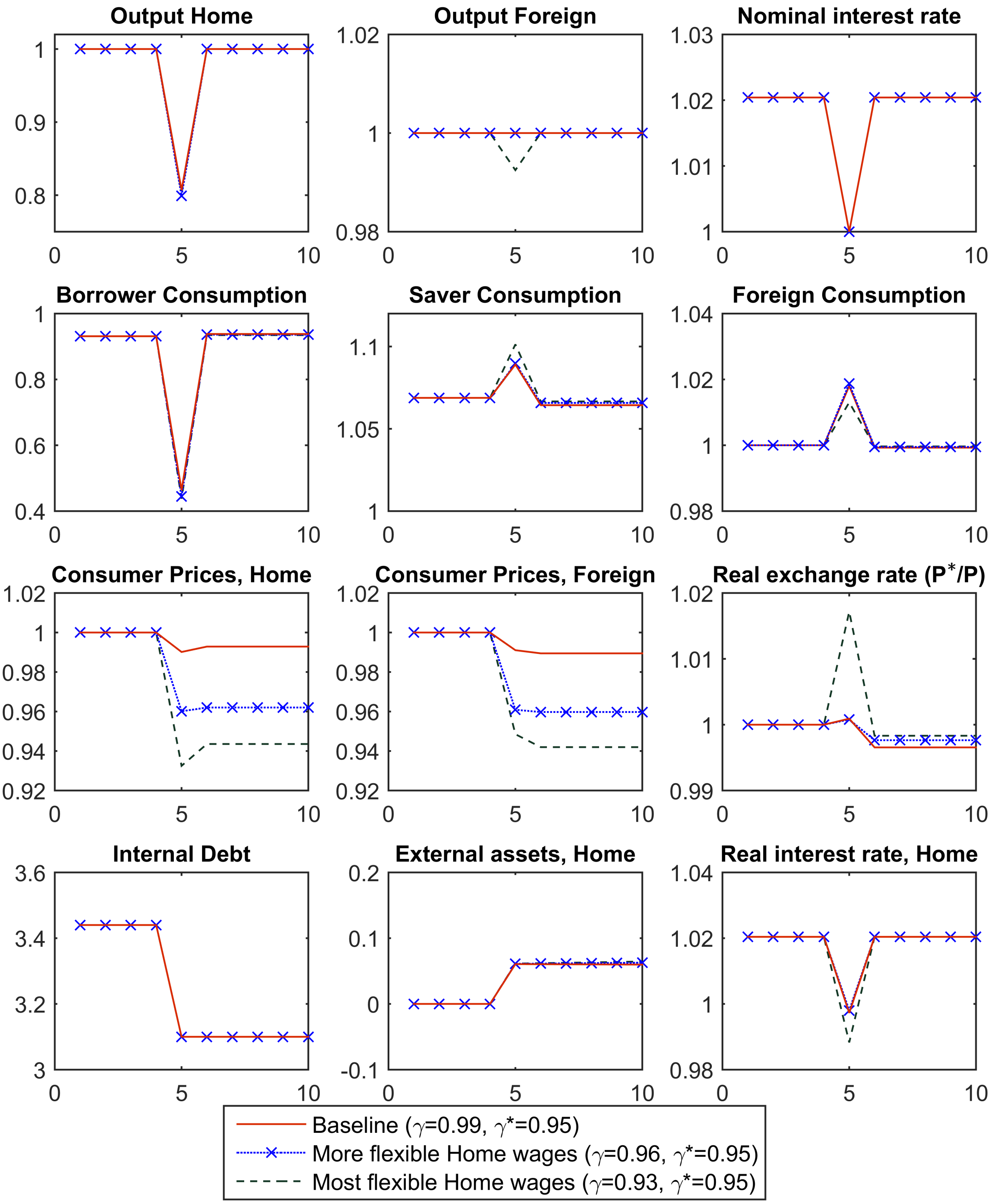

Figure 6: Deleveraging in a medium-sized union member, $n = 0.37$. Baseline model. Parameter values in Table 1. One period is a year.

wage rigidity constraint in (3.15), but because Home wages are more flexible, prices in Home fall by more (third row). The real exchange rate depreciates, more so than in the baseline scenario. Turning to output (top row), Foreign now enters a mild recession because its prices cannot fall by enough to fully offset the negative demand spillover from Home. Even though aggregate demand and Home output are boosted by the real depreciation and a lower real interest rate, the benefits of these are roughly offset by higher debt deflation, and Home output falls by roughly the same amount as in the baseline scenario.

On reflection, the results in this section resemble the $n \to 1$ case of Section 4.2 more than they do the $n \to 0$ case of Section 4.1. The deleveraging is associated with a large output drop in Home and deflationary spillovers to Foreign, which increase in size with higher wage flexibility. Unlike Home output and prices, the real exchange rate and Foreign output change little in all three of our simulated scenarios.

In the next section, we examine a fuller model which allows us to relate our quantitative results to the dynamics of the post-crisis slump in the euro area. To do this, we allow the deleveraging to take place gradually, and over a longer time period.

5.2 Dynamic Deleveraging

Euro-area deleveraging has been taking place for the best part of six years, and the pace of the deleveraging has been quite gradual. To account for this, we modify our model in one key way. Borrowers no longer face a fixed borrowing limit $\bar{B}_t$. Instead, when debts in the economy exceed some level perceived as "safe"—denoted $\bar{B}_{S,t}$—the savers (through financial intermediaries) start charging higher interest rates on borrowing. A deleveraging shock is then a reduction in this safe level of debt, which increases interest spreads and incentivises borrowers to reduce their indebtedness. In this regard we mimic the set-up of Benigno et al. (2014) who consider a closed economy.[21]

Formally, borrowers now maximise

$$\max_{\{C_t^b\}_{t=0}^{\infty}} \sum_{t=0}^{\infty} (\beta^b)^t \ln(C_t^b)$$

subject to

$$P_t C_t^b + B_{t-1}^b = (R_t^b)^{-1} B_t + W_t L_t, \tag{5.1}$$

[21] Our set-up differs only slightly from Benigno et al. (2014), in the following ways. First, we examine two open economies. Second, to improve tractability, the spread is a function of nominal, not real, debt. Last, we provide a full non-linear solution of the model under perfect foresight.

where R_t^b is the gross borrowing rate determined according to

$$R_t^b = R_t \left(B_t^{\mathrm{Av}} / \bar{B}_{S,t} \right)^{\phi} \tag{5.2}$$

$$R_t^b \geq R_t. \tag{5.3}$$

Note that the spread is determined by the average debt per borrower throughout the economy, rather than individual debt, even though in equilibrium $B_t^{\mathrm{Av}} = B_t^b$. Here, ϕ is the elasticity of the gross borrowing rate with respect to excessive debt. As argued in Benigno et al. (2014), one can interpret this set-up as capturing financial intermediation in a very stylised way: banks lend to borrowers at rate R_t^b and pay the savers R_t. The premium for excessive borrowing can be interpreted as a charge for default risk in the presence of asymmetric information, or as compensating for fraud. And the profits of these transactions are distributed to savers, who own the banks.[22]

Optimality requires that borrowers satisfy an Euler equation

$$\left(C_t^b\right)^{-1} = \beta^b R_t^b \left(C_{t+1}^b\right)^{-1} \frac{P_t}{P_{t+1}}. \tag{5.4}$$

Furthermore, as in Benigno et al. (2014), we let $\beta^b \to \beta^s = \beta$, such that in steady state $B^b = \bar{B}_S$ from combining (5.2) and (5.4), and so banks make zero profits.

Lastly, for our numerical analysis of the gradual deleveraging model, we assume that the central bank implements its inflation target via a Taylor-type rule of the form

$$R_t = (\beta^s)^{-1} (\Pi_t^u)^{\varphi^{\pi}} \text{ subject to } R_t \geq 1, \tag{5.5}$$

where we assume $\varphi^{\pi} > 1$.

We measure periods in quarters, and adjust the model parameters accordingly. All parameter values are listed in Table 2. Values for n, ω and χ are unchanged from before. The quarterly discount factor $\beta = 0.995$ is equivalent to a 2% annualised real interest rate in steady state. Debt limits of 344% and 310% annual GDP per borrower translate to 1376% and 1240% of quarterly GDP respectively. We set the Taylor rule parameter φ^{π} to the conventional value of 1.5. Wage rigidity occupies a range of values, allowing for maximum wage falls of between 4% and 8% per year.[23] Finally, we set the spread elasticity to 0.38 in order to target a zero-lower-bound episode which lasts 6 quarters.

[22] The savers' budget constraint is now given by $P_t C_t^s + R_t^{-1} B_t^s + R_t^{*-1} D_t = W_t L_t + B_{t-1}^s + D_{t-1} + \frac{\chi}{1-\chi}\left(R_t^b - R_t\right) B_t^b$, where the last term are banking profits distributed to savers in a lump-sum manner.

[23] Note that we examine a slightly narrower range than in Section 5.1, to preserve stability of the numerical solution. Because of dynamic feedback effects, large changes in wage flexibility can have dramatic effects on model outcomes: they trigger a deflationary spiral, such that a stable model solution does not exist.

Parameter:	n	$\bar{B}^H$	$\bar{B}^L$	β	ω	χ	γ	ϕ	φ^π
Value:	0.37	13.76	12.4	0.995	0.2	0.5	0.98–0.99	0.38	1.5

Table 2: Parameters for quantitative analysis: dynamic deleveraging model

We now turn to our quantitative analysis. We assume that at time t, the safe debt limit unexpectedly and permanently tightens from $\bar{B}^H$ to $\bar{B}^L$. From (5.2), this opens up a spread between borrowing and lending rates, since the level of debt becomes judged as "excessive": $\bar{B}^H > \bar{B}^L = \bar{B}_{S,t}$, triggering a deleveraging from equation (5.4), until the new safe debt level $\bar{B}^L$ is reached in the long run. The dynamic response to the shock is depicted in Figure 7. We compare the response in a baseline scenario of rigid wages in stressed countries (red solid lines) to a counterfactual scenario of more flexible wages (black dashed lines). Throughout this experiment, we assume that wages in Foreign are flexible enough to support full employment. From the top row, we can see that the deleveraging shock generates a recession and pushes the union to the zero lower bound. Looking at the mechanism in more detail, the second row shows the associated increase in spreads which triggers a slow and gradual debt reduction towards the new, lower, safe level. The debt repayment acts as a long-lasting drag on borrower consumption (third row, left), which lowers aggregate demand and generates deflationary pressures. In the early stages of the adjustment, the central bank is unable to cut interest rates sufficiently in order to offset these pressures, and the economy enters a recession.

Turning to the focus of our experiment, the bottom row depicts the movements in relative prices. In every period of the recession, Home wages fall by the maximum amount permitted by the rigidity constraint, which leads to a fall in Home prices. Under the baseline scenario, Foreign prices fall slightly due to the deflationary spillovers, dampening the movement in the real exchange rate. Prices in both countries fall by around 1% in the first quarter such that there is no real depreciation. Thereafter prices in Home fall by around 5.5% and in Foreign—by 3%, so just over half of the price fall in Home is dampened by the deflationary spillover to Foreign, and the resulting real depreciation is rather modest in size.

The absolute price movements under the counterfactual scenario of more flexible wages (bottom row, dashed lines) are much more pronounced. However this does not mean that *relative* prices adjust by more—in fact, they adjust by less and even move in the "wrong" direction. This is because the sharp fall in Home prices triggers a destabilising deflationary spiral and increases the negative demand spillover to Foreign, and with it the pressure on Foreign prices to drop *relative* to Home prices. Debt deflation in Home forces borrowers to reduce con-

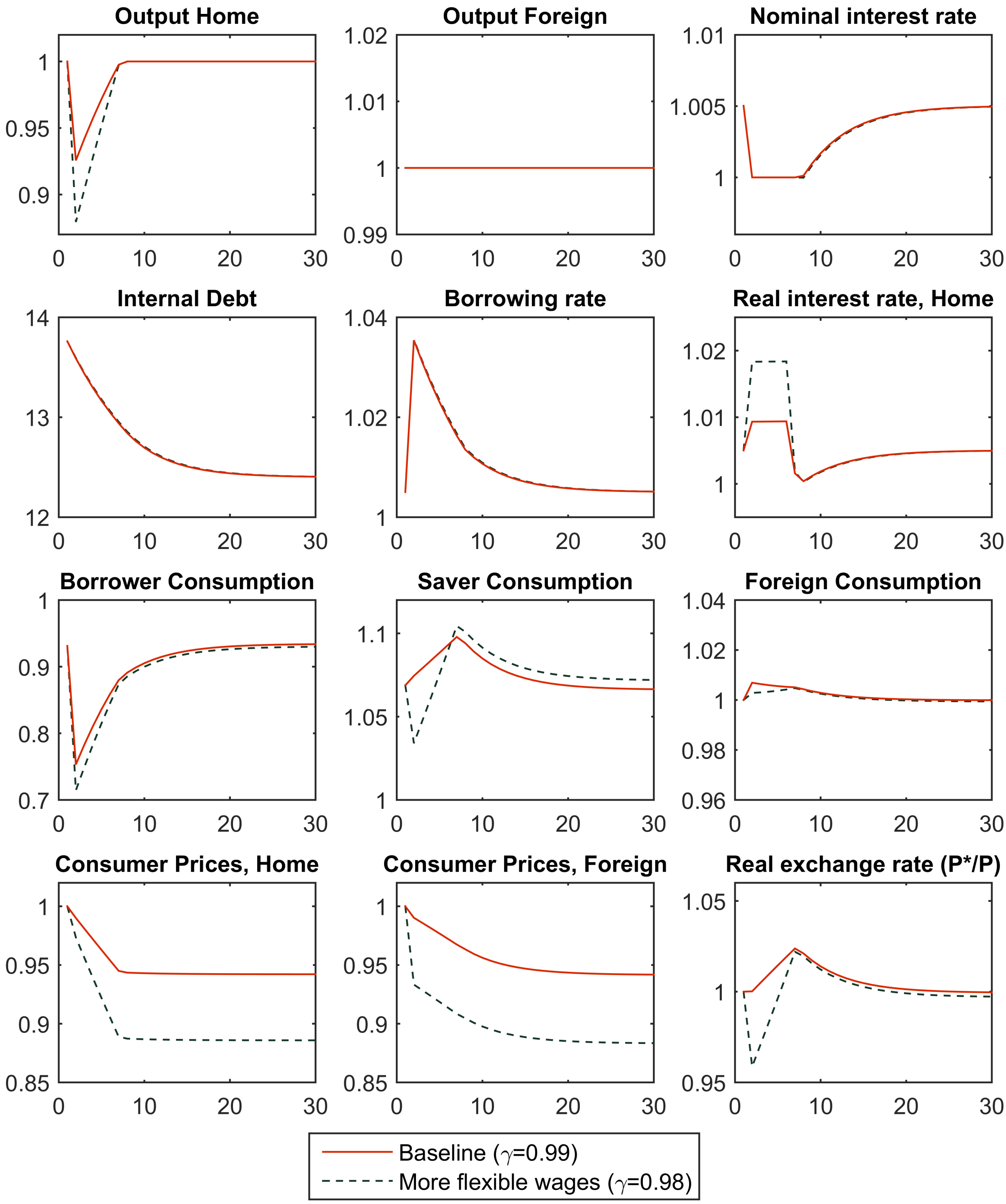

Figure 7: Dynamic deleveraging in a medium-sized union member, $n = 0.37$. Parameter values in Table 2. One period is a quarter. Interest rates shown on a quarterly basis.

sumption further (third row, left). At the same time, expected deflation raises real interest rates (second row, right), such that even savers cut back on consumption (third row, middle). This in turn reduces real incomes throughout the Home economy and deepens the recession. The resulting fall in demand for Foreign exports pushes down Foreign prices to the extent that the real exchange rate appreciates sharply at first, further depressing real incomes and exacerbating the deleveraging in Home. The paradox of flexibility is in full force, and the recession is deeper than under the baseline scenario of more rigid wages.

The discussion above supports the conclusions of our analytical model in Section 4 and the one-period deleveraging experiment in Section 5. Further, the impact of higher wage flexibility—and the associated paradoxes—seem to be much more pertinent in the dynamic setting. More precisely, the differences in output and real exchange rate movements between the baseline and flexible-wage counterfactuals are much larger. Whilst under one-period deleveraging higher wage flexibility was not helpful but largely harmless, under dynamic deleveraging it can be highly destabilising. The reason for this is twofold. First, because deleveraging is more gradual, the adverse effects of debt deflation can persist for longer. Second, because some of the deflation is anticipated, it raises real interest rates and reduces consumption and demand further.

Even though the model in this section remains fairly stylised, the impulse responses for our baseline scenario match a number of facts which characterise the post-crisis slump in the euro area (Section 2). Just like the stressed euro-area economies, there is a deep recession in the Home country, triggered by a long and gradual deleveraging process. The recession persists for several periods and pushes the union to the zero lower bound. Prices in Home and Foreign move in a synchronised manner, which means that the real depreciation is small, around 2.4% at its peak. This is still somewhat larger than the "raw" 0.6% real depreciation we observe for the stressed euro countries in the data, but broadly in line with the 2% depreciation calculated using tax-adjusted price data.[24] The fall in the Foreign price level helps the rest of the union remain at full employment, so whilst prices move together, the output performance of the two country groups diverges. Our counterfactual simulation suggests that had wages in stressed economies been more flexible, we would have still seen little relative price adjustment within the euro area. Indeed, the real exchange rate may have even appreciated, deepening the recession.

[24] Figures quoted are for 2008Q4—2014Q4.

6 Conclusion

Why—given the heterogenous economic performance across the euro area after 2009—has there been no significant adjustment of intra-euro-area real exchange rates? In this paper, we address the question by setting up a model of a currency union in which one of the two member countries experiences a large deleveraging shock. A key feature of the model is that the size of countries may differ. This allows us to study polar cases for which we are able to solve the model in closed form. In doing so we clarify aspects of the transmission mechanism, which also turn out to be relevant in the quantitative assessment of the model.

In case the domestic economy is small, there are no spillovers from the deleveraging shock into the rest of the union. Also, monetary policy remains unchanged. In this case, in line with much of the received wisdom, the extent of real deprecation is inversely related to the degree of wage rigidities. In the context of our model, this is simply the result of prices fluctuating one-for-one with wages. More flexibility in wages not only makes the real exchange rate more flexible, it also makes the economy more resilient in the face of adverse country-specific shocks. This result is notable because more wage flexibility also leads to more debt deflation in response to the deleveraging shock. Still, this effect is not strong enough to offset the benefits from increased flexibility for a small member country of a currency union.

A different picture emerges once we consider a large currency union member. Monetary policy lowers interest rates in response to the shock, and reaches the zero lower bound if the shock is large enough. The output response in the domestic economy and in the rest of the union will generally be different, in particular if there is sufficient wage flexibility in the rest of the union: while domestic output collapses, it may stay at its initial level in the other countries. At the same time there is deflationary pressure across the entire union, and the response of the real exchange rate to the deleveraging shock is dampened if the zero lower bound binds. In fact, the real exchange rate may even appreciate if the shock is large and/or wages are relatively flexible in the rest of the union. More wage flexibility in the domestic economy can also be destabilising and induce an even stronger real appreciation—an instance of the paradox of flexibility, established by Eggertsson and Krugman (2012) in a closed-economy context.

In our quantitative assessment we assume that the domestic economy accounts for 37 percent of the currency union, corresponding to the GDP weight of the stressed economies prior to the crisis. In response to a deleveraging shock of 34 percent GDP, we find the model able to account for some of the basic facts of the post-crisis slump in the euro area: there is large divergence in terms of output performance, there is deflationary pressure in the entire union, and the real exchange rate response is muted.

We also find that the paradox of flexibility reigns in our calibrated model. In a counterfactual scenario where wages are more flexible, the real exchange rate appreciates and the output loss is even larger than in the baseline case. Hence, we think that our analysis has some relevance for the ongoing policy debate in the euro area. Namely, similarly to Eggertsson et al. (2014), we find that measures geared towards raising the flexibility of the economies hit hardest by the crisis are likely to be ineffective—unless they are accompanied by expansionary monetary policy measures which may make up for the zero-lower-bound constraint on interest rates.

References

Benigno, P., Eggertsson, G. B., and Romei, F. (2014). Dynamic Debt Deleveraging and Optimal Monetary Policy. NBER Working Papers 20556, National Bureau of Economic Research, Inc.

Benigno, P. and Romei, F. (2014). Debt deleveraging and the exchange rate. *Journal of International Economics*, 93(1):1–16.

Berka, M., Devereux, M. B., and Engel, C. (2015). Real Exchange Rates and Sectoral Productivity in the Eurozone. Working Papers 26970, Department of Economics, The University of Auckland.

Broda, C. (2004). Terms of trade and exchange rate regimes in developing countries. *Journal of International Economics*, 63(1):31–58.

Cochrane, J. H. (2015). The New-Keynesian Liquidity Trap. mimeo.

Cook, D. and Devereux, M. B. (2013). Sharing the burden: Monetary and fiscal responses to a world liquidity trap. *American Economic Journal: Macroeconomics*, 5(3):190–228.

Cook, D. and Devereux, M. B. (2014). Exchange rate flexibility under the zero lower bound. Globalization and Monetary Policy Institute Working Paper 198, Federal Reserve Bank of Dallas.

Corsetti, G., Kuester, K., and Müller, G. J. (2013). Floats, pegs and the transmission of fiscal policy. In Céspedes, L. F. and Galí, J., editors, *Fiscal Policy and Macroeconomic Performance*, volume 17 of *Central Banking, Analysis, and Economic Policies*, chapter 7, pages 235–281. Central Bank of Chile.

De Paoli, B. (2009). Monetary policy and welfare in a small open economy. *Journal of International Economics*, 77(1):11–22.

Eggertsson, G., Ferrero, A., and Raffo, A. (2014). Can structural reforms help Europe? *Journal of Monetary Economics*, 61(C):2–22.

Eggertsson, G. B. and Krugman, P. (2012). Debt, Deleveraging, and the Liquidity Trap: A Fisher-Minsky-Koo Approach. *The Quarterly Journal of Economics*, 127(3):1469–1513.

Engel, C. (1999). Accounting for U.S. Real Exchange Rate Changes. *Journal of Political Economy*, 107(3):507–538.

Fisher, I. (1933). The debt-deflation theory of great depressions. *Econometrica*.

Fornaro, L. (2015). International Debt Deleveraging. CEPR Discussion Papers 10469, C.E.P.R. Discussion Papers.

Friedman, M. (1953). *Essays in Positive Economics*, chapter The case for flexible exchange rates.

Galí, J. and Monacelli, T. (2005). Monetary policy and exchange rate volatility in a small open economy. *Review of Economic Studies*, 72:707–734.

Galí, J. and Monacelli, T. (2013). Understanding the gains from wage flexibility: The exchange rate connection. mimeo.

Gilchrist, S., Schoenle, R., Sim, J., and Zakrajsek, E. (2015). Financial Heterogeneity and Monetary Union. mimeo.

Kollmann, R. (2001). The exchange rate in a dynamic-optimizing business cycle model with nominal rigidities: a quantitative investigation. *Journal of International Economics*, 55(2):243–262.

Kollmann, R., Enders, Z., and Müller, G. J. (2011). Global banking and international business cycles. *European Economic Review*, 55(3):407–426.

Martin, P. and Philippon, T. (2014). Inspecting the Mechanism: Leverage and the Great Recession in the Eurozone. NBER Working Papers 20572, National Bureau of Economic Research, Inc.

Monacelli, T. (2004). Into the Mussa puzzle: monetary policy regimes and the real exchange rate in a small open economy. *Journal of International Economics*, 62(1):191–217.

Reinhart, C. M. and Rogoff, K. S. (2014). Recovery from Financial Crises: Evidence from 100 Episodes. *American Economic Review*, 104(5):50–55.

Schmitt-Grohé, S. and Uribe, M. (2015). Downward Nominal Wage Rigidity, Currency Pegs, and Involuntary Unemployment. Journal of Political Economy, forthcoming.

Sutherland, A. (2005). Incomplete pass-through and the welfare effects of exchange rate variability. *Journal of International Economics*, 65(2):375–399.

Appendix

This Appendix presents proofs of Propositions 1—4.

A1 Proposition 1 proof

For $n \to 0$, we can reduce the system of equations (3.1)—(3.21) to the following 7 equations:

$$P_t C_t^s = P_{t+1} C_{t+1}^s \tag{A1.1}$$

$$P_t C_t^b = \beta^s \bar{B}_t - \bar{B}_{t-1} + P_{H,t} Y_t \tag{A1.2}$$

$$C_t = (1-\chi) C_t^s + \chi C_t^b \tag{A1.3}$$

$$(1-\chi)(\beta^s D_t - D_{t-1}) = P_{H,t} Y_t - P_t C_t \tag{A1.4}$$

$$P_t = P_{H,t}^{1-\omega} \tag{A1.5}$$

$$P_{H,t} Y_t = (1-\omega) P_t C_t + \omega Y \tag{A1.6}$$

$$0 = (Y_t - Y)(P_{H,t} - \gamma P_{H,t-1}), \tag{A1.7}$$

as well as two inequalities:

$$Y_t \leq Y \tag{A1.8}$$

$$P_{H,t} \geq \gamma P_{H,t-1} \tag{A1.9}$$

The first equation is derived from the savers' Euler equation (3.2), the second—from the borrowers' budget constraint (3.4) (combined with (3.11), (3.12)), and the third—from aggregate consumption (3.6). Equation (A1.4) is the country's budget constraint, obtained by combining the saver's and borrower's budget constraints (3.1) and (3.4), and substituting for nominal incomes from (3.11) and (3.12). The remaining equations are the price index (from 3.9), Home market clearing (from 3.18), and the complementary slackness condition (from 3.15). The inequality in (A1.9) is obtained by combining (3.14) and (3.12). Additionally, we have used that $R_t = (\beta^s)^{-1}$, $P_t^* = P_{F,t} = 1$, and $C_t^* = Y_t^* = Y$.

The perfect foresight solution is a sequence of endogenous variables $\{C_t^s, C_t^b, C_t, Y_t, P_t, P_{H,t}, D_t\}$ that solves equations (A1.1)—(A1.9), given the initial conditions (stated in Section 3.5), and an exogenous path for $\{\bar{B}_t\}$, known in the initial period.

We can solve for real and nominal variables separately. The system in nominal variables can be reduced further to three equations

$$P_t C_t^s = P_{t+1} C_{t+1}^s \tag{A1.10}$$

$$(1-\chi)(\beta^s D_t - D_{t-1}) = (1-\chi) P_{H,t} Y_t - (1-\chi) P_t C_t^s - \chi(\beta^s \bar{B}_t - \bar{B}_{t-1}) \tag{A1.11}$$

$$(1-(1-\omega)\chi) P_{H,t} Y_t = (1-\omega)(1-\chi) P_t C_t^s + (1-\omega)\chi(\beta^s \bar{B}_t - \bar{B}_{t-1}) + \omega Y \tag{A1.12}$$

in three unknowns $(P_t C_t^s), (P_{H,t} Y_t)$ and D_t.
We solve this system forward analytically. The full solution is presented in the proof of Proposition 2. Here, we only present the solution for Home nominal income $(P_{H,t} Y_t)$:

$$P_{H,t} Y_t = Y + (1-\beta^s)\frac{(1-\chi)(1-\omega)}{\omega} D_{t-1} - \eta(\bar{B}_{t-1} - \bar{B}_t),$$

where $\eta = (\beta^s(1-\omega)\chi)/(1-(1-\omega)\chi) > 0$.
In the period of deleveraging, $D_{t-1} = 0$, $\bar{B}_{t-1} = \bar{B}^H$ and $\bar{B}_t = \bar{B}^L$, which yields

$$P_{H,t} Y_t = Y - \eta(\bar{B}^H - \bar{B}^L), \tag{A1.13}$$

Under full employment, $Y_t = Y$, and hence

$$P_{H,t} = 1 - \eta(\bar{B}^H - \bar{B}^L)/Y. \tag{A1.14}$$

If there is unemployment, $Y_t < Y$ and from (A1.7)

$$P_{H,t} = \gamma P_{H,t-1} = \gamma$$

From (A1.9),

$$P_{H,t} = \max(\gamma, 1 - \eta(\bar{B}^H - \bar{B}^L)/Y \le 1. \tag{A1.15}$$

The inequality holds because $\gamma \le 1$.
From (3.10) and (A1.5), $Q_t = P_{H,t}^{\omega-1}$, where $\omega - 1 < 0$. Combining this with (A1.15) gives us the real exchange rate formula in Propostion 1:

$$Q_t = P_t^*/P_t = \min\left(\gamma^{\omega-1}, [1 - \eta(\bar{B}^H - \bar{B}^L)/Y]^{\omega-1}\right) \ge 1.$$

A2 Proposition 2 proof

We first present the full solution to the system of equations (A1.10)—(A1.12):

$$\begin{aligned}
P_t C_t^s &= Y + (1-\beta^s)\left(\frac{\chi}{1-\chi}\bar{B}_{t-1} + \frac{1-(1-\omega)\chi}{\omega} D_{t-1}\right) \\
P_t C_t^b &= Y - (1-\beta^s)\left(\bar{B}_{t-1} - \frac{(1-\chi)(1-\omega)}{\omega} D_{t-1}\right) - \frac{\beta^s}{1-(1-\omega)\chi}(\bar{B}_{t-1} - \bar{B}_t) \\
P_{H,t} Y_t &= Y + (1-\beta^s)\frac{(1-\chi)(1-\omega)}{\omega} D_{t-1} - \frac{\beta^s(1-\omega)\chi}{1-(1-\omega)\chi}(\bar{B}_{t-1} - \bar{B}_t) \\
D_t &= D_{t-1} + \frac{\omega\chi}{(1-(1-\omega)\chi)(1-\chi)}(\bar{B}_{t-1} - \bar{B}_t).
\end{aligned}$$

In the period of deleveraging, $D_{t-1} = 0$, $\bar{B}_{t-1} = \bar{B}^H$ and $\bar{B}_t = \bar{B}^L$. Substituting for this gives the following expressions for nominal spending and incomes:

$$\begin{aligned} P_t C_t^s &= Y + (1-\beta^s)\frac{\chi}{1-\chi}\bar{B}^H \\ P_t C_t^b &= Y - (1-\beta^s)\bar{B}^H - \frac{\eta}{(1-\omega)\chi}(\bar{B}^H - \bar{B}^L) \\ P_{H,t} Y_t &= Y - \eta(\bar{B}^H - \bar{B}^L), \end{aligned}$$

where $\eta = (\beta^s(1-\omega)\chi)/(1-(1-\omega)\chi) > 0$, as above.

We can immediately see that nominal spending and incomes are independent of wage flexibility γ. Savers' nominal spending is independent of the deleveraging shock, whilst borrowers' nominal spending and incomes fall proportionately with the deleveraging shock $\bar{B}^H - \bar{B}^L$. Because of this, the smaller the nominal adjustment in prices, the larger the adjustment of real spending and incomes will be, which gives us some intuition for the benefits of higher wage flexibility. We establish this more formally below.

Suppose first that the wage rigidity condition (A1.9) is binding. Then $P_{H,t} = \gamma$ and $P_t = \gamma^{1-\omega}$ from (A1.5). Real spending and output, as well as real incomes $W_t L_t / P_t$, are given below

$$C_t^s = \gamma^{\omega-1}\left[Y + (1-\beta^s)\frac{\chi}{1-\chi}\bar{B}^H\right] \tag{A2.1}$$

$$C_t^b = \gamma^{\omega-1}\left[Y - (1-\beta^s)\bar{B}^H - \frac{\eta}{(1-\omega)\chi}(\bar{B}^H - \bar{B}^L)\right] \tag{A2.2}$$

$$Y_t = \gamma^{-1}\left[Y - \eta(\bar{B}^H - \bar{B}^L)\right] \tag{A2.3}$$

$$W_t L_t / P_t = P_{H,t} Y_t / P_t = \gamma^{\omega} Y_t = \gamma^{\omega-1}\left[Y - \eta(\bar{B}^H - \bar{B}^L)\right], \tag{A2.4}$$

where we have used (3.11) and (3.12) in the last equation. Note that all increase with lower γ, or higher wage flexibility (which follows from $\omega - 1 < 0$).

When γ is low enough such that the wage rigidity condition in (A1.9) is not binding, from (A1.15), prices are at their lowest level given by (A1.14), output is at full employment level, $Y_t = Y$, and consumption and real incomes are at their maximum level.[25] If γ is lowered further beyond this point, the real variables no longer change. This concludes the proof of Proposition 2.

[25] That is, the maximum when varying wage flexibility and taking all other parameters as given.

A3 Proposition 3 proof

For $n \to 1$, we can reduce the system of equations (3.1)—(3.21) to the following 11 equations:

$$\beta^s R_t P_t C_t^s = P_{t+1} C_{t+1}^s \tag{A3.1}$$

$$P_t C_t^b = R_t^{-1} \bar{B}_t - \bar{B}_{t-1} + P_t Y_t \tag{A3.2}$$

$$C_t = Y_t \tag{A3.3}$$

$$C_t = (1-\chi) C_t^s + \chi C_t^b \tag{A3.4}$$

$$\beta^s R_t P_t^* C_t^* = P_{t+1}^* C_{t+1}^* \tag{A3.5}$$

$$R_t^{-1} D_t^* - D_{t-1}^* = \omega (P_t C_t - P_t^* C_t^*) \tag{A3.6}$$

$$P_{F,t} Y_t^* = (1-\omega) P_t^* C_t^* + \omega P_t C_t \tag{A3.7}$$

$$P_t^* = P_{F,t}^{1-\omega} P_t^{\omega} \tag{A3.8}$$

$$0 = (R_t - 1)(P_t - P_{t-1}) \tag{A3.9}$$

$$0 = (Y_t - Y)(P_t - \gamma P_{t-1}), \tag{A3.10}$$

$$0 = (Y_t^* - Y)(P_{F,t} - \gamma^* P_{F,t-1}), \tag{A3.11}$$

as well as these inequalities:

$$Y_t \leq Y, \qquad Y_t^* \leq Y \tag{A3.12}$$

$$P_t \geq \gamma P_{t-1}, \qquad P_{F,t} \geq \gamma^* P_{F,t-1} \tag{A3.13}$$

$$R_t \geq 1 \tag{A3.14}$$

The first five equations are the Home savers' Euler equation (3.2), the borrowers' budget constraint (3.4), Home goods market clearing (3.18), Home aggregate consumption (3.6), and Foreign saver's Euler equation (3.8) respectively. Equation (A3.6) is the Foreign country's budget constraint, which we obtain by combining the Foreign saver's budget constraint (3.7) with the Foreign goods market clearing condition (3.19), and using the expressions in (3.10), (3.11) and (3.12). The remaining equations are, in order, the Foreign goods market clearing (3.19), Foreign price index (3.9), monetary policy (equations 3.16 and 3.17), and the complementary slackness conditions in Home and Foreign, in (3.15). Inequalities refer to the full employment constraint on output, downward wage rigidity and the zero lower bound. Throughout, we have used $P_t^u = P_t = P_{H,t} = W_t$, and $D_t = 0$ due to the large size of Home.[26] Additionally, we write down a simplified expression for the savers' budget constraint in Home, which is not necessary to compute the solution, but provides a useful stepping stone in parts

[26] $P_t^u = (P_t)^n (P_t^*)^{1-n}$ is the union-wide price level.

of the proof:

$$P_t C_t^s = \frac{\chi}{1-\chi}(\bar{B}_{t-1} - R_t^{-1}\bar{B}_t) + P_t Y_t. \tag{A3.15}$$

The perfect foresight solution is a sequence of endogenous variables $\{C_t^s, C_t^b, C_t, Y_t, P_t, R_t, P_t^*, C_t^*, D_t^*, Y_t^*, P_{F,t}\}$ that solves equations (A3.1)—(A3.14), given the initial conditions (stated in Section 3.5), and an exogenous path for $\{\bar{B}_t\}$, known in the initial period. The solution will depend on whether the union is at the zero lower bound or not. We consider each of these two cases in turn.

(a) Outside of the zero lower bound

For the remainder of the proof, we adopt the same time notation as in Proposition 3: rather than using the general time subscript t, we denote t as the period of deleveraging, and $t-1$ as the initial steady state. Equation (A3.9) then yields $P_t = P_{t-1} = 1$, and hence $Y_t = Y$ from equation (A3.10): we have zero inflation and full employment in Home during the deleveraging period. At this point, it is helpful to write down the savers' budget constraint (A3.15) at t and $t+1$:

$$\begin{aligned} C_t^s &= Y + \frac{\chi}{1-\chi}\left(\bar{B}^H - R_t^{-1}\bar{B}^L\right) \\ C_{t+1}^s &= Y + \frac{\chi}{1-\chi}(1-\beta^s)\bar{B}^L \end{aligned}$$

where we have used that $Y_t = Y_{t+1} = Y$, $\bar{B}_{t-1} = \bar{B}^H$ and $\bar{B}_t = \bar{B}^L$.

Plugging the above expressions into the savers' Euler equation (A3.1), we obtain an expression for the nominal interest rate

$$R_t = (\beta^s)^{-1}\frac{(1-\chi)Y + \chi\bar{B}^L}{(1-\chi)Y + \chi\bar{B}^H} \tag{A3.16}$$

Intuitively, the central bank cuts the nominal interest rate sufficiently such that the savers consume all of the debt repayments they receive from the borrowers to maintain full employment.

We now turn to the developments in Foreign. In the period before deleveraging $D_{t-1}^* = 0$, and thereafter $D_{t+1}^* = D_t^*$ and $R_{t+1} = (\beta^s)^{-1}$. Also, there is no downward pressure on Foreign prices, and it remains at full employment, $Y^* = Y$.[27] We can then write down a 4x4 equation

[27] One can verify this ex post by checking the real exchange rate formula in (A3.16). Since $Q_t > 1$, $P_{F,t} > 1$ and thus $P_{F,t} > \gamma^* P_{F,t-1}$ since $\gamma^* \leq 1$ and $P_{F,t-1} = 1$. From (A3.11), this implies $Y_t^* = Y$.

system which allows us to solve for the Foreign price level, and hence, the real exchange rate.

$$R_t P_t^* C_t^* = (\beta^s)^{-1} P_{t+1}^* C_{t+1}^* \tag{A3.17}$$

$$R_t^{-1} D_t^* = \omega(P_t C_t - P_t^* C_t^*) \tag{A3.18}$$

$$(1-\beta^s) D_t^* = \omega(P_{t+1}^* C_{t+1}^* - P_{t+1} C_{t+1}) \tag{A3.19}$$

$$P_{F,t} Y = (1-\omega) P_t^* C_t^* + \omega P_t C_t \tag{A3.20}$$

The first equation is Foreign saver's Euler in (A3.5), second and third—the Foreign country budget constraint (A3.6) at t and $t+1$ respectively, and the last equation is the Foreign goods market clearing in (A3.7). R_t is exogenous to Foreign and given by (A3.16). Using that $P_t C_t = Y$, we first combine (A3.17)—(A3.19) to solve for $P_t^* C_t^*$

$$P_t^* C_t^* = Y(1 - \beta^s + R_t^{-1}), \tag{A3.21}$$

and then substitute this expression into (A3.20) to yield

$$Q_t = P_t^*/P_t = P_{F,t}^{1-\omega} = \left[(1-\omega)(1-\beta^s + R_t^{-1}) + \omega\right]^{1-\omega}, \tag{A3.22}$$

which, combined with the interest rate expression in (A3.16), gives us the real exchange rate formula in Proposition 3(a):

$$Q_t \; (:= Q_t^{\text{NoZLB}}) = \left[(1-\omega)\left(1 - \beta^s\left(1 - \frac{(1-\chi)Y + \chi\bar{B}^H}{(1-\chi)Y + \chi\bar{B}^L}\right)\right) + \omega\right]^{1-\omega}. \tag{A3.23}$$

Because $\bar{B}^H > \bar{B}^L$, $Q_t^{\text{NoZLB}} > 1$, hence the real exchange rate depreciates.

(b) At the zero lower bound

Suppose that stabilising union-wide inflation (and therefore economic activity from (A3.10)) would require $R_t < 1$. From equation (A3.16), this requires a shock large enough, such that

$$\beta^s \bar{B}^H - \bar{B}^L > \frac{(1-\chi)}{\chi}(1-\beta)Y =: \underline{\zeta}.$$

In this case, we know that union-wide inflation cannot be stabilised (for if it were, output would be at potential as argued above, and thus the implied R_t would be negative). We shall demonstrate in Section A4 that in equilibrium, the price level will fall to its lower bound provided by downward wage rigidity, from (A3.10), $P_t = \gamma P_{t-1} = \gamma$, and that $Y_t < Y$ so that output strictly drops below potential. Further note that in period $t+1$, the economy is in steady state, hence $R_{t+1} = (\beta^s)^{-1}$, $P_{t+1} = P_t = \gamma$, and $Y_{t+1} = Y$.

Before doing so, however, we establish the response of the real exchange rate by taking for granted the equilibrium at the zero lower bound described above, and to be established in the following section. To do so, first focus on the developments in Foreign. Suppose first that Foreign wages are fully flexible. This means Foreign remains at full employment from (A3.11), and $Y_t^* = Y$. Furthermore, we have $R_t = 1$ from (A3.9), $P_t C_t = \gamma Y_t$ and $P_{t+1} C_{t+1} = \gamma Y$ (where we use $P_t = P_{t+1} = \gamma$ in (A3.3), and that $Y_{t+1} = Y$). Plugging these values into the 4x4 system of equations in (A3.17)—(A3.20) yields an expression for Foreign nominal consumption

$$P_t^* C_t^* = \gamma Y (1 + (1 - \beta^s) \frac{Y_t}{Y}).$$

Combining this with the market clearing condition in (A3.7) gives us an expression for the real exchange rate under flexible Foreign prices:

$$Q_t = \left[\frac{P_{F,t}}{P_t} \right]^{1-\omega} = \left[1 - \omega + (1 - (1 - \omega)\beta^s) \frac{Y_t}{Y} \right]^{1-\omega} \tag{A3.24}$$

We refer the reader to equation (4.4) and the proof of Proposition 4 for the precise expression for Y_t.

Suppose now that Foreign wages are downwardly rigid, and furthermore, the rigidity is sufficiently high such that (A3.11) binds and hence $P_{F,t} = \gamma^*$. Then, trivially,

$$Q_t = \left[\frac{\gamma^*}{\gamma} \right]^{1-\omega}.$$

We now combine the cases of rigid and flexible wages in Foreign. From (A3.13), we know that Foreign price level falls to either the flex-price level, or the maximum amount permitted by downward wage rigidity:

$$P_{F,t} = \max \left(\gamma^*, 1 - \omega + (1 - (1 - \omega)\beta^s) \frac{Y_t}{Y} \right),$$

Putting this expression into the real exchange rate formula in (3.10) yields

$$Q_t \ (:= Q_t^{\text{ZLB}}) = \max \left(\left[\frac{\gamma^*}{\gamma} \right]^{1-\omega}, \left[1 - \omega + (1 - (1 - \omega)\beta^s) \frac{Y_t}{Y} \right]^{1-\omega} \right), \tag{A3.25}$$

which is the formula in Proposition 3(b).

To establish the inequality $Q_t^{\text{ZLB}} < Q_t^{\text{NoZLB}}$, we require that both parts of the max() operator above are smaller in magnitude than Q_t^{NoZLB}. This first of all requires that Foreign wages are not excessively rigid relative to Home:

$$\left[\frac{\gamma^*}{\gamma} \right]^{1-\omega} < Q_t^{\text{NoZLB}},$$

which from (A3.23) requires

$$\gamma^*/\gamma < 1 + \kappa, \text{ where}$$
$$\kappa = (1 - \omega)\beta^s \left(\frac{(1 - \chi)Y + \chi \bar{B}^H}{(1 - \chi)Y + \chi \bar{B}^L} - 1 \right) > 0$$

Turning to the second part of the max() operator, since $Y_t < Y$ and $1 - (1-\omega)\beta^s > 0$, this is bounded above by

$$\left[1 - \omega + (1 - (1-\omega)\beta^s)\frac{Y_t}{Y}\right]^{1-\omega} < \left[2 - \omega - (1-\omega)\beta^s\right]^{1-\omega}.$$

Because $\beta^s \bar{B}^H - \bar{B}^L > \underline{\zeta}$, we know that the counterfactual $R_t < 1$ in (A3.16), and hence $R_t^{-1} > 1$. Applying this inequality to equation (A3.22), we get

$$Q_t^{\text{NoZLB}} > \left[2 - \omega - (1-\omega)\beta^s\right]^{1-\omega},$$

Therefore, the inequality is satisfied for both parts of the max() operator, and $Q_t^{\text{ZLB}} < Q_t^{\text{NoZLB}}$.

To get a real appreciation, we need both parts of the max() operator to be less than 1. For the first part, we simply require $\gamma^*/\gamma < 1$. For the second part, we require

$$\beta^s \bar{B}^H - \bar{B}^L > \frac{(1-\chi)}{\chi}\left(\frac{1-\beta}{1-(1-\omega)\beta}\right)\gamma Y =: \widetilde{\zeta}$$

which is obtained by substituting for Y_t in (A3.24) (using the formula in (4.4), derived in the Proposition 4 proof), and setting the resulting expression to be less than 1. Finally, a real appreciation requires that the deleveraging shock is both large enough to push the union to the zero lower bound ($\beta^s \bar{B}^H - \bar{B}^L > \underline{\zeta}$), and large enough to trigger a real appreciation once at the zero lower bound ($\beta^s \bar{B}^H - \bar{B}^L > \widetilde{\zeta}$). Thus it is required that

$$\beta^s \bar{B}^H - \bar{B}^L > \max(\widetilde{\zeta}, \underline{\zeta}) =: \bar{\zeta}.$$

A4 Proposition 4 proof

Recall that from our definition of monetary policy, Section 3.3, monetary policy cuts the nominal interest rate all the way to $R_t = 1$ if due to deflationary pressure, the inflation target $\Pi_t^u = 1$ cannot be reached. Here we show in a first step that for a large enough shock ($\beta^s \bar{B}^H - \bar{B}^L > \underline{\zeta}$), $P_t = P_{t+1} = \gamma$ along with $Y_t < Y$ is such an equilibrium, and in a second step, that it is the only one possible at the ZLB. Finally, we provide a proof of Proposition 4.

First note that as $n = 1$, real wage income and economic activity are directly related (as shown in the main text)

$$\frac{W_t L_t}{P_t} = Y_t.$$

Substituting this and the fact that $P_t = P_{t+1} = \gamma$ into the borrowers' budget constraint in (A3.2), and the savers' Euler equation in (A3.1) gives

$$C_t^b = -\frac{\bar{B}^H - \bar{B}^L}{\gamma} + Y_t \tag{A4.1}$$

$$C_t^s = (\beta^s)^{-1} C_{t+1}^s \tag{A4.2}$$

Turning to savers' budget constraint (equation A3.15) at $t+1$, and knowing that $\bar{B}_t = \bar{B}^L$ in the new steady state, we get

$$C_{t+1}^s = Y + \frac{\chi}{1-\chi}\left(\frac{(1-\beta^s)\bar{B}^L}{\gamma}\right) \tag{A4.3}$$

Combining (A3.3) in (A3.4) yields

$$Y_t = \chi C_t^s + (1-\chi) C_t^b,$$

and substituting for saver and borrower consumption using equations (A4.1)—(A4.3) gives us the expression for output in (4.4):

$$Y_t = (\beta^s)^{-1}\left[Y - \frac{\chi}{1-\chi}\left(\frac{\beta \bar{B}^H - \bar{B}^L}{\gamma}\right)\right]. \tag{A4.4}$$

From this equation, it follows that $Y_t < Y$ whenever

$$\beta^s \bar{B}^H - \bar{B}^L > \gamma \frac{(1-\chi)}{\chi}(1-\beta) Y = \gamma \underline{\zeta},$$

which must always hold, because $\gamma \leq 1$ and $\beta^s \bar{B}^H - \bar{B}^L > \underline{\zeta}$ at the ZLB. Thus we have established that $P_t = P_{t+1} = \gamma$ along with $Y_t < Y$ is an equilibrium.

To see that no other deflationary equilibrium at the ZLB exists, assume that prices fall to some level $1 = P_{t-1} > P_t = P_{t+1}(=: \tilde{P}) > \gamma$. In this case, from slackness condition (A3.10), output must be at potential in the period of deleveraging, $Y_t = Y$. However as prices fall to $\tilde{P}$ equation (4.4), by setting $Y_t = Y$, can be written as

$$\beta^s \bar{B}^H - \bar{B}^L = \tilde{P}\underline{\zeta}.$$

The fact that at the ZLB, $\beta^s \bar{B}^H - \bar{B}^L > \underline{\zeta}$, then leads to a contradiction because under deflationary pressure, $\tilde{P} < 1$ as mentioned before. Thus, there can be no other equilibrium at the ZLB where due to deflationary pressure, the central bank cuts its interest rate to $R_t = 1$.[28]

[28] Note that an inflationary equilibrium at the ZLB exists, $\tilde{P} > 1$, such that $Y_t = Y$ and $R_t = 1$ in the period of deleveraging. Thus we rule out this equilibrium by maintaining that the central bank would only cut its interest rate to $R_t = 1$ in the case of deflationary pressure in the period of deleveraging. This would be strictly implied if—as we do in our numerical implementation of the model—the central bank *implemented* its strict inflation target via a Taylor-type feedback rule.

We can see that from equation (4.4), output Y_t falls with lower γ. This also means that real incomes fall, and from (A4.1), that borrower consumption falls—since real incomes Y_t are lower and real debt repayments $(\bar{B}^H - \bar{B}^L)/\gamma$, which enter negatively, are higher. This completes the proof of Proposition 4(a). As a side note, from (A4.2) and (A4.3) we can see that saver consumption increases slightly in the period of deleveraging with more flexibility (and thus lower prices), because saver consumption becomes higher in the new steady state. This is because the value of saver assets (equal to borrower debt) rises in real terms as prices decline by more. However, this increase in saver consumption comes at the expense of the borrowers (both during the deleveraging period and in the new steady state), and is not enough to offset the fall in borrower consumption during the deleveraging period (because output falls, from equation 4.4).

We now turn to part (b) of Proposition 4. If the Foreign wage rigidity constraint is not binding, the real exchange rate is given by expression (A3.24). Then γ only enters this expression via Y_t. We can see that higher wage flexibility lowers output Y_t, and, since $1 - (1-\omega)\beta^s > 0$, lowers the real exchange rate Q_t. If $\bar{\zeta} > \beta^s \bar{B}^H - \bar{B}^L > \underline{\zeta}$, $Q_t > 1$ and the real exchange rate depreciates by less as γ declines. If $\beta^s \bar{B}^H - \bar{B}^L > \bar{\zeta}$, $Q_t < 1$ and the real exchange rate appreciates by more as γ declines. This completes the proof of the Proposition.