Monetary Policy in the Age of Automation

Luca Fornaro and Martin Wolf*

November 2021
PRELIMINARY, COMMENTS WELCOME

Abstract

We provide a framework in which monetary policy affects firms' automation decisions (i.e. how intensively capital and labor are used in production). This new feature has far-reaching consequences for monetary policy. Monetary expansions can increase output by inducing firms to invest and automate more, while having little impact on inflation and employment. A protracted period of weak demand might translate into less investment and de-automation, rather than into deflation and involuntary unemployment. Running the economy hot, through expansionary monetary and fiscal policies, may have a positive long run impact on labor productivity and wages. Technological advances that increase the scope for automation may give rise to persistent unemployment, unless they are accompanied by expansionary macroeconomic policies.

JEL Codes: E32, E43, E52, O31, O42

Keywords: monetary policy, automation, fiscal expansions, hysteresis, liquidity traps, secular stagnation, endogenous productivity, wages

*Luca Fornaro: CREI, Universitat Pompeu Fabra, Barcelona School of Economics and CEPR; LFornaro@crei.cat. Martin Wolf: University of St. Gallen and CEPR; martin.wolf@unisg.ch. We thank Daron Acemoglu, Ricardo Caballero, Guido Cozzi, Ben Moll, Lukasz Rachel, Ludwig Straub and seminar/conference participants at the University of St. Gallen, University of Konstanz, CREI, Boston Fed, University of Lancaster, LACEA meeting, Toulouse University, National Bank of Ukraine, CEMFI and the BoE/MMF/NIESR/TPI Workshop on productivity and structural change for very helpful comments. We thank Diego Bohorquez for excellent research assistance. Luca Fornaro acknowledges financial support from the European Research Council under the European Union's Horizon 2020 research and innovation program, Starting Grant (851896-KEYNESGROWTH) and the Spanish Ministry of Economy and Competitiveness, through the Severo Ochoa Programme for Centres of Excellence in R&D (CEX2019-000915-S), and from the Generalitat de Catalunya, through CERCA and SGR Programme (2017-SGR-1393).

1 Introduction

Technological progress often takes the form of automation, that is capital replacing labor in performing production tasks. What are the implications of this phenomenon for monetary policy? This question is attracting a growing attention from economic commentators and policymakers. In a recent book, for instance, Sandbu (2020) argues that maintaining a high pressure economy is necessary to exploit the gains from automation. In absence of expansionary macroeconomic policies, the reason is, the process of replacing labor with capital may cause chronically high unemployment. A related debate is surrounding Biden's fiscal stimulus. Some argue that the Fed should react to the stimulus by tightening monetary policy, to prevent overheating on the labor market and the emergence of a wage-price spiral (Summers, 2021). Others suggest that maintaining monetary policy accommodative - and so running the economy hot - will induce firms to invest in labor-saving technologies, ultimately leading to higher productivity (Konczal and Mason, 2021). In spite of the importance of these debates, however, we still lack a framework connecting monetary policy and automation.[1]

This paper provides a model to study monetary policy in the age of automation. The key novelty of our theory is that monetary policy affects firms' technological decisions, in particular about how intensively capital and labor are used in production. As we will see, this feature implies that monetary policy interventions may have unconventional effects on the economy. For instance, in our framework monetary expansions may generate output booms by fostering firms' use of automation technologies and labor productivity, while having little impact on employment and inflation. In addition, changes in macroeconomic conditions may pose unconventional challenges for monetary policy. For example, during protracted periods of weak demand, the central bank may face a trade-off between sustaining employment or automation. Moreover, technological advances that increase the scope for automation might give rise to persistent unemployment, unless they are accompanied by expansionary macroeconomic policies.

The backbone of our framework is a simplified version of the Acemoglu and Restrepo (2018) model of automation. In this model, capital and labor are highly substitutable in performing some production tasks. This feature implies that movements in factor prices affect how production tasks are allocated between capital and labor. In particular, a drop in the cost of capital relative to wages induces firms to replace labor with capital in performing some tasks, thereby increasing their use of automation in production.

To study the implications of automation for monetary policy, we modify the Acemoglu and Restrepo (2018) model in two dimensions. First, and most importantly, we introduce nominal wage rigidities. The presence of wage rigidities implies that monetary policy has real effects, and that output can deviate from its potential level. Second, following a recent literature in monetary

[1] Another policy debate touching on the relationship between monetary policy and automation is the one surrounding the so-called UK productivity puzzle. Pessoa and Van Reenen (2014) suggest that insufficient demand, rather than manifesting itself into high unemployment, might induce firms to disinvest and use capital less intensively in production. In their view, this effect explains why the recovery from the 2008 financial crisis in the UK was characterized by a dramatic drop in firms' investment and productivity growth.

economics (Mian et al., 2021; Michaillat and Saez, 2021; Michau, 2018), in our framework agents derive utility from holding wealth. This feature allows us to think about cases in which aggregate demand in the long-run is downward sloping in the real interest rate.

Our theory therefore connects monetary policy to firms' adoption of automation technologies. The reason is that monetary policy affects firms' cost of capital - relative to wages - and so the relative profitability of employing capital or labor in performing production tasks. To understand this link, consider a monetary policy easing. The monetary easing reduces the real interest rate, and thus firms' cost of capital. Under certain circumstances, cheaper access to capital may induce firms to reallocate production tasks from labor to capital, thus triggering an increase in firms' use of automation technologies.[2] When this happens, a monetary policy expansion generates a surge in labor productivity. We call this link between monetary policy, firms' automation decisions and labor productivity the *automation effect* of monetary policy.

A first implication of the automation effect is that monetary interventions may have an unconventional impact on employment. The conventional view, captured by the New Keynesian model, is that monetary expansions increase employment and inflation. The reason is that easier monetary policy stimulates demand for consumption and investment, and firms need to employ more workers to satisfy the higher demand for their products. In our framework, however, a monetary easing may also increase labor productivity due to the automation effect. For given aggregate demand, higher productivity reduces firms' demand for labor. Depending on the circumstances, either effect can dominate the other. Hence, the relationship between the policy rate and firms' labor demand (i.e. the labor demand curve) may very well be non-monotonic.

A consequence of the non-monotonic labor demand curve is that in our economy there may be multiple strategies through which full employment can be attained. In fact, there may be two stable steady states consistent with full employment.[3] In the first one, the interest rate is high and aggregate demand is weak. Both forces induce firms to keep investment and automation low. Full employment is reached through depressed wages, so that firms are willing to employ all the labor force in spite of weak demand for their products. In the second full employment steady state, the interest rate is low and aggregate demand is strong. Due to the low interest rate - and the associated low cost of capital - firms' use of automation in production is high. High automation, in turn, translates into high labor productivity and high wages. Full employment is achieved through strong aggregate demand, which sustains firms' labor demand.[4]

[2] Of course, only monetary interventions affecting interest rates in the medium run are likely to have an impact on firms' investment decisions. Therefore, throughout the paper we consider monetary interventions persistent enough to move interest rates in the medium run. Nakamura and Steinsson (2018) provide evidence in favor of an impact of monetary policy on medium-run real rates.

[3] Throughout the paper, we say that the economy operates at full employment when equilibrium employment is equal to households' labor supply (assumed to be fixed). In our framework full employment also corresponds to the natural allocation, i.e. the allocation that would prevail under flexible wages. So when the economy operates at full employment the output gap is equal to zero and output is equal to its potential level.

[4] Caballero et al. (2006) also study a framework in which two steady states, one with low capital and high interest rate and one with high capital and low interest rate, are possible. However, in their model steady state multiplicity does not depend on the possibility that some production tasks may be automated, which instead plays a crucial role in our framework.

Our model also implies that the macroeconomic impact of monetary interventions may depend on the time horizon considered. Once again, imagine that the central bank engineers a persistent reduction in the real interest rate. In the short run, our economy responds in a perfectly conventional way, that is through a rise in employment. This happens because it takes time for firms to invest in order to change their use of automation in production. In the medium run, however, firms may react to the monetary expansion by automating part of their production process. In this case, running the economy hot increases automation and labor productivity in the medium run. A monetary expansion may even trigger a transition from the low to the high automation steady state. When this occurs, a period of accommodative monetary policy and overheating is followed by a permanent rise in productivity. Relatedly, in our economy the inflation response to a monetary expansion may change over time. In the short run, overheating on the labor market puts upward pressure on wage and price inflation. In the medium run, the rise in labor productivity associated with higher automation operates as a disinflationary force. Therefore, a persistent monetary expansion may cause first a rise and then a drop in inflation.

Not only our framework describes novel effects from monetary policy interventions, but it also implies that changes in macroeconomic conditions may have unconventional implications for monetary policy. We start by considering the impact of a persistent slowdown in demand, perhaps as a consequence of a severe financial crisis or of the factors studied by the secular stagnation literature (Eggertsson et al., 2019). If demand is sufficiently weak, the high automation full employment steady state becomes unattainable, because the associated interest rate would violate the effective lower bound on the policy rate. Monetary policy might then face a trade off between employment and automation. It could either maintain automation high, by ensuring a low cost of capital, but at the cost of some chronic unemployment. Otherwise, it could sustain employment by making capital expensive. A high cost of capital, in fact, pushes firms to use capital less intensively and labor more intensively in production. But then full employment would be associated with low labor productivity and low wages.

Against a background of weak demand, fiscal policy may help reconcile full employment with high automation. As highlighted by the Keynesian literature, indeed, fiscal expansions sustain aggregate demand and the equilibrium interest rate. The novel implication of our framework, however, is that fiscal expansions may primarily induce increases in automation and labor productivity, while having little impact on inflation and employment. Once again, this happens because firms may react to the rise in demand mainly by increasing the use of automation in production.

In the final part of the paper, we consider the impact of technological advances that increase the scope for automation. Imagine a scenario in which the range of tasks in which capital can substitute labor increases. For given aggregate demand, this rise in automation might result in higher unemployment. In this case, as suggested by Sandbu (2020), sustaining aggregate demand through expansionary macroeconomic policies may be needed to ensure that the rise in automation does not translate into higher unemployment.

Taking stock, our framework suggests that the effects of monetary interventions may go well beyond the usual employment and inflation targets. In particular, monetary policy may have important implications for firms' investment in automation technologies, labor productivity and real wages. Therefore, central banks may need to take into account this broader set of macro indicators to inform their monetary policy decisions.

Our paper is connected to the literature on the macroeconomic impact of automation. In particular, we build on the framework proposed by Acemoglu and Restrepo (2018). As in Moll et al. (2021), we consider an environment in which the capital supply curve is finitely elastic. Other examples of this literature are Acemoglu and Restrepo (2019), Aghion et al. (2017), Jaimovich et al. (2020), Korinek and Trammel (2020), Restrepo (2015) and Zeira (1998). All these works focus on economies in which monetary policy is neutral and output is always equal to its potential (or natural) level. Instead, we consider an environment in which monetary policy has real effects, and in which movements in aggregate demand may induce deviations of output from potential, either by causing Keynesian unemployment or overheating. These features allow us to study the implications of automation for monetary policy, a topic which has not been considered by the literature before.

Our paper is also related to the recent Keynesian growth literature studying the impact of monetary policy and aggregate demand on firms' investment in innovation and productivity (Benigno and Fornaro, 2018; Fornaro and Wolf, 2020; Garga and Singh, 2020; Moran and Queralto, 2018).[5] In our framework too, monetary policy can affect firms' technological choices and productivity. However, this happens because firms endogenously choose how to allocate production tasks between capital and labor (instead, we abstract from the impact of monetary policy on investment in innovation). As we argued in the introduction, this new feature opens the door to a whole new set of results, such as the existence of a non-monotonic relationship between the policy rate and labor demand, the existence of multiple full employment steady states, and the possibility that monetary and fiscal expansions may mainly lead to increases in productivity and wages, with little impact on employment and inflation.

Moreover, our paper is linked to theories in which weak aggregate demand gives rise to long periods of depressed economic activity (Acharya et al., 2021; Benigno and Fornaro, 2018; Caballero and Farhi, 2018; Eggertsson et al., 2019; Schmitt-Grohé and Uribe, 2017). In all these works, depressed economic activity is associated with high unemployment. In our framework, instead, weak aggregate demand may persistently depress output by inducing firms to de-automate production. In this case, low aggregate demand causes low labor productivity, rather than high unemployment.

Finally, some recent empirical works have explored the connection between macroeconomic policies, aggregate demand and productivity. Jordà et al. (2020) and Ilzetzki (2021) provide evidence in favor of a positive impact of respectively monetary and fiscal expansions on productivity. The empirical works of Bertolotti et al. (2021) and Furlanetto et al. (2021) point toward a positive impact of increases in aggregate demand on productivity. Our model provides a possible explanation for these empirical findings.

[5] See Cerra et al. (2021) for an excellent review of this literature.

The rest of the paper is composed of five sections. Section 2 describes the baseline model. Section 3 derives the labor demand curve and discusses the conditions under which multiple full employment steady states are possible. Section 4 studies the impact of monetary interventions. Section 5 considers the implications of long periods of weak demand, and the economy's response to a rise in automation. Section 6 concludes. The appendix provides all the mathematical proofs, as well as some derivations and model extensions.

2 Baseline model

This section lays down our baseline model. The economy has two key elements. First, as in Acemoglu and Restrepo (2018), firms can choose whether to automate some production tasks. This decision determines the intensity with which labor and capital are used in production. Second, the presence of nominal rigidities implies that output can deviate from its potential level. In order to illustrate transparently our key results, the framework in this section is kept voluntarily simple. Throughout the paper, however, we will extend this baseline framework in several directions.

Consider an infinite-horizon closed economy. Time is discrete and indexed by $t \in \{0, 1, 2, ...\}$. The economy is inhabited by households, firms, and by a government that sets monetary policy. For simplicity, we assume perfect foresight.

2.1 Households

There is a continuum of measure one of identical households deriving utility from consumption of a homogeneous "final" good. The lifetime utility of the representative household is

$$\sum_{t=0}^{\infty} \beta^t \left(\log C_t + \xi \left(\frac{B_{t+1}}{P_t} + K_{t+1} \right) \right), \tag{1}$$

where C_t denotes consumption and $0 < \beta < 1$ is the subjective discount factor. Each household is endowed with $\bar{L}$ units of labor and there is no disutility from working. However, due to the presence of nominal wage rigidities to be described below, a household might be able to sell only $L_t < \bar{L}$ units of labor on the market.

Households can trade in one-period bonds B_t. Bonds are denominated in units of currency and pay the nominal interest rate i_t. Households can also invest in physical capital K_t, which pays a real return r_t^k and depreciates at rate δ. $B_{t+1}/P_t + K_{t+1}$ denotes the stock of wealth, in real terms, held by the household at the end of the period. The parameter $\xi > 0$ thus determines the utility that households derive from holding wealth. This utility function, which is a special case of the one studied by Mian et al. (2021), implies that steady state consumption demand is decreasing in the real interest rate.[6] The presence of this 'long-run IS curve' allows us to derive many results in

[6]In absence of wealth in utility, instead, steady state consumption would be perfectly elastic to changes in the interest rate. The households' discount factor would then be the only determinant of the equilibrium interest rate in the long run.

closed form, using steady state analyses.[7]

The problem of the representative household consists in choosing C_t, B_{t+1} and K_{t+1} to maximize expected utility, subject to a no-Ponzi constraint and the budget constraint

$$P_t C_t + \frac{B_{t+1}}{1+i_t} + P_t K_{t+1} = W_t L_t + P_t(r_t^k + 1 - \delta)K_t + B_t,$$

where P_t is the nominal price of the final good, B_{t+1} is the stock of bonds purchased by the household in period t, and B_t is the payment received from its past investment in bonds. K_{t+1} is the stock of capital held by the household at the end of period t, and used in production in period $t+1$. W_t denotes the nominal wage, so that $W_t L_t$ is the household's labor income.

Households have two decisions to make. First, their optimal saving decision is described by

$$\frac{1}{C_t P_t} = \frac{\beta(1+i_t)}{P_{t+1}C_{t+1}} + \frac{\xi}{P_t} \tag{2}$$

This is just a standard Euler equation, except for the additional incentive to save caused by the presence of wealth in utility, captured by the last term on the right-hand side. Second, households need to allocate their savings between bonds and capital. No arbitrage between these two assets implies

$$\frac{(1+i_t)P_t}{P_{t+1}} = r_{t+1}^k + 1 - \delta.$$

Finally, households' optimal saving behavior obeys the transversality condition[8]

$$\lim_{T\to\infty} \beta^T \frac{B_{t+T+1}/P_{t+T+1} + K_{t+T+1}}{C_{t+T}} = 0. \tag{3}$$

2.2 Final good production

The final good is produced by competitive firms using a continuum of measure one of intermediate inputs, or tasks, $y_{j,t}$, indexed by $j \in [0,1]$. Denoting by Y_t the output of the final good, the production function is

$$\log Y_t = \int_0^1 \log y_{j,t} dj.$$

Profit maximization implies the demand functions

$$p_{j,t} y_{j,t} = Y_t,$$

where $p_{j,t}$ is the price of intermediate input j in terms of the final good.

[7] A growing literature is exploring the implications of agents' deriving utility from wealth in Keynesian models (Michaillat and Saez, 2021; Michau, 2018; Mian et al., 2021). A long-run IS curve also arises in other widely-used environments, such as overlapping generations of finitely-lived agents (Eggertsson et al., 2019), or economies in which agents face uninsurable idiosyncratic risk (Aiyagari, 1994).

[8] Since $\beta < 1$, the transversality condition implies that in the long run the wealth to consumption ratio does not tend to infinity. This optimality condition follows from the fact that the marginal utility from consuming tends to infinity as consumption tends to zero, while the marginal utility from holding wealth is constant.

2.3 Intermediate inputs production

Intermediate inputs are produced by competitive firms, and are heterogeneous in their production technologies. Following Acemoglu and Restrepo (2018), we model technological constraints on automation by assuming that production tasks indexed by $j > J^h$ can be performed only with labor, while the remaining tasks $j \leq J^h$ may be performed with capital. An increase in J^h thus captures technological progress expanding the potential for automation in the production process. As we will see, however, technology is not the only determinant of how intensively automation is used in production.

More precisely, the production function of a generic intermediate input j is

$$y_{j,t} = \gamma_j^k k_{j,t} + \gamma_j^l l_{j,t},$$

where $k_{j,t}$ is the capital used to perform task j, $l_{j,t}$ denotes labor employed in task j, and γ_j^k and γ_j^l denote respectively the productivity of capital and labor in task j.

Intermediates indexed by $j \leq J^l$ are characterized by $\gamma_j^k = \gamma^k > 0$ and $\gamma_j^l = 0$, and thus can be produced with capital only. These intermediate inputs should be thought as those production tasks in which capital is vastly more productive than labor. Intermediates indexed by $j > J^h \geq J^l$ can instead be produced with labor only, and are characterized by $\gamma_j^k = 0$ and $\gamma_j^l = \gamma^l > 0$. These are the production tasks for which automation is not available due to technological constraints. The remaining intermediates $J^l < j \leq J^h$ can be produced with capital or labor, and they are characterized by $\gamma_j^k = \gamma^k$ and $\gamma_j^l = \gamma^l$.

Perfect competition implies that the price of intermediate inputs is equal to their marginal cost, so that

$$p_{j,t} = \begin{cases} \frac{r_t^k}{\gamma^k} & \text{if } j \leq J^l \\ \min\left(\frac{r_t^k}{\gamma^k}, \frac{w_t}{\gamma^l}\right) & \text{if } J^l < j \leq J^h \\ \frac{w_t}{\gamma^l} & \text{if } j > J^h, \end{cases}$$

where $w_t \equiv W_t/P_t$. The interesting implication is that to produce intermediates $J^l < j \leq J^h$ firms employ the cheapest (productivity-adjusted) factor of production. Factor prices thus play a key role in determining the intensity with which capital and labor are used in production.

To see this point more clearly, define $J_t^* \leq J^h$ such that all the intermediate inputs with $j \leq J_t^*$ are produced with capital, while the rest are produced with labor. There are three possible cases to consider. First, suppose that $r_t^k/\gamma^k > w_t/\gamma^l$. In this case capital is expensive compared to labor, and so firms employ capital only to perform production tasks for which it is essential, so that $J_t^* = J^l$. This corresponds to a low automation equilibrium. A second possibility is that $r_t^k/\gamma^k = w_t/\gamma^l$. In this case firms producing intermediates are indifferent between using labor or capital, and the economy is in a partial automation equilibrium with $J^l < J_t^* \leq J^h$. Finally, if $r_t^k/\gamma^k < w_t/\gamma^l$ capital is cheap compared to labor. Firms thus exploit automation as much as possible, and so the economy is in a high automation equilibrium with $J_t^* = J^h$. Summing up, J_t^*

is such that

$$J_t^* \begin{cases} = J^l & \text{if } r_t^k/\gamma^k > w_t/\gamma^l \\ \in \left[J^l, J^h\right] & \text{if } r_t^k/\gamma^k = w_t/\gamma^l \\ = J^h & \text{if } r_t^k/\gamma^k < w_t/\gamma^l. \end{cases} \tag{4}$$

This production function, which is a simplified version of the one posited by Acemoglu and Restrepo (2018),[9] captures in a tractable way the notion that capital and labor are highly substitutable in performing some production tasks. The distance between J^l and J^h encapsulates the easiness with which capital can substitute labor in production. For instance, the monetary economics literature commonly focuses on the case $J^l = J^h$, meaning that the allocation of production tasks between capital and labor is exogenous and independent of macroeconomic conditions. As the distance between J^l and J^h rises, there is more scope for firms to adapt their production technology in response to changes in the macroeconomic environment.

2.4 Aggregate production function

A useful property of this model is that aggregate output can be written as

$$Y_t = \left(\frac{\gamma^k K_t}{J_t^*}\right)^{J_t^*} \left(\frac{\gamma^l L_t}{1 - J_t^*}\right)^{1-J_t^*}. \tag{5}$$

This is a standard Cobb-Douglas production function in capital and labor, with a twist. The twist is that the intensity with which labor and capital are used in production is endogenous and depends on firms' decisions, as encapsulated by the term J_t^*. That said, the model preserves all the usual properties of Cobb-Douglas production functions. Hence, equilibrium prices satisfy the condition

$$1 = \left(\frac{r_t^k}{\gamma^k}\right)^{J_t^*} \left(\frac{w_t}{\gamma^l}\right)^{1-J_t^*}. \tag{6}$$

Moreover, the capital and labor share are respectively given by

$$\frac{r_t^k K_t}{Y_t} = J_t^* \tag{7}$$

$$\frac{w_t L_t}{Y_t} = 1 - J_t^*. \tag{8}$$

Hence, factors' shares are endogenous and depend on the intensity with which automation is used in production. A higher use of automation, that is a higher J_t^*, is associated with a higher capital share.

[9] In Appendix D, we study a version of the model in which, as in Acemoglu and Restrepo (2018), the productivity of labor varies smoothly across different production tasks.

2.5 Wages, prices and monetary policy

We consider an economy with frictions in the adjustment of nominal wages.[10] The presence of nominal wage rigidities plays two roles in the model. First, it creates the possibility that employment may deviate from households' labor supply. Second, it opens the door to real effects of monetary policy interventions.

In particular, we assume that employment and wage inflation are related by a simple Phillips curve

$$W_t = \left(\frac{L_t}{\bar{L}}\right)^{\psi} W_{t-1},$$

where $\psi \geq 0$ denotes the slope of the Phillips curve. According to this expression, wage inflation is decreasing in involuntary unemployment ($\bar{L} - L_t$), which is a common feature of standard models used to study monetary policy.[11]

Monetary policy controls the nominal rate i_t. Because of wage stickiness, movements in the nominal rate affect the real interest rate and other real variables. To see this point, notice that the nominal price of the final good can be written as

$$P_t = \left(\frac{r_t^k}{\gamma^k}\right)^{\frac{J_t^*}{1-J_t^*}} \frac{W_t}{\gamma^l}.$$

This equation implies that the nominal wage rigidity is partly inherited by the price of the final good. Combining the previous two expressions with the no arbitrage condition between bonds and capital gives

$$(1+i_t)\frac{\left(\frac{r_t^k}{\gamma^k}\right)^{\frac{J_t^*}{1-J_t^*}}}{\left(\frac{r_{t+1}^k}{\gamma^k}\right)^{\frac{J_{t+1}^*}{1-J_{t+1}^*}}}\left(\frac{\bar{L}}{L_{t+1}}\right)^{\psi} = r_{t+1}^k + 1 - \delta. \tag{9}$$

This expression shows that movements in the policy rate have real effects, either by influencing the return to capital, employment, or the intensity with which capital is used in production. Throughout the paper, we will consider different assumptions about how monetary policy is set.

2.6 Market clearing and definition of equilibrium

Since households are identical, equilibrium on the bonds market requires $B_t = 0$. Market clearing for the final good then implies

$$Y_t = C_t + K_{t+1} - (1-\delta)K_t. \tag{10}$$

[10] A growing body of evidence emphasizes how nominal wage rigidities represent an important transmission channel through which monetary policy affects the real economy. For instance, this conclusion is reached by Olivei and Tenreyro (2007), who show that monetary policy shocks in the US have a bigger impact on output in the aftermath of the season in which wages are adjusted. Micro-level evidence on the importance of nominal wage rigidities is provided by Fehr and Goette (2005), Gottschalk (2005), Barattieri et al. (2014) and Gertler et al. (2020).

[11] Typically, in the modern monetary literature an additional term capturing forward looking expectations appears on the right-hand side of the Phillips curve. It would be straightforward to incorporate this feature in our analysis.

The output of the final good is thus either consumed or invested in capital.

Turning to the labor market, in absence of nominal rigidities equilibrium employment would satisfy $L_t = \bar{L}$. Hence, we can think of $\bar{L}$ as the natural level of employment. For future reference, we say that when $L_t = \bar{L}$ the economy is operating at full employment. In presence of involuntary unemployment, i.e. when $L_t < \bar{L}$, we say that the economy operates below potential and features a negative output gap.

We are now ready to define an equilibrium of the baseline model

Definition 1 *An equilibrium of the baseline model is a path of real allocations* $\{C_t, L_t, K_{t+1}, Y_t, J_t^*\}_t$ *and prices* $\{w_t, r_t^k\}_t$*, satisfying (2)-(10) and* $L_t \leq \bar{L}$*, given a path for the policy rate* $\{i_t\}_t$ *and an initial value for the capital stock* $K_0 > 0$.

3 Monetary policy, employment and productivity

A distinctive feature of our framework is that monetary policy affects firms' technological decisions, in particular about how production tasks are allocated between labor and capital. This feature implies that the impact of monetary policy interventions on employment can be quite different compared to standard frameworks, such as the New Keynesian model (Galí, 2009). This also means that employment - or even inflation - might not be a good guidance for the conduct of monetary policy.

To make these points, in this section we start by deriving firms' labor demand as a function of the policy rate. We then show that guaranteeing that the economy operates at full employment, or even that an inflation target is achieved, may not pin down a unique equilibrium. Later on, in Section 4, we will explore the impact of monetary interventions on employment and labor productivity.

3.1 The labor demand curve

In our framework, monetary policy interventions trigger contrasting effects on firms' labor demand and employment. We start by illustrating how these effects operate in steady state. To streamline the analysis, in this section we will consider the case of a flat Phillips curve ($\psi \to 0$), so that in steady state the nominal and real interest rates coincide. We will also impose the following parametric restrictions.

Assumption 1 *The parameters and the policy rate* i *satisfy*

$$-\delta(1 - J^h) < i < \frac{1}{\beta} - 1 \tag{11}$$

$$\tilde{\delta} < \delta < \gamma^k, \tag{12}$$

where the threshold $\tilde{\delta}$*, defined in the proof to Lemma 1, is increasing in* J^h *and decreasing in* β.

Intuitively, condition (11) guarantees that steady state consumption and investment are positive and finite. The role of condition (12), instead, will become clear later on.

In steady state, firms' labor demand can be written as[12]

$$L = \underbrace{\frac{1-J^*}{\gamma^l}}_{\text{automation}} \underbrace{\left(\frac{i+\delta}{\gamma^k}\right)^{\frac{J^*}{1-J^*}}}_{\text{capital deepening}} \underbrace{Y}_{\text{aggregate demand}} . \tag{13}$$

This expression encapsulates the three effects through which changes in the steady state interest rate affect labor demand: aggregate demand, capital deepening and automation.

Let us start from the aggregate demand effect. In steady state consumption and investment are respectively equal to[13]

$$C = \frac{1-\beta(1+i)}{\xi} \tag{14}$$

$$\delta K = \frac{\delta J^*}{i+\delta} Y. \tag{15}$$

Combining these equations with the market clearing condition (10) gives an expression relating aggregate demand for final output to the interest rate

$$Y = \frac{1-\beta(1+i)}{\xi\left(1-\frac{\delta J^*}{i+\delta}\right)}, \tag{16}$$

where condition (11) guarantees that output demand is positive and finite. According to this expression, a drop in the interest rate boosts aggregate demand for final output, because it stimulates both consumption and investment. Higher aggregate demand, in turn, sustains firms' demand for labor. The aggregate demand effect thus points toward a negative relationship between the interest rate and labor demand.

Next turn to the capital deepening effect, captured by the second term on the right-hand side of (13). A lower interest rate is associated with a lower cost of capital. As capital becomes cheaper, firms producing the final good react by relying more intensively on the production tasks performed by capital, at the expenses of the production tasks performed by labor (more formally, after a drop in i, y_j rises for $j \leq J^*$ and falls for $j > J^*$). Therefore, the capital deepening effect points toward a positive relationship between the interest rate and labor demand.

Both the aggregate demand and the capital deepening effects are standard, and well studied by the literature on monetary economics. In this literature, the aggregate demand effect typically dominates the capital deepening one, so that a lower interest rate is associated with higher demand for labor.

[12]To obtain this expression, combine (5) and (7) with the no arbitrage condition between bonds and capital $i + \delta = r^k$.

[13]Consumption demand is just given by the households' Euler equation in steady state. Steady state investment, instead, follows from (7) and $r^k = i + \delta$.

The automation effect, encapsulated by the first term on the right-hand side of (13), is instead a distinguishing feature of our framework. In our model, in fact, changes in the interest rate can have an impact on the *allocation* of tasks between capital and labor, that is on J^*. For instance, a drop in the interest rate may induce firms to increase their use of automation technologies, leading to a rise in J^*. In turn, a higher use of automation boosts labor productivity. Holding constant aggregate demand, higher productivity depresses labor demand. Similar to the capital deepening effect, the automation effect thus points toward a positive relationship between the interest rate and firms' labor demand.

What makes the automation effect particularly interesting is that it may be highly non linear and, under certain circumstances, strong enough to dominate the aggregate demand effect. To understand this point, we need to trace firms' technological decisions as a function of the steady state interest rate. There are three cases to consider.

Low automation. First consider a case in which the steady state is associated with low automation ($J^* = J^l$). This happens if capital is expensive compared to labor, i.e. if $r^k/\gamma^k > w/\gamma^l$. Using equations (6) and (9), one can see that this is the case if the interest rate is sufficiently high, precisely if

$$i > \gamma^k - \delta \equiv \bar{i}. \tag{17}$$

The reason is that a high interest rate is associated with a high cost of capital, and thus with a low use of automation in production. The production function then takes a Cobb-Douglas form, with capital share equal to J^l.

In this regime only the standard aggregate demand and capital deepening effects operate. Following the literature on monetary economics, we consider scenarios in which the aggregate demand effect dominates over the capital deepening one. Condition (12) ensures that this is the case, so that for $i > \bar{i}$ a marginal drop in the interest rate leads to a rise in firms labor demand.[14]

Partial automation. A second possibility is that in steady state automation is partial ($J^l < J^* < J^h$). This happens if $r^k/\gamma^k = w/\gamma^l$ and so if $i = \bar{i}$. For this value of the interest rate, firms are indifferent between any degree of automation between J^l and J^h, and their labor demand becomes

$$L = \frac{1 - J^*}{\gamma^l} Y. \tag{18}$$

Hence firms can satisfy the demand for their products through different combinations of L and J^*. Intuitively, in a partial automation steady state firms can easily substitute labor with capital in production, because the automation effect operates at full force.

High automation. The last case corresponds to high automation ($J^* = J^h$). High production automation happens when capital is sufficiently cheap, precisely when $r^k/\gamma^k < w/\gamma^l$ and so $i < \bar{i}$. In this case the production function takes a Cobb-Douglas form with capital share J^h.

[14] See the proof to Lemma 1 in the Appendix for a derivation of this result. Intuitively, the aggregate demand effect is stronger when the elasticity of consumption and investment to changes in the interest rate are higher. These two elasticities are increasing respectively in β and δ. The capital deepening effect is instead stronger the higher the capital share in production, i.e. the higher J^l. These considerations explain why the labor demand curve slopes down when condition (12) holds.

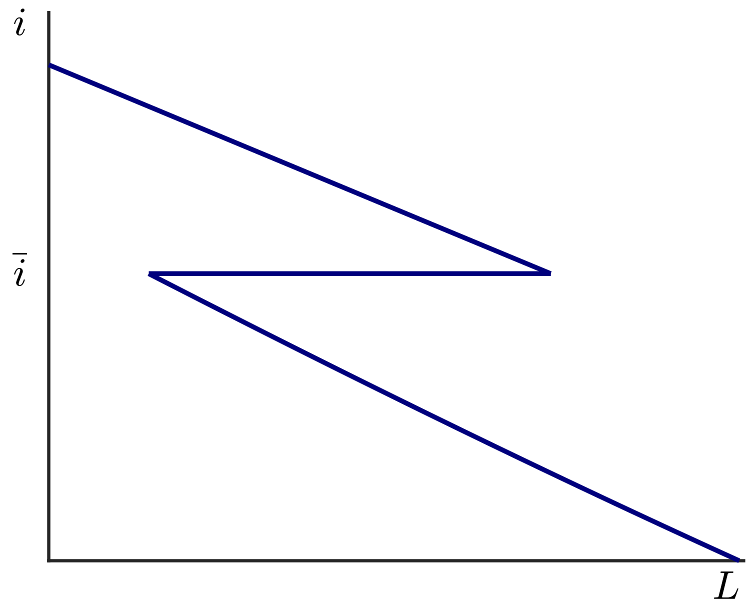


Figure 1: The labor demand curve.

Once again, in this regime only the standard aggregate demand and capital deepening effects operate. Condition (12) then ensures that the aggregate demand effect dominates the capital deepening one, so that for $i < \bar{i}$ a marginal drop in the interest rate increases firms' labor demand.

The labor demand curve. We are now ready to trace the employment response to changes in the steady state interest rate. This is shown by Figure 1, which plots the relationship between L and i implied by firms' labor demand. The line is downward sloped for $i > \bar{i}$, since under low automation the aggregate demand effect dominates. It has a horizontal segment corresponding to the point $i = \bar{i}$, because under partial automation firms can easily reallocate production tasks between capital and labor, and the automation effect is strong. Under high automation, that is for $i < \bar{i}$, the labor demand curve is downward sloped, because the aggregate demand effect once again dominates.

The interesting result is that the labor demand curve is non-monotonic, due to the fact that firms endogenously decide the intensity with which automation is used in production. A particular important role is played by the threshold $\bar{i}$. Above that threshold capital is expensive and automation is low, below $\bar{i}$ capital is cheap and automation high. Around that threshold the automation effect is especially forceful, and dominates the classic aggregate demand channel.

For instance, consider a drop in the interest rate. On the one hand, a lower interest rate fosters aggregate demand, pointing toward higher labor demand and employment. On the other hand, if the interest rate drops from above to below the threshold $\bar{i}$, it induces firms to switch from the low to the high automation technology. In turn, this triggers a discrete increase in labor productivity, and so a fall in firms' labor demand. Under certain circumstances, the automation effect is so strong so that - running against conventional wisdom - a drop in the policy rate may depress labor demand and employment. As we will show throughout the rest of the paper, taking into account this effect may change substantially our view of the impact of monetary policy actions and macroeconomic shocks on the economy.

Remarks on the automation technology. In our model, the automation technology is such that there is a threshold value for the interest rate that determines whether firms operate the low or high automation technology. Of course, the presence of such a stark threshold is unrealistic. However, it is a useful abstraction to illustrate the notion that - under certain circumstances - the impact of monetary policy on firms' use of automation technologies can be strong, and the automation effect may dominate the aggregate demand one. Moreover, it is not hard to modify the framework to allow for a more gradual, and realistic, relationship between the interest rate and firms' use of automation. As we discuss in Appendix D, following Acemoglu and Restrepo (2018) this could be achieved by assuming that labor productivity is heterogeneous across different production tasks. Even in this case, it is not hard to imagine scenarios in which the automation effect sometimes outweighs the aggregate demand one.

3.2 The full employment steady state(s)

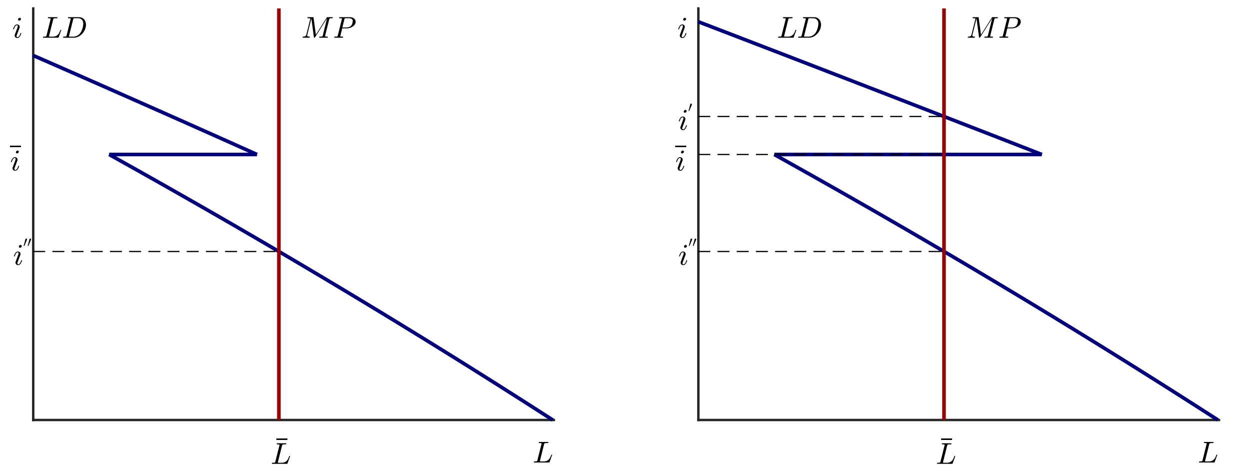
Let us now assume, as commonly made by the literature, that monetary policy ensures that the economy operates at full employment in steady state ($L = \bar{L}$). This is equivalent to assuming that in the long run monetary policy replicates the natural allocation, i.e. the allocation that would prevail under flexible wages. To do so, the central bank must set the interest rate at a level that guarantees that firms' demand for labor is exactly equal to $\bar{L}$. This is known as the natural interest rate. In standard monetary models, such as the New Keynesian framework, there is typically a unique steady state in which the economy operates at full employment and the interest rate is equal to its natural value. In presence of automation, however, this may not be the case.[15]

Figure 2 shows graphically how the steady state is determined. The LD schedule captures firms' labor demand. The MP schedule, which corresponds to a vertical line at $L = \bar{L}$, captures the monetary policy stance. A steady state equilibrium corresponds to a point in which the labor demand and monetary policy schedules intersect. The left panel of Figure 2 displays a case in which the two schedules intersect once, and so in which the full employment steady state is unique.[16] In the right panel of Figure 2, instead, the labor demand and monetary policy schedules intersect three times. In this case, there are three different full employment steady states, each one associated with a different natural rate.

The top one corresponds to a case in which the interest rate is sufficiently high ($i = i' > \bar{i}$), so that firms use capital only to perform tasks in which it is essential ($J^* = J^l$). In this steady state demand is so low that firms can satisfy it without automating much their production processes. Low

[15]To be clear, even in the New Keynesian framework multiple steady states are possible (Benhabib et al., 2001; Benigno and Fornaro, 2018). However, in the New Keynesian model there is a unique steady state in which employment is equal to its natural level and inflation is on target. In our model, due to firms' endogenous automation decisions, multiple steady states with employment equal to its natural level and inflation on target are possible.

[16]The particular steady state shown in the figure features high automation. However, one can easily think of scenarios in which there is a unique full employment steady state characterized by low automation.



(a) Unique full employment steady state. **(b) Multiple full employment steady states.**

Figure 2: Full employment steady state(s).

automation, in turn, depresses labor productivity and real wages.[17] The middle one corresponds to an interior equilibrium with $i = \bar{i}$, in which automation is only partial ($J^l < J^* < J^h$). Here aggregate demand is strong enough to induce firms to substitute labor with capital in some production tasks, but not high enough to induce them to fully automate production. Partial automation sustains wages, which are higher than in the low automation steady state. The lower intersection corresponds to a case in which the interest rate is sufficiently low ($i = i'' < \bar{i}$) so that firms have an incentive to push automation to its full potential ($J^* = J^h$). High automation sustains labor productivity, and allows firms to satisfy the strong aggregate demand associated with a low interest rate. Moreover, high labor productivity leads to high wages, which are higher than in the two other steady states.

The three steady states can also be ranked in terms of their capital stock. In particular the high-automation steady state features a higher capital stock, as well as higher investment and saving rates, compared to the other two. In fact, in our model multiple full employment steady states are possible because a high use of automation sustains both the demand and the supply of capital.[18] On the one hand, as it is intuitive, a high use of automation is associated with a

[17] Rearranging equation (6), one can see that in steady state the (log of the) real wage is given by

$$\log w = \log \gamma^l + \frac{J^*}{1-J^*} \log \frac{\gamma^k}{r^k}.$$

Now recall that in the low automation steady state $r^k > \gamma^k$, in the partial automation steady state $r^k = \gamma^k$ and in the high automation steady state $r^k < \gamma^k$. It follows that the high automation steady state is the one featuring the highest w, while the low automation one features the lowest w.

[18] Formally, desired investment by firms as a fraction of output is equal to $\delta J^*/(i+\delta)$. Households' saving rate, conditional on full employment, is instead given by

$$1 - \frac{C}{Y} = 1 - \frac{1-\beta(1+i)}{\xi \left(\frac{\gamma_k}{i+\delta}\right)^{\frac{J^*}{1-J^*}} \frac{\gamma^l \bar{L}}{1-J^*}}.$$

high demand for capital by firms. On the other hand, a high use of automation boosts labor productivity, and so households' income. In the spirit of Mian et al. (2021), our assumption of non-homothetic preferences implies that households' saving rate is increasing in income. Therefore, the productivity gains offered by automation sustain households' savings. This effect explains why in our model there can be multiple full employment steady states, across which capital and the interest rate are negatively related.[19]

The following proposition specifies the conditions under which steady state multiplicity arises.

Proposition 1 *Suppose that Assumption 1 holds, then at least one full employment steady state exists. Moreover, there exists a threshold Γ such that if $J^h < \Gamma$ the full employment steady state is unique and features low automation, if $J^l > \Gamma$ the full employment steady state is unique and is characterized by high automation, and if $J^l < \Gamma < J^h$ there are three full employment steady states, one associated with low automation, one with partial automation and one with high automation.*

Proposition 1 states that multiple full employment steady states are more likely to arise when the distance between J^l and J^h is bigger, that is when there are many production tasks that can be performed either with capital or labor. For instance, consider the case $J^l = J^h$, in which it is not possible for firms to substitute capital for labor in any production task. Then the production function becomes Cobb-Douglas, and there is a unique full employment steady state.[20] The more we deviate from this benchmark, by increasing the number of tasks in which capital and labor are highly substitutable, the more likely it is that multiple full employment steady states appear. So the existence of multiple full employment steady states is a direct consequence of firms' option to automate some production tasks.

What are the economic implications of this result? The most direct one is that there can be multiple strategies through which an economy can sustain full employment. For instance, full employment could be reached through a combination of weak aggregate demand, depressed investment, as well as low use of automation and low wages. But it could also be reached through a mix of strong aggregate demand and buoyant investment, leading to a high use of automation in production and high labor productivity. This second case reconciles full employment with high real wages.

The second implication is about the conduct of monetary policy. Major central banks, such as the Fed or the ECB, usually consider labor market indicators to judge how far the economy is from the potential (or natural) level of production. Underlying this practice is the idea that there is a single long run level of output consistent with full employment. But, as we have just seen, this notion may not apply when monetary policy affects firms' use of automation. Therefore, a

[19]There is a parallel with the model studied by Caballero et al. (2006). The key difference is that in their framework steady state multiplicity is the result of the particular saving function assumed, while in our case it is the outcome of the productivity gains offered by automation.

[20]Kurz (1968) shows that economies in which agents derive utility from wealth and the production function is Cobb-Douglas may feature steady state multiplicity. Condition (12) rules out this possibility in our framework.

commitment by the central bank to maintain the economy at full employment may not be enough to pin down a unique long run equilibrium.

A similar reasoning applies to inflation, the other major indicator on which central banks base their policy decisions. In fact, the three full employment steady states share exactly the same wage and price inflation rates (normalized to zero). The reason is that these three steady states feature the same level of employment, and hence - consistent with the wage Phillips curve logic - the same wage inflation.[21] This result implies that even a commitment to achieve an inflation target in the long run may not be enough to pin down a unique steady state.

But then, how can monetary policy determine the long run equilibrium? To do so, the central bank should take into account other macroeconomic indicators. One option is to consider investment, since strong investment is needed to sustain a high level of automation. Another possibility is to consider real wages, since in steady state the real wage is increasing in the intensity with which capital is used in production. In particular, in our simple model, the parameter γ^l acts as a threshold for the real wage, since in a high automation steady state $w > \gamma^l$, in a partial automation steady state $w = \gamma^l$, while in a low automation steady state $w < \gamma^l$.

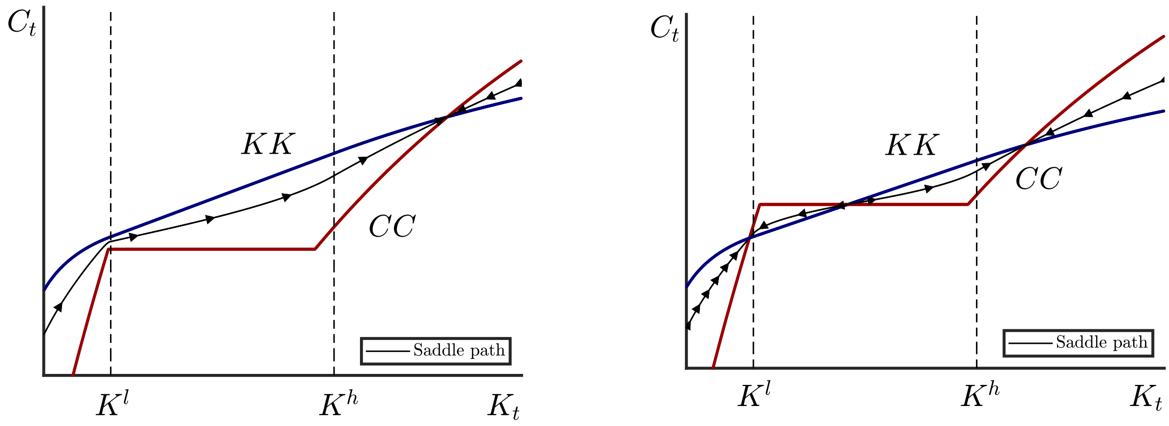
Corollary 1 *Suppose that the central bank targets full employment and either $w < \gamma^l$, $w = \gamma^l$ or $w > \gamma^l$. Then there is at most a single steady state in which the central bank achieves its targets.*

To give an example of what this result implies, consider an economy settled on the low automation full employment steady state. Now imagine that the central bank engineers a permanent increase in aggregate demand by lowering the policy rate. If $J^l < \Gamma < J^h$, this policy action may move the economy to the high automation full employment steady state (we will be more precise about this point later on, in Section 4.3). In this case, the monetary expansion will not affect employment or inflation in the long run, but rather cause an increase in investment and real wages. The message is that, compared to the traditional focus on inflation and the labor market, central banks may need to take into account a broader set of macroeconomic indicators, including firms' investment and real wages, when thinking about the impact of their actions on the economy.

3.3 Dynamics under full employment

So far, we have shown what happens if full employment is achieved in the long run. But what if monetary policy is committed to maintaining full employment at all times? To answer this question, we now solve for the dynamics under full employment, i.e. for the natural allocation. The main result is that the economy may exhibit hysteresis, in the sense that the long run equilibrium may depend on the initial conditions and on the shocks that hit the economy. Hence, even a commitment from the monetary authority to maintaining full employment at all times may not uniquely pin down the long-run equilibrium of the economy.

[21] Notice that, since we are focusing on steady states, an identical result would hold if we had assumed a forward looking New Keynesian Phillips curve.



(a) Unique steady state. **(b) Multiple steady states.**

Figure 3: Phase diagram and saddle path in the natural allocation.

We start by showing that, as in Acemoglu and Restrepo (2018), the capital stock K_t is a sufficient statistic for the state of automation at time t. To see this, combine the factor price equations (7)-(8) and $L_t = \bar{L}$ to obtain

$$\frac{w_t}{r_t^k} = \frac{1 - J_t^*}{J_t^*} \frac{K_t}{\bar{L}}, \tag{19}$$

Also, recall that when $r_t^k/\gamma^k > w_t/\gamma^l$ firms automate as little as possible and hence $J_t^* = J^l$, while when $r_t^k/\gamma^k < w_t/\gamma^l$ firms automate as much as possible and $J_t^* = J^h$. From equation (19), this implies that a critical threshold K^l exists, such that if $K_t < K^l$ firms find it optimal to engage in low automation. Moreover, a second critical threshold $K^h \geq K^l$ exists, such that if $K_t > K^h$ firms engage in high automation. Finally, when the capital stock lies in between these two thresholds, automation is partial. Formally,

$$J_t^* \begin{cases} = J^l & \text{if } K_t \leq K^l \equiv \frac{\gamma^l}{\gamma^k} \frac{J^l}{1-J^l} \bar{L} \\ \in [J^l, J^h] & \text{if } K^l < K_t \leq K^h \\ = J^h & \text{if } K_t > K^h \equiv \frac{\gamma^l}{\gamma^k} \frac{J^h}{1-J^h} \bar{L}. \end{cases} \tag{20}$$

The intuition is that when the capital stock is high wages are high (due a high marginal product of labor) and the return to capital is low (due to a low marginal product of capital). Hence, when the capital stock is high labor is expensive and capital is cheap, leading firms to intensively automate their production.

With this result, we can study the dynamics of the model using a phase diagram in consumption-capital space, shown in Figure 3. In the interest of space, here we just provide the intuition behind the phase diagram, while we explain in detail how it is derived in Appendix C. We study both the case where the steady state is unique (Figure 3a) and where the economy exhibits three full employment steady states (Figure 3b).

The KK schedule in Figure 3 shows the capital locus, that is the set of points in consumption-capital space for which $K_{t+1} = K_t$. The capital locus derives from the resource constraint of the economy, which implies that capital is constant if $C_t = Y_t - \delta K_t$. In the region shown, the capital locus is upward sloping, because a higher capital stock is associated with higher output and consumption.[22] Note that the capital locus is initially concave, then linear, then concave again. The reason is that the production function is a Cobb Douglas with low intensity of capital in production when the capital stock is low ($K_t < K^l$), it is linear in the middle range for capital ($K^l < K_t < K^h$), reflecting perfect substitutability of factors of production, and it is a Cobb Douglas with high intensity of capital when the capital stock is high ($K_t > K^h$).

The CC schedule in Figure 3 shows the consumption locus, that is the set of points in consumption capital space for which $C_{t+1} = C_t$. Intuitively, it captures the fact that, according to households' Euler equation, consumption is constant when the return to capital satisfies $C_t = (1 - \beta(r^k_{t+1} + 1 - \delta))/\xi$. The consumption locus is upward sloping, because a higher capital stock depresses the return to capital, inducing households to consume more. Moreover, the consumption locus is also a piece wise function of the capital stock, with a flat component in the region where automation is partial.[23] Intuitively, this happens because in this region the return to capital is constant.

An intersection of the two loci corresponds to a steady state of the model. Regardless of whether the steady state is unique or multiple steady state exists, for any initial level of the capital stock $K_0 > 0$, the economy converges to a steady state through a unique equilibrium path. For instance, Figure 3a shows the transitional dynamics when the steady state is unique and characterized by high automation.

The interesting case to consider is one in which three steady states are possible, which is shown in the right panel of Figure 3. The first result is that, while the partial automation steady state is unstable, both the low and high automation steady states are locally stable.[24] The second result is that whether the economy transits toward the low or high automation steady state depends on the initial capital stock. More precisely, there exists a thresholds $\bar{K}$ such that if $K_0 < \bar{K}$ the economy ends up in the low automation steady state, while if $K_0 > \bar{K}$ the economy transits toward the high automation steady state. $\bar{K}$ corresponds to the capital stock in the partial automation steady state.

This result implies that our economy can feature hysteresis effects, since its long run behavior may be affected by initial conditions and shocks. For example, imagine that the economy starts

[22] Zooming out, the capital locus has the hump shape that is familiar from the neoclassical growth model. That is, it becomes downward sloped for sufficiently high levels of capital, once output net of capital depreciation becomes decreasing in the capital stock.

[23] Unlike for the capital locus, the kinks in the consumption locus occur not exactly at K^l and K^h. This happens because the kink in the return r^k_{t+1} depends on the capital stock K_{t+1}, whereas what is drawn on the axis is the capital stock K_t. See the Appendix C for the details.

[24] See the Appendix C for a proof.

in the high automation steady state. Also imagine that the economy is hit by a (previously unexpected) recession or a financial crisis, leading to a reduction in the capital stock.[25] If the shock is sufficiently large, so that the capital stock falls below $\bar{K}$, it will induce a shift in the economy from the high to the low automation steady state. In this case, a temporary negative shock may permanently affect the economy's behavior by triggering a process of de-automation.

The important implication of these results is that even a commitment by the central bank to maintain the economy at full employment at all times may not be enough to pin down a unique long run equilibrium. Once again, to maintain control of the evolution of the economy monetary policy may need to take into account a broader set of macroeconomic indicators, in particular firms' investment and real wages.

4 Monetary policy interventions

We now consider monetary policy interventions that induce deviations from the full employment equilibrium (i.e. from the natural allocation). To do so, we slightly generalize the baseline model to include the possibility that the economy may operate above potential, by replacing households' utility function (1) with

$$\sum_{t=0}^{\infty} \beta^t \left(\log C_t - \frac{\left(L_t/\bar{L}\right)^{1+\eta}}{1+\eta} + \xi \left(\frac{B_{t+1}}{P_t} + K_{t+1} \right) \right),$$

where the second term in brackets denotes the disutility from supplying labor, and the parameter $\eta > 0$ is the inverse of the Frisch elasticity of labor supply. To keep the analysis simple, we focus on the limit $\eta \to +\infty$, and we maintain the assumption that arrangements in the labor markets are such that households satisfy firms' labor demand. In this case, households' labor supply is constant and equal to $\bar{L}$. If wages were flexible, equilibrium employment would be always equal to $\bar{L}$. Since wages are rigid, however, L_t may deviate from $\bar{L}$. As in the baseline model, when $L_t < \bar{L}$ there is involuntary unemployment and output is below potential. The difference is that now we can also consider the case $L_t > \bar{L}$, which corresponds to a "hot economy" with output above potential.[26]

4.1 A temporary monetary expansion

In our first experiment, we consider an economy that starts in the low automation steady state (for the results in this section, it does not matter whether the full employment steady state is unique or not). In period 0, the central bank engineers a previously unexpected drop in the real interest

[25] In fact, several studies have captured the impact of the 2008 global financial crisis precisely with a shock the destroys part of the capital stock (see, for example, Gertler and Karadi (2011)).

[26] The definition of the equilibrium for this variant of the model follows closely Definition 1. The only difference is that the constraint $L_t \leq \bar{L}$ does not apply to this version of the model. In particular, all the results of the previous section extend to this version of the model.

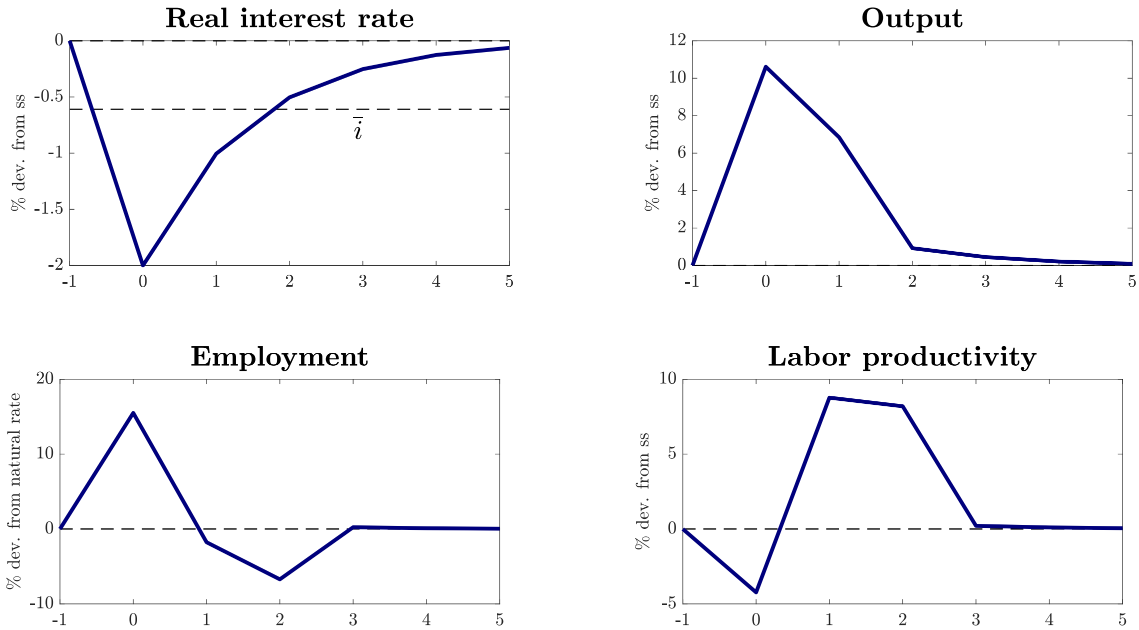


Figure 4: Response to a temporary monetary expansion.

rate.[27] The interest rate then gradually returns to its initial value according to

$$\log(1+r_t) = \rho \log(1+r_{t-1}) + (1-\rho)\log\left(1+r'\right),$$

where $1+r_t \equiv (1+i_t)P_t/P_{t+1}$ denotes the real rate, r' is the real rate in the initial steady state, and $0 < \rho < 1$ is a parameter governing the persistence of the shock. From period 0 on, agents perfectly anticipate these monetary policy actions.

Figure 4 shows the economy's response to a persistent monetary expansion, sufficiently large to push the interest rate initially below $\bar{i}$.[28] As in standard models, on impact (i.e. in $t = 0$) the monetary expansion causes a rise in consumption and investment. In turn, the increase in aggregate demand generates a boom characterized by overheating on the labor market ($L_0 > \bar{L}$).

While these results are standard, what comes next is not. In periods $t = 1, 2$, the economy experiences a large rise in labor productivity. This is due to the fact that the drop in the interest rate is large enough to induce firms to switch to a high use of automation technologies in production.

[27] To streamline the exposition, in this section we frame monetary policy as a path for the real interest rate. The implied path for the nominal rate and inflation can be then derived using the expressions presented in Section 2.5. More formally, we assume that the central bank follows the rule

$$1 + i_t = (1+i')\frac{P_{t+1}}{P_t}(1+\epsilon_t),$$

where i' denotes the interest rate in the low automation steady state and ϵ_t is a monetary policy shock.

[28] To construct the figure we set $\beta = .9$, $\delta = 1$, $\xi = .25$, $\gamma^l = .1$, $\gamma^k = 1.073$, $J^l = .3$, $J^h = .35$, $\bar{L} = 1$ and $\rho = .5$. Of course, this parametrization is purely illustrative and not meant to be realistic. In particular, we have set the depreciation rate δ to an unrealistically high value. Setting δ to a realistic value, in fact, would lead to an extremely large impact of changes in the interest rate on investment. To get a realistic response of investment to changes in the interest rate, the literature typically assumes the presence of capital adjustment costs. We are currently working on extending the model in this direction.

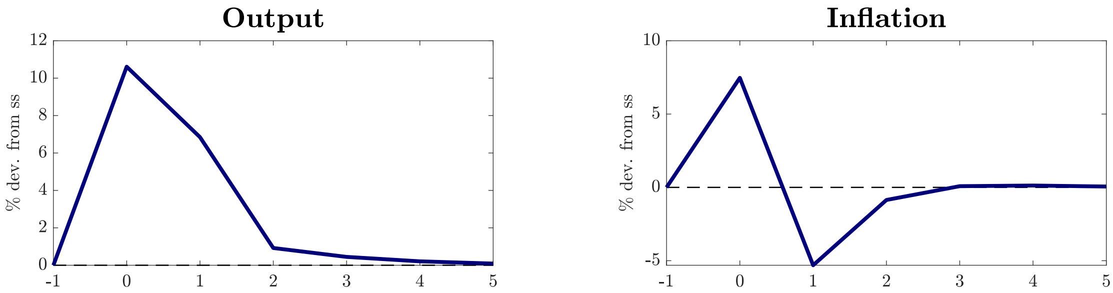


Figure 5: Output boom and ketchup-bottle inflation.

This effect shows up only in the medium run, because in order to use the high automation technology firms need to increase sufficiently their capital stock. Since in the short run the capital stock is fixed, it takes some time for the monetary expansion to have a positive impact on productivity.

The presence of this automation effects has also some interesting implications for employment. In periods $t = 1, 2$ output remains above its steady state value, because of the loose monetary conditions. Employment, however, falls below its natural level, meaning that during the monetary expansion the economy operates below potential. The reason is that the increase in the use of automation depresses firms' labor demand. Because of this effect, in the medium run a monetary expansion can generate a boom without overheating on the labor market.

A similar reasoning applies to inflation, as shown in Figure 5.[29] The monetary expansion leads to an increase in inflation in period 0, because overheating on the labor market puts upward pressure on nominal wages. But in periods 1 and 2, once the capital stock adjusts, loose monetary policy is associated with inflation below target. In fact, on the one hand, involuntary unemployment leads to a drop in wage inflation. On the other hand, a lower cost of capital also decreases firms' marginal cost.[30] Both effects point toward a drop in inflation below its steady state value, even though monetary policy is looser than in steady state. Therefore, a monetary stimulus may very well generate a rise in inflation in the short run, followed by a period of low inflation in the medium run. Martin Sandbu has coined the term ketchup-bottle theory of inflation to describe this effect (Sandbu, 2021).

This experiment reveals that, when firms react by adjusting their production technology, monetary interventions can have unconventional effects. As we have just seen, a monetary expansion may lead to an output boom, accompanied by involuntary unemployment and inflation below target. Instead, a monetary tightening may have exactly the opposite effect. By increasing the cost of capital, in fact, a monetary tightening may lead firms to de-automate production. If this effect is sufficiently strong, a period of low demand may be associated with overheating on the labor market and inflation above target.

[29] To generate this figure we have set $\psi = .2$.

[30] This second effect is reminiscent of the cost channel of monetary policy, that is the idea interest rates have a direct impact on firms' marginal costs (Christiano and Eichenbaum, 1992). However, in contrast with that literature, in our model a drop in the interest rate can lead to a fall in inflation also by causing involuntary unemployment and lower wage inflation.

4.2 Understanding the unconventional impact of monetary interventions

While monetary actions may induce firms to change their use of automation technologies, this does not mean that any monetary intervention will have this effect. As we discuss in this section, the implication is that in our model the macroeconomic impact of monetary actions can be highly non linear, and feature time, size and state dependency.

Time dependency. It takes times for firms to change their production technologies. For instance, increasing the use of robots in a manufacturing plant is not something that can be done over one or two quarters. Our model captures this notion in a stylized fashion, through the standard assumption that it takes one period to transform investment into capital to be used in production. Hence, in the short run monetary interventions have a perfectly conventional impact on the economy. It is only in the medium run, i.e. once the capital stock has adjusted, that a monetary intervention may affect firms' use of automation technologies and labor productivity.

A related point is that only monetary interventions that are persistent enough are likely to affect firms' use of automation. The reason is that firms' investment decisions are typically based on medium-term interest rates, and they are likely to be unresponsive to short-lived changes in monetary conditions.[31] While strictly speaking our baseline model does not embed this effect, it is not difficult to extend it so as to take it into account. For instance, one possibility is to add capital investment costs. In this case, the investment response to a monetary intervention is going to depend also on its persistence.

Size dependency. Another distinctive feature of our framework is that the macroeconomic impact of monetary interventions may be size-dependent. Small monetary shocks, in fact, are unlikely to affect firms' production technologies. Only large interventions are going to affect significantly firms' use of automation. Going back to the example shown in Figure 4, the impact of the monetary expansion on automation and labor productivity arises only because we consider an intervention large enough to push the interest rate below $\bar{i}$. Had we considered a small drop in the policy rate, instead, the response of the economy would have been entirely conventional.

State dependency. Finally, the impact of monetary interventions depend on the state of the economy, in particular on how close the economy is operating to the automation frontier, i.e on how close J_t^* is to J^h. For instance, consider a case in which firms are already fully exploiting existing automation technologies, so that $J_t^* = J^h$. In this case, since automation is against its technological constraint, monetary expansions will not be able to further increase the use of automation in production. It is only when a backlog of unexploited automation technologies is present, that is when $J_t^* < J^h$ that an expansionary policy may have a positive impact on firms' use of automation. Indeed, to illustrate the unconventional effects of monetary policy, in Section 4.1 we have considered an economy starting from the low automation steady state ($J_0^* = J^l$).

[31] In their empirical analysis, Nakamura and Steinsson (2018) show that monetary policy actions can affect real interest rates in the medium run.

4.3 Hysteresis effects from running the economy hot

Now suppose that multiple steady states are possible, and that the economy has settled on the low automation one. Can monetary policy engineer a transition to the high automation steady state? It turns out that the answer is yes, at least if the central bank implements a sufficiently large monetary expansion. In this case, running the economy hot for a while has a permanent impact on firms' use of automation, investment, labor productivity and wages.

Consider the following scenario. The economy is initially in the low automation steady state. In period 0 the central bank starts lowering the interest rate, until it reaches its value in the high automation steady state in the long run. As before, we assume that the initial drop in the interest rate in $t = 0$ is previously unexpected by agents, but that there is perfect foresight from then on.

Figure 6 illustrates the dynamics triggered by this monetary intervention.[32] Initially, the drop in the interest rate generates overheating on the labor market. Intuitively, lower interest rates stimulate demand for consumption and investment. Holding constant the production technology, firms satisfy this higher demand by employing more labor in production.[33]

At some point, however, the interest rate falls below the $\bar{i}$ threshold (or, equivalently, the capital stock rises above the threshold K^*). When that happens, firms switch from the low to the high automation technologies, which results in a sharp increase in productivity. What is interesting, is that in this scenario the switch to high automation becomes self-sustaining, in the sense that in the long run the economy reaches a new steady state in which employment is equal to its natural level, but firms' use of automation and labor productivity rise permanently. Therefore, temporarily overheating the labor market through a monetary expansion may lead to a permanent rise in labor productivity (and real wages).

One important qualification is in order. Whether temporarily running the economy hot leads to a long run impact on labor productivity and wages depends on the structural features of the economy. In particular, if the economy is already operating in the high automation regime, monetary expansions are likely to lead to overheating on the labor market and inflation, while having little effect on long run labor productivity. If, instead, the only possible steady state features low automation, then running the economy hot may foster automation and lead to higher labor productivity. But this effect lasts only as long as the central bank maintains an expansionary monetary policy, and disappears if the central bank reverts to targeting the natural rate.

Notwithstanding these caveats, the model offers a new perspective on the effects of running the economy hot. Traditionally, keeping a high pressure economy by stimulating demand is thought to lead to overheating on the labor market and inflation. But traditional monetary models usually abstract from the possibility that firms may adapt to demand conditions by changing their production technology. Once this possibility is taken into account, one can see that running the

[32] To construct the figure we assumed that the interest rates converges linearly to its new steady state value, and we have set $\beta = 0.9$, $\delta = 0.1$, $\xi = .25$, $\gamma^l = .1$, $\gamma^k = 0.173$, $J^l = 0.45$, $J^h = 0.475$, $\bar{L} = 1$ and $\rho = 0.85$.

[33] In the figure, there are two peaks in the employment response. The first one occurs right at the start of the transition, and it is associated with a sharp rise in consumption due to the positive wealth effect associated with the permanent increase in labor productivity. The second one occurs at the moment when the threshold $\bar{i}$ is crossed, and it is due to the surge in investment needed for firms to switch from the low to the high automation technology.

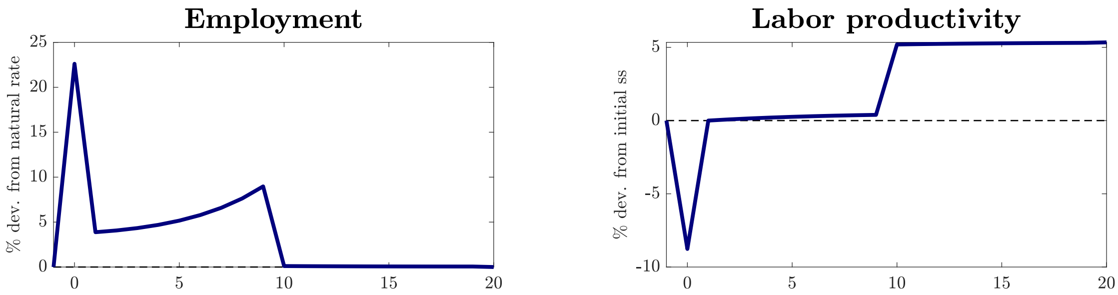


Figure 6: Long run effects from running the economy hot.

economy hot may lead to a long run rise in automation, labor productivity and real wages.

5 Unconventional monetary policy challenges

Not only our framework describes novel effects from monetary policy interventions, but it also implies that changes in macroeconomic conditions may have unconventional implications for monetary policy. In this section we consider three different scenarios. First, we study the economy's response to a protracted period of weak demand. We then turn to the interactions between monetary and fiscal policy. Finally, we consider the macroeconomic implications of a rise in automation, driven by technological change.[34]

5.1 Weak demand, liquidity traps and automation

An interesting question to ask is what happens if the economy experiences a persistent drop in aggregate demand. This scenario could capture the long lasting effects on consumption and investment of a severe financial crisis, such as the 2008 one. Or it may capture secular trends negatively affecting global demand, such as those emphasized by the literature on secular stagnation (Summers, 2016; Eggertsson et al., 2019; Benigno and Fornaro, 2018).

A simple way to introduce exogenous variations in demand is to assume that households have a preference for investing in bonds over capital, perhaps because bonds are more liquid than capital.[35] To do so, we generalise the utility function (1) to

$$\sum_{t=0}^{\infty} \beta^t \left(\log C_t + \xi \left(\frac{e^{\zeta_t} B_{t+1}}{P_t} + K_{t+1} \right) \right),$$

where $\zeta_t \geq 0$ determines households' preference for investing in liquid bonds. The households'

[34] To facilitate the exposition, in this section we restrict attention to the case of a flat Phillips curve ($\psi \to 0$).

[35] See Fisher (2015) for an early example of macroeconomic model in which households derive utility from holding bonds. Del Negro et al. (2017) use a shock to bonds' liquidity to model the 2008 financial crisis.

Another possibility would be to consider variations in ξ, the parameter capturing the utility that households derive from holding wealth. Though some details would differ, the main results would hold also under this alternative source of demand shocks. We chose to consider shocks to the demand for bonds because they give rise to a spread between the return to capital and bonds, which is consistent with the fact that the decline in the interest rate over the last 20 years has not been matched by a comparable fall in the return to capital.

Euler equation (2) then becomes

$$\frac{1}{C_t P_t} = \frac{\beta(1+i_t)}{P_{t+1}C_{t+1}} + \frac{\xi e^{\zeta_t}}{P_t}, \tag{21}$$

while the no arbitrage condition between bonds and capital is replaced by

$$\frac{(1+i_t)P_t}{P_{t+1}} - \left(r^k_{t+1} + 1 - \delta\right) = \frac{\xi C_{t+1}}{\beta}\left(1 - e^{\zeta_t}\right). \tag{22}$$

The baseline model corresponds to the case $\zeta_t = 0$ for all t. When $\zeta_t > 0$, the model deviates from the baseline in two dimensions. First, according to (21), a rise in ζ_t induces a drop in households' demand for consumption. The reason is that the higher ζ_t the higher the incentives that households have to forego consumption and accumulate liquid bonds. Second, according to (22), a rise in ζ_t causes a drop in the real interest rate $(1+i_t)P_t/P_{t+1}$. Intuitively, a higher ζ_t increases households' preference for liquid bonds over illiquid capital. To ensure that the no arbitrage condition holds, the real rate on bonds has to fall until agents are indifferent between investing in the two assets.

To make the analysis interesting, let us also assume that the central bank faces a lower bound i^{lb} on its policy rate. As in Section 3, monetary policy seeks to maintain the economy at full employment. The presence of a lower bound on the policy rate, however, may prevent the central bank from attaining its full employment target. More formally, monetary policy is now described by

$$(L_t - \bar{L})(i_t - i^{lb}) = 0,$$

with $L_t \leq \bar{L}$ and $i_t \geq i^{lb}$. In words, either the economy operates at full employment, or monetary policy is constrained by the lower bound.

Throughout this section we focus on steady states, and consider the response of a permanent drop in aggregate demand, captured with a permanent increase in ζ. We can thus employ the diagram introduced in Section 3.2.[36] As shown in Figure 7, the lower bound on the policy rate is represented by the presence of a horizontal segment corresponding to the point $i = i^{lb}$ in the MP schedule.

As can be seen by comparing the two panels of Figure 7, in response to a permanent rise in ζ the LD schedule shifts down and to the left. This shift is driven by two distinct effects. First, an increase in households' preference for liquid bonds causes a drop in demand for consumption. This effect reduces firms' labor demand for any given level of the policy rate, and is captured graphically by the leftward shift of the LD curve. Second, again holding constant the policy rate,

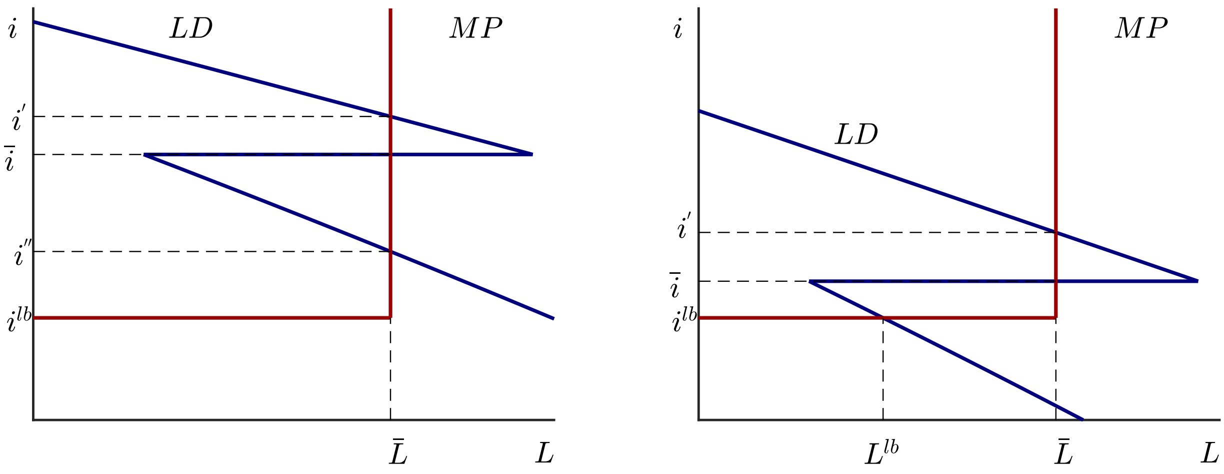
[36] To derive the labor demand schedule for this version of the model, we use the fact that in steady state

$$C = \frac{1-\beta(1+i)}{\xi e^{\zeta}}$$

$$r^k = \frac{1}{\beta} - (1-\delta) - \frac{1-\beta(1+i)}{\beta e^{\zeta}}$$

$$\bar{i} = (1-e^{\zeta})\frac{1-\beta}{\beta} + e^{\zeta}(\gamma^k - \delta).$$

The rest of the derivations follow closely the ones described in Section 3.1.



(a) High demand. **(b) Low demand.**

Figure 7: Impact of permanent drop in aggregate demand.

a rise in ζ drives up the required return from investing in illiquid capital (r^k). This effect reduces the threshold $\bar{i}$ below which firms find it optimal to engage in high automation, as captured by the downward shift of the LD schedule.[37] If the rise in ζ is sufficiently large, the full employment steady state with high automation is no longer attainable, because it would require a policy rate below i^{lb} (right panel of Figure 7). Rather, the high automation steady state is now characterized by a permanent liquidity trap ($i = i^{lb}$) with involuntary unemployment ($L^{lb} < \bar{L}$).[38]

Proposition 2 *Suppose that the economy starts from a full employment high automation steady state in which $\zeta = 0$ and the policy rate is equal to $i'' > i^{lb}$, where i^{lb} denotes the lower bound on the policy rate. Consider a permanent rise in households' liquidity preference for bonds to $\zeta > 0$. Then the full employment high automation steady state is no longer attainable if ζ satisfies*

$$e^{\zeta} > \frac{1 - \beta(1 + i^{lb})}{1 - \beta(1 + i'')}. \tag{23}$$

Now imagine an economy that starts from the full employment steady state with high automation (point $(\bar{L}, i'')$ in the left panel of Figure 7). After a sufficiently large drop in demand, the central bank may face a trade off between automation and employment (right panel of Figure 7). If it wants to keep automation high, it needs to stimulate demand as much as possible, and accept some unemployment (point (L^{lb}, i^{lb})). If it wants to maintain the economy at full employment, it

[37] For completeness, let us mention that there is another effect which tends to push the LD curve to the right. Keeping constant the degree of automation and the policy rate, a higher required return r^k makes firms substitute out of capital and into labor, as labor has now become a cheaper factor of production. However, this effect is dominated by the aggregate demand effect, which pushes the LD curve to the left.

[38] In fact, if the fall in demand is so large so that $\bar{i} < i^{lb}$ the high automation steady state disappears, and the only steady state possible features low automation. In this case, demand may be so weak so that even under low automation there may be some involuntary unemployment.

needs to engineer a fall in wages through tight monetary policy, so as to push the economy toward the low automation steady state (point $(\bar{L}, i')$). This scenario suggests that the macroeconomic response to negative demand shocks may be more complex than what is commonly thought. Traditionally, low inflation and high unemployment are seen as the counterpart of demand shortages. But, weak demand may very well show up into a lower use of automation in production, while having little effects on employment and inflation.

These results connect to the so-called UK productivity puzzle (Pessoa and Van Reenen, 2014). In the aftermath of the 2008 financial crisis, the UK experienced a relatively swift recovery of employment. But it also experienced a prolonged period of low investment, dismal productivity growth and weak real wage growth. Through the lenses of our model, a possible explanation is that low aggregate demand might have led firms to substitute capital for labor in production through a process of de-automation (in fact, this is the hypothesis put forward by Pessoa and Van Reenen (2014)). The model also suggests, as we will see briefly, that this process of de-automation could have been avoided through expansionary fiscal policies, aiming at sustaining aggregate demand.

5.2 Fiscal expansions

Once the lower bound on the policy rate binds, there is little that conventional monetary interventions can do to stimulate demand. For this reason, it is often argued that when the policy rate is close to its lower bound monetary policy should be complemented with fiscal policy. We now take a brief detour from our focus on monetary policy, and study the response of the economy to fiscal interventions during periods of persistently low demand.

We will consider a fiscal expansion, that is a rise in government expenditure. Imagine that in a given period t the government consumes G_t units of the final good, and that government expenditure is fully financed with lump-sum taxes.[39] The only equilibrium condition affected is the market clearing for the final good (10), which is replaced by

$$Y_t = C_t + G_t + K_{t+1} - (1-\delta)K_t.$$

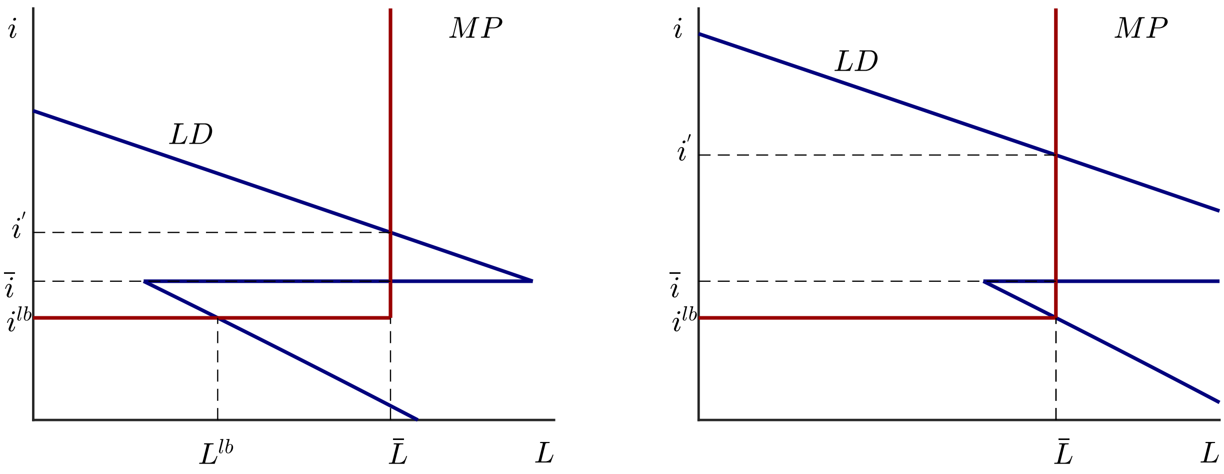
Hence, changes in government consumption act as demand shifters. For instance, a fiscal expansion (i.e. a rise in G_t) produces an increase in aggregate demand.

Now consider a scenario in which demand is so low that the full employment high automation steady state is not attainable (left panel of Figure 8). Starting from this scenario, the government implements a permanent rise in government expenditure. Graphically, a fiscal expansion produces a rightward shift of the labor demand curve (right panel of Figure 8), because for a given value of

[39]Hence, households' budget constraint becomes

$$P_tC_t + \frac{B_{t+1}}{1+i_t} + K_{t+1} + P_tT_t = W_tL_t + P_t(r_t^k + 1 - \delta)K_t + B_t,$$

where T_t denotes the taxes paid to the government. The government budget constraint is instead $G_t = T_t$.



(a) No fiscal response. **(b) Fiscal expansion.**

Figure 8: Impact of fiscal expansion.

the policy rate firms now have to satisfy a higher demand for their products.[40]

As shown in the figure, a sufficiently large fiscal expansion restores the existence of a full employment high automation steady state. This happens because the increase in demand caused by the fiscal expansion translates into a rise of the interest rate consistent with full employment. The following proposition states this result formally.

Proposition 3 *Suppose that condition (23) is satisfied, so that if $G = 0$ a full employment high automation steady state does not exist. Then a full employment high automation steady state exists if the government sets $G = \tilde{G} > 0$ (see the proof for the definition of $\tilde{G}$).*

What about the response of automation and labor productivity to a fiscal expansion? Consider an economy in which demand is too low to sustain a full employment steady state with high automation, as depicted by the left panel of Figure 8. Also assume that the central bank is strictly committed to full employment, so that the economy starts from the low automation full employment steady state. Starting from this scenario, the government increases fiscal expenditure to $\tilde{G}$. With this fiscal policy in place, the full employment high automation steady state becomes attainable. Hence, by running the type of expansionary monetary policy described in Section 4.3, the central bank can now push the economy to the high automation steady state. In this case the fiscal expansion, combined with expansionary monetary policy, causes an increase in firms' use of

[40]To see this result, consider that in steady state the goods market clearing condition becomes

$$Y\left(1 - \frac{\delta J^*}{r^k}\right) = \frac{1 - \beta(1+i)}{\xi e^{\zeta}} + G,$$

where we have used $K = J^* Y / r^k$ and $C = (1 - \beta(1+i))/(\xi e^{\zeta})$. Hence, holding constant i, a rise in G causes an increase in demand and a rightward shift of the LD curve.

automation. In doing so, the fiscal expansion crowds in private investment, leading to a rise in long run labor productivity and wages.[41]

These insights contribute to the debate on the macroeconomic impact of large fiscal expansions, such as the one implemented by the Biden administration. The perspective offered by the model is that, as argued by Konczal and Mason (2021), a fiscal stimulus may lead to higher labor productivity and wages, by inducing firms to adopt labor-saving technologies. In particular, this can happen if the economy is stuck in an equilibrium in which - due to lack of aggregate demand - firms are not fully exploiting the productivity gains offered by automation technologies.

5.3 Rise in automation

An important driver of technological progress is the discovery of new methods to replace labor with capital in some production tasks. In fact, some recent technological advances, such as digital technologies, robotics and artificial intelligence, have dramatically expanded the number of tasks that capital can perform (Acemoglu and Restrepo, 2019). This fact has sparked a lively debate about whether rising automation may displace labor from production, and cause an increase in unemployment. Usually, the debate on the displacement effect caused by automation is framed in terms of models in which demand is strong enough to maintain full employment. Displacement, in these models, occurs because automation affect the supply of labor.

In this section we revisit this debate with the help of our model, which allows for the possibility of involuntary unemployment driven by weak demand.[42] We show that the ability of monetary (and fiscal) policy to sustain full employment can be a key determinant of the labor displacement effect caused by rising automation. Our main insight is that the labor displacement effect may be particularly strong in times of weak demand, when monetary policy is constrained by the lower bound on its policy rate.

We capture a rise in automation through a permanent increase in J^h. An increase in J^h, in fact, expands the set of tasks for which automation is possible. For simplicity, we focus on a steady state analysis, and go back to our baseline model in which bonds do not carry a liquidity premium over capital (i.e. we assume $\zeta_t = 0$ for all t). The following proposition describes the impact of a rise in J^h on firms' labor demand in steady state.

Proposition 4 *Suppose that the following condition holds*

$$J^h < \frac{i^{lb} + (i^{lb} + \delta) \log \left(\frac{\gamma^k}{i^{lb}+\delta} \right)}{i^{lb} + \delta \log \left(\frac{\gamma^k}{i^{lb}+\delta} \right)}. \tag{24}$$

[41] The fiscal expansion also affects investment and labor productivity if the economy starts from the high automation steady state (i.e. from point (L^{lb}, i^{lb}) in the left panel of Figure 8). If the fiscal expansion is not too large, so that it does not affect the policy rate (i.e. if $G \leq \tilde{G}$), it will crowd in private investment. Holding constant the policy rate, the reason is, a rise in demand induces firms to increase their investment in capital. Further rises in government expenditure above the threshold $\tilde{G}$, however, induce the central bank to increase the policy rate so as to prevent overheating on the labor market. Hence increasing G above $\tilde{G}$ crowds out private investment, because a higher policy rate increases firms' cost of capital. In fact, private investment in steady state achieves its maximum when $G = \tilde{G}$.

[42] Our framework, instead, abstracts from the impact of automation on households' labor supply.

Then a rise in J^h lowers firms' labor demand in steady state for given $i \in [i^{lb}, \bar{i})$.

A rise in J^h triggers two contrasting effects on firms' labor demand, conditional on firms fully exploiting the new automation possibilities (i.e. if $i < \bar{i}$). On the one hand, an increase in J^h lowers firms' labor demand because, now that capital performs more production tasks, less labor is needed to satisfy a given level of demand. On the other hand, higher automation requires more investment in physical capital, which sustains aggregate demand and firms' demand for labor. When condition (24) is satisfied the first effect dominates, and a rise in J^h depresses labor demand for $i \leq \bar{i}$.[43] We will focus on this scenario throughout the rest of the section.[44]

As shown in Figure 9, an increase in J^h causes a leftward shift of the LD schedule for values of $i \leq \bar{i}$. This drop in labor demand is not a challenge to full employment, as long as monetary policy is not constrained by the lower bound. In this case, by lowering the policy rate, the central bank can expand demand enough to ensure that a higher automation of production is consistent with full employment.

But now consider the scenario shown in Figure 9. After the rise in J^h, sustaining a level of aggregate demand consistent with full employment and high automation would require a policy rate below i^{lb}. In this case, the high automation steady state is associated with a liquidity trap ($i = i^{lb}$) and involuntary unemployment ($L = L^{lb} < \bar{L}$). Moreover, once the lower bound on the policy rate binds, further improvements in automation technologies translate into higher unemployment. Therefore, the displacement effect on employment caused by a rise in automation is particularly strong when monetary policy is not able to stimulate demand.

Corollary 2 *Suppose that condition (24) holds and that J^h is high enough to satisfy*

$$\bar{L} > \frac{1 - J^h}{\gamma^l} \left(\frac{i^{lb} + \delta}{\gamma^k} \right)^{\frac{J^h}{1-J^h}} \frac{1 - \beta(1 + i^{lb})}{\xi \left(1 - \frac{\delta J^h}{i^{lb}+\delta} \right)}. \tag{25}$$

Then a marginal rise in J^h lowers employment in the high automation steady state.

So far, we have assumed that fiscal policy does not react to rising automation. However, following the logic outlined in Section 5.2, one can see that the displacement effect on employment caused by automation can be counteracted through fiscal expansions. Higher government expenditure, in fact, sustains aggregate demand and firms' demand for labor. Hence, in times of weak demand, fiscal expansions can complement monetary policy to ensure that technological advances pushing forward the automation frontier do not cause involuntary unemployment.

These results lend support to the view put forward by Sandbu (2020). In times of fast-growing automation, expansionary macroeconomic policies may play a key role in maintaining the econ-

[43] Condition (24) is satisfied as long as J^h is sufficiently small or i^{lb} is sufficiently high. For instance, if $i^{lb} = 0$ it is satisfied for any J^h.

[44] Moll et al. (2021), instead, consider the opposite scenario. In their model, in fact, a rise in the automation frontier increases firms' labor demand and the natural interest rate.

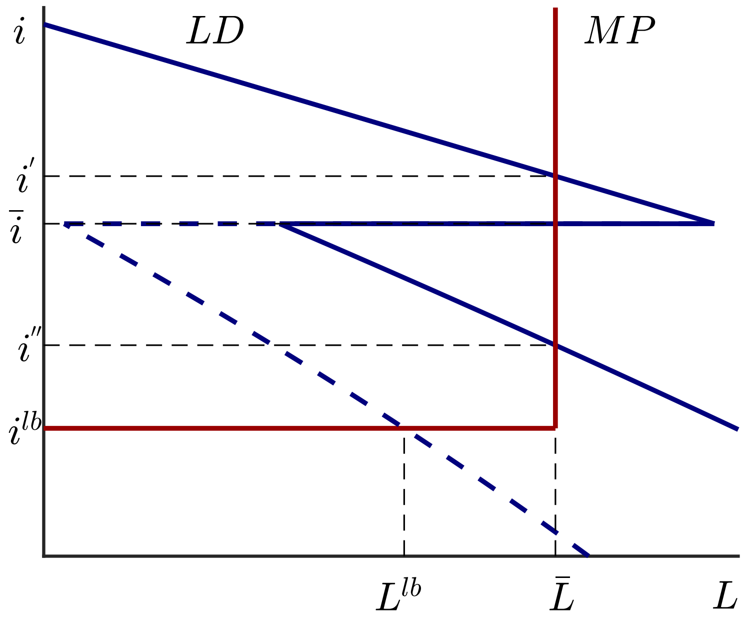


Figure 9: Rise in automation. Notes: solid LD line denotes low J^h, dashed LD line denotes high J^h.

omy at full employment. Otherwise, rising automation may very well cause a chronic increase in involuntary unemployment, due to scarcity of jobs.

6 Conclusion

In this paper, we have provided a theory to study monetary policy in the age of automation. The overarching theme of the paper is that monetary policy interventions are likely to affect firms' use of automation in production. In fact, we have shown that under certain circumstances monetary (as well as fiscal) policy interventions may mainly affect firms' automation decisions and productivity, while having little effects on employment and inflation. These results suggest that - when designing their policy - central banks may need to deviate from their traditional focus on labor market indicators and inflation, and consider a broader set of macroeconomic variables, including firms' technological choices, investment and real wages.

To conclude, let us mention two avenues of future research that could build on this paper. First, in this paper we have taken a purely positive perspective, and abstracted from deriving normative implications. However, in future work we plan to use our framework to study the impact on welfare of monetary and fiscal interventions. Second, in this paper we have taken the automation frontier, determining the set of tasks that can be performed with capital, as an exogenous factor. However, as argued by Acemoglu and Restrepo (2018), the automation frontier is shaped by technological progress. It would thus be interesting to study monetary policy in a framework in which the automation frontier is the endogenous outcome of firms' innovation activities.

Appendix

A Additional lemmas

Lemma 1 *Suppose that condition* (12) *holds, then firms' labor demand is strictly decreasing in* i *for* $i > \bar{i}$ *and* $i < \bar{i}$.

Proof. We start by considering the case $i < \bar{i}$, in which firms' labor demand is

$$L = \frac{1-J^h}{\gamma^l}\left(\frac{i+\delta}{\gamma^k}\right)^{\frac{J^h}{1-J^h}} \frac{1-\beta(1+i)}{\xi\left(1-\frac{\delta J^h}{i+\delta}\right)}. \tag{A.1}$$

Taking the derivative

$$L'(i) = \frac{1-J^h}{\gamma^l}\left(\frac{i+\delta}{\gamma^k}\right)^{\frac{J^h}{1-J^h}} \frac{1}{\xi\left(1-\frac{\delta J^h}{i+\delta}\right)^2}\left[\frac{J^h}{1-J^h}\frac{i(1-\beta(1+i))}{(i+\delta)^2} - \beta\left(1-\frac{\delta J^h}{i+\delta}\right)\right].$$

Hence $L'(i) < 0$ if and only if the term in brackets is negative in the relevant range. Multiplying by $(i+\delta)^2$, the term in brackets becomes a quadratic in i. The condition for this to be negative is

$$g(i) \equiv i^2 + i\left[\delta(1-J^h)(2-J^h) - \frac{1-\beta}{\beta}J^h\right] + \delta^2(1-J^h)^2 > 0 \tag{A.2}$$

for all admissible values of i.

Since $g(i)$ is a U-shaped parabola, a sufficient condition for $g(i) > 0$ is that the minimum of $g(i)$ is larger than zero. The minimum of $g(i)$ is reached when $i = i^*$ given by

$$i^* = -\frac{1}{2}\left(\delta(1-J^h)(2-J^h) - \frac{1-\beta}{\beta}J^h\right).$$

Inserting this in (A.2) we obtain that $g(i) > 0$ for any admissible value of i if

$$g(i^*) = \delta^2(1-J^h)^2(4-J^h)\frac{\beta}{1-\beta} + 2\delta(1-J^h)(2-J^h) - \frac{1-\beta}{\beta}J^h > 0. \tag{A.3}$$

This condition is a quadratic in δ. Note that the terms multiplying δ^2 and δ are both positive, whereas the last summand is negative. This means that there exists a unique positive value of δ, call it $\tilde{\delta}$, for which $g(i^*) = 0$. This also implies that condition (A.3) is satisfied if and only if $\delta > \tilde{\delta}$, meaning that - conditional on high automation - the labor demand curve slopes down if δ exceeds the threshold $\tilde{\delta}$. Moreover, it is easy to see that the threshold $\tilde{\delta}$ is decreasing in β and increasing in J^h.

Turning to the case $i > \bar{i}$, corresponding to low automation equilibria, following the steps outlined above one can see that the labor demand curve is downward sloping if and only if

$$\delta^2(1-J^l)^2(4-J^l)\frac{\beta}{1-\beta}+2\delta(1-J^l)(2-J^l)-\frac{1-\beta}{\beta}J^l>0, \tag{A.4}$$

which is satisfied when $\delta > \tilde{\delta}$. ■

B Proofs

B.1 Proof of Proposition 1

Start by recalling that assumption (12) guarantees that the labor demand curve takes the shape illustrated by Figure 1, downward sloped for $i > \bar{i}$, horizontal and backward bending for $i = \bar{i}$, and downward sloped for $i < \bar{i}$ (see the proof to Lemma 1). Moreover, (11) guarantees that for the range of i that we will consider steady state consumption is positive and finite.

We now derive a condition for the high automation steady state characterized by $L = \bar{L}$ to exist. Notice first that

$$\lim_{i\to-\delta(1-J^h)} \tilde{L}(i) = +\infty,$$

where $\tilde{L}(i)$ denotes labor demand conditional on high automation.[45] Second, recall from the text that the partial automation region starts at $i = \bar{i} \equiv \gamma^k - \delta$. Hence, an intersection of labor demand with the full employment level $\bar{L}$ exists whenever $\tilde{L}(\bar{i}) < \bar{L}$, that is if and only if

$$h(J^h) \equiv \frac{1-J^h}{\gamma^l}\frac{1-\beta(1+\gamma^k-\delta)}{\xi\left(1-\frac{\delta J^h}{\gamma^k}\right)} < \bar{L}. \tag{B.1}$$

We next derive a condition for the low automation steady state with $L = \bar{L}$ to exist. Notice first that

$$\lim_{i\to\frac{1}{\beta}-1} \hat{L}(i) = 0,$$

where $\hat{L}(i)$ denotes labor demand conditional on low automation (equation (A.1), but J^h replaced by J^l).[46] Second, recall again that the partial automation region starts at $i = \bar{i} \equiv \gamma^k - \delta$. Hence, an intersection of labor demand with the full employment level $\bar{L}$ exists whenever $\hat{L}(\bar{i}) > \bar{L}$, that is if and only if

$$h(J^l) > \bar{L}. \tag{B.2}$$

We now state the conditions for existence of the full and low automation steady states in a more compact manner. Recall first that we assume $\gamma^k > \delta$, from condition (12). Moreover, assume for now that $\gamma^k < 1/\beta - 1 + \delta$, hence that γ^k lies in the open interval

$$\delta < \gamma^k < \frac{1}{\beta} - 1 + \delta.$$

[45] Recall that we restrict our analysis to $i > -\delta(1-J^h)$, hence the limit is taken from above.
[46] Recall that we restrict our analysis to $i < \frac{1}{\beta} - 1$, hence the limit is taken from below.

In this case, as it can be easily verified, the function h is strictly downward sloping. Moreover, observe that h satisfies

$$h(0) = \frac{1}{\gamma^l \xi}(1 - \beta(1 + \gamma^k - \delta)),$$

which is strictly positive by assumption, and furthermore $h(1) = 0$. We now make a case distinction according to the level of $\bar{L}$.

In case $\bar{L} > h(0)$, then $\bar{L} > h(J)$ for all $J \in [0, 1]$, because h is strictly downward sloping. But then equation (B.1) is satisfied for all J^h, hence the high automation steady state always exists in this case. Conversely, equation (B.2) is not satisfied for any J^l, hence the low automation steady state never exists in this case.

The second case to consider is $0 < \bar{L} < h(0)$. In this case, we can invert the function h to define a threshold level $\tilde{\Gamma}$

$$\tilde{\Gamma} = h^{-1}(\bar{L}) = \frac{1 - \beta(1 + \gamma^k - \delta) - \xi \gamma^l \bar{L}}{1 - \beta(1 + \gamma^k - \delta) - \xi \gamma^l \bar{L} \frac{\delta}{\gamma^k}}.$$

By construction, it must be that $0 < \tilde{\Gamma} < 1$.[47] By conditions (B.1) and (B.2), under this parameter constellation, there is therefore scope for multiplicity. Indeed, in case of $J^h \geq J^l > \tilde{\Gamma}$, condition (B.1) holds but not (B.2), so that the high automation steady state is unique. Conversely, if $\tilde{\Gamma} > J^h \geq J^l$, then condition (B.1) does not hold but (B.2) holds, so that the low automation steady state is unique. Finally, in the case $J^h > \tilde{\Gamma} > J^l$, both (B.1) and (B.2) are satisfied, and hence both steady states exist.

We last turn to the case where $\gamma^k > 1/\beta - 1 + \delta$. In this case, h is strictly upward sloping. Moreover, as it is easy to see, $h(0)$ is now negative (and $h(1) = 0$ is the same as before). But this implies that $h(J) < 0$ for any level of $J \in [0, 1]$, hence (B.1) is satisfied for any level of J^h in this case, whereas (B.2) is violated for any level of J^l in this case. Hence, under this parameter constellation, the high automation steady state is again unique.

We may summarize this as follows. Define the parameter Γ such that

$$\Gamma = \begin{cases} \tilde{\Gamma} & \text{if } \gamma^k < \frac{1}{\beta} - 1 + \delta \text{ and } \bar{L} < \frac{1}{\gamma^l \xi}(1 - \beta(1 + \gamma^k - \delta)) \\ 0 & \text{else.} \end{cases}$$

Then, the high automation steady state exists if and only if $J^h > \Gamma$. The low automation steady state exists if and only if $J^l < \Gamma$. Finally, looking at Figure 2 makes it clear that a partial automation steady state with $L = \bar{L}$ exists if and only if the low and high automation steady states also exist. Hence, if $J^l < \Gamma < J^h$ the model features three full employment steady states.

[47] To see this, use that $\bar{L} > h(0)$ to see that the numerator is positive. Moreover, use that $\gamma^k > \delta$ to see that the denominator is also positive, and in fact larger than the numerator. Hence the overall expression is positive and smaller than 1.

B.2 Proof of Corollary 1

By equation (6), in a low automation steady state $w < \gamma^l$, in a partial automation steady state $w = \gamma^l$ and in a high automation steady state $w > \gamma^l$. Using the results in Proposition 1, one can then see that the conditions stated in Corollary 1 pin down at most a unique steady state consistent with the central bank's targets.[48]

B.3 Proof of Proposition 2

The economy starts with $\zeta = 0$, and so the policy rate in the initial full employment steady state i'' solves

$$\bar{L} = \frac{1-J^h}{\gamma^l}\left(\frac{i''+\delta}{\gamma^k}\right)^{\frac{J^h}{1-J^h}} \frac{1-\beta(1+i'')}{\xi\left(1-\frac{\delta J^h}{i''+\delta}\right)}. \tag{B.3}$$

By assumption $i'' > i^{lb}$. Now consider a permanent rise in the liquidity preference for bonds to $\zeta > 0$. Suppose that a full employment steady state with high automation exists. Then in this steady state the policy rate has to satisfy

$$\bar{L} = \frac{1-J^h}{\gamma^l}\left(\frac{r^k}{\gamma^k}\right)^{\frac{J^h}{1-J^h}} \frac{1-\beta(1+i)}{\xi e^{\zeta}\left(1-\frac{\delta J^h}{r^k}\right)}. \tag{B.4}$$

$$r^k = \frac{1}{\beta} - (1-\delta) - \frac{1-\beta(1+i)}{\beta e^{\zeta}}. \tag{B.5}$$

Let's call $\tilde{i}''$ the value of the policy rate satisfying (B.4) and (B.5). Using (B.3), (B.4) and (B.5) one can see that $\tilde{i}''$ satisfies

$$e^{\zeta} = \frac{1-\beta(1+\tilde{i}'')}{1-\beta(1+i'')}. \tag{B.6}$$

Since the right hand side of (B.6) is decreasing in $\tilde{i}''$ then if

$$e^{\zeta} > \frac{1-\beta(1+i^{lb})}{1-\beta(1+i'')},$$

then $\tilde{i}'' < i^{lb}$ and so no full employment high automation steady state exists because it would violate the lower bound on the policy rate.

B.4 Proof of Proposition 3

Suppose that $i = i^{lb}$ and $G = \tilde{G}$. Then a full employment high automation steady state exists if $\tilde{G}$ satisfies

$$\bar{L} = \frac{1-J^h}{\gamma^l}\left(\frac{r^k}{\gamma^k}\right)^{\frac{J^h}{1-J^h}} \left(\frac{1-\beta(1+i^{lb})}{\xi e^{\zeta}} + \tilde{G}\right)\frac{1}{1-\frac{\delta J^h}{r^k}}.$$

[48] The "at most" part comes from the fact that if $J^l > \Gamma$ ($J^h < \Gamma$) then a steady state consistent with the central bank's targets exists only if the central bank targets $w > \gamma^l$ ($w < \gamma^l$).

$$r^k = \frac{1}{\beta} - (1-\delta) - \frac{1-\beta(1+i^{lb})}{\beta e^{\zeta}}.$$

Condition (23) ensures that $\tilde{G} > 0$.

B.5 Proof of Proposition 4

For $i < \bar{i}$ firms' labor demand is given by

$$L = \frac{1-J^h}{\gamma^l}\left(\frac{i+\delta}{\gamma^k}\right)^{\frac{J^h}{1-J^h}} \frac{1-\beta(1+i)}{\xi\left(1-\frac{\delta J^h}{i+\delta}\right)}.$$

Taking logs and differentiating with respect to i yields

$$\frac{\partial \log L}{\partial J^h} = -\frac{1}{1-J^h} - \frac{1}{(1-J^h)^2}\log\left(\frac{\gamma^k}{i+\delta}\right) + \frac{\delta}{i+\delta(1-J^h)}.$$

Rearranging, one finds that this expression is negative if

$$J^h < \frac{i+(i+\delta)\log\left(\frac{\gamma^k}{i+\delta}\right)}{i+\delta\log\left(\frac{\gamma^k}{i+\delta}\right)}.$$

One can check that the numerator of the right hand side of this expression is increasing in i for $i \leq \bar{i}$, while the denominator is increasing in i for $i > 0$ and decreasing in i for $i < 0$. Moreover, the right hand side is ≥ 1 for $i \geq 0$. Because $J^h \leq 1$, this implies that if the condition above is satisfied for $i = i^{lb}$ it is also satisfied for any $i^{lb} \leq i < \bar{i}$.

B.6 Proof of Corollary 2

Since condition (24) holds, labor demand is decreasing in J^h for $i \in [i^{lb}, \bar{i})$. Hence, if J^h is high enough to satisfy condition (25) maintaing full employment in the high automation steady state would require setting $i < i^{lb}$. This means that the lower bound on the policy rate binds in the high automation steady state. Therefore, employment in the high automation steady state is given by

$$L = \frac{1-J^h}{\gamma^l}\left(\frac{i^{lb}+\delta}{\gamma^k}\right)^{\frac{J^h}{1-J^h}} \frac{1-\beta(1+i^{lb})}{\xi\left(1-\frac{\delta J^h}{i^{lb}+\delta}\right)} < \bar{L},$$

and it is decreasing in J^h (since condition (24) is assumed to hold).

C The natural allocation

C.1 Deriving the capital and consumption locus

We start by deriving the capital locus, that is, the set of points in the consumption-capital space for which $K_{t+1} = K_t$. To do so, we use the resource constraint $K_{t+1} = Y_t - C_t + (1-\delta)K_t$, to obtain

$$C^{cap}(K_t) = Y(K_t) - \delta K_t. \tag{C.1}$$

Define the parameter $\phi^m \equiv (\gamma^k/J^m)^{J^m}(\gamma^l \bar{L}/(1-J^m))^{1-J^m}$ for $m \in \{l,h\}$. Then the function $Y(K_t)$ can be derived using (5) and (20)

$$Y(K_t) = \begin{cases} \phi^l K_t^{J^l} & \text{if } K_t \leq K^l \\ \gamma^k K_t + \gamma^l \bar{L} & \text{if } K^l < K_t \leq K^h \\ \phi^h K_t^{J^h} & \text{if } K_t > K^h, \end{cases} \tag{C.2}$$

where we have also used (19) and the fact that $w_t/\gamma^l = r_t^k/\gamma^k$ to derive that output is linear in the partial automation region.

As argued in the main text, output is a piece-wise function of the capital stock. It is initially concave, then linear, then concave again. The capital locus (C.1), shown in the left panel of Figure 10, inherits these properties. The arrows in the figure illustrate the transitional dynamics implied by this locus. When consumption is above $C^{cap}(K_t)$, the capital stock shrinks over time, while the opposite occurs when consumption is below $C^{cap}(K_t)$.

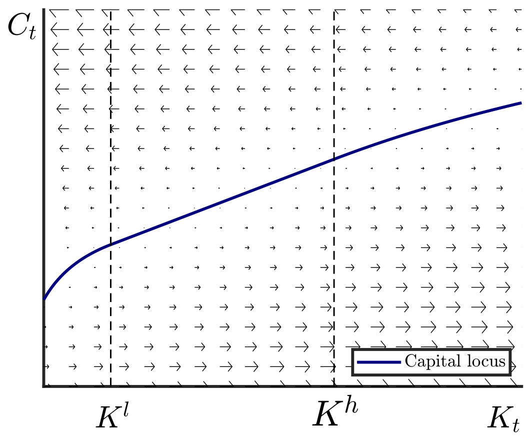
We next derive the consumption locus, defined as the set of points in consumption-capital space for which $C_{t+1} = C_t$. To do so, we use the Euler equation (2) and the no arbitrage condition between bonds and capital, to obtain

$$\frac{C_{t+1}}{C_t} = \beta(r_{t+1}^k + 1 - \delta) + \xi C_{t+1}.$$

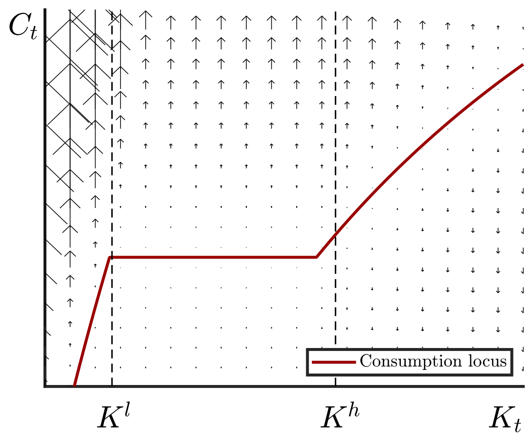
As is usual, the return to capital r_{t+1}^k is a function of the capital stock. However, notice that in our model, the return to capital also depends on the state of automation. Luckily, the capital stock K_t is a sufficient statistic also for the return to capital. Namely, as one can see from equation (C.2), the return to capital is given by

$$r^k(K_t) = \begin{cases} J^l \phi^l K_t^{J^l - 1} & \text{if } K_t \leq K^l \\ \gamma^k & \text{if } K^l < K_t \leq K^h \\ J^h \phi^h K_t^{J^h - 1} & \text{if } K_t > K^h. \end{cases} \tag{C.3}$$

A detail to consider is that the return to capital in the Euler equation is dated at time $t+1$, rather than at time t. We thus insert the resource constraint (10), in order to express K_{t+1} as a



(a) Capital locus.



(b) Consumption locus.

Figure 10: Capital and consumption locus and dynamics.

function of the current capital stock. This yields

$$1 = \beta(r^k(Y(K_t) - C^{cons}(K_t) + (1-\delta)K_t) + 1 - \delta) + \xi C^{cons}(K_t). \tag{C.4}$$

The consumption locus $C^{cons}(K_t)$ is thus defined implicitly and, in contrast to the capital locus, cannot be expressed in closed form. However, it can easily be obtained numerically by iterating over equation (C.4). We show the result in Figure 10, in the right panel. The consumption locus is upward sloping. As explained before, the intuition is that a higher capital stock depresses the return to capital and the interest rate, inducing households to consume more. Moreover, the consumption locus is a piece wise function of the capital stock, with a flat component in the region where the economy operates under partial automation, where the return to capital is a constant (see equations (C.2) and (C.3)). Again, we add arrows to the figure to indicate the transitional dynamics implied by the consumption locus. As can be seen from equation (C.4), when $C_t > C^{cons}(K_t)$, then $C_{t+1} \geq C_t$, such that above the locus, the arrows are pointing (weakly) upwards.[49]

Taken together the two loci and the associated dynamics, one can infer graphically that in the long run the economy converges either to the low or the high automation steady state. In fact, one can see that in case all three steady states exist the partial automation steady state must be unstable. Indeed, the partial automation steady state level of capital (which we denote $\bar{K}$) separates two regions of initial conditions. If initially, $K_0 < \bar{K}$, then the economy converges to the low automation steady state over time. Conversely, the economy converges to the high automation steady state over time in case initially $K_0 > \bar{K}$. Moreover, we verified numerically that the saddle

[49]The inequality is weak because there is a region where r^k_{t+1} is flat in the capital stock K_{t+1}, see equation (C.3). Hence even when $C_t > C^{cons}(K_t)$, implying that K_{t+1} lies below the level implied by $C_t = C^{cons}(K_t)$, this does not imply that the return to capital rises in this region. Hence consumption growth is still equal to zero.

path leading to the steady state is unique.

C.2 Local dynamics

Next, we study the local dynamics around the steady state(s). We show that under the conditions stated in Assumption 1, in case the three steady states exist, both the low and high automation steady states are locally stable and determinate, whereas the partial automation steady state is locally unstable.

The budget constraint $K_{t+1} = Y_t - C_t + (1-\delta)K_t$, in log-linear terms, is given by

$$K\hat{K}_{t+1} = Y\hat{Y}_t - C\hat{C}_t + (1-\delta)K\hat{K}_t$$

where variables without time subscripts denote steady state values of a variable and where hats indicate log-deviation from steady state. In turn, the Euler equation $C_{t+1}/C_t = \beta(r^k_{t+1} + 1 - \delta) + \xi C_{t+1}$ becomes

$$\hat{C}_{t+1} - \hat{C}_t = \beta r^k \hat{r}_{t+1} + \xi C\hat{C}_{t+1}.$$

We now make a case distinction according to the respective regimes.

High automation. Recall that in this case, $Y_t = \phi^h K_t^{J^h}$ as well as $r^k_{t+1} = \phi^h J^h K_{t+1}^{J^h-1}$. By using that $r^k = \phi^h J^h \bar{K}^{J^h-1}$ and $\beta(r^k + 1 - \delta) + \xi\bar{C} = 1$, we can write the system as

$$\hat{K}_{t+1} = (r^k + 1 - \delta)\hat{K}_t - \left(\frac{r^k}{J^h} - \delta\right)\hat{C}_t$$
$$\beta(r^k + 1 - \delta)\hat{C}_{t+1} = \beta r^k (J^h - 1)\hat{K}_{t+1} + \hat{C}_t.$$

By equation (11), Assumption 1, it holds that $i > -\delta(1 - J^h)$. From $r^k = i + \delta$, it then follows that the term $r^k/J^h - \delta$ is strictly positive.

We write this in matrix notation as

$$\begin{pmatrix} \hat{K}_{t+1} \\ \hat{C}_{t+1} \end{pmatrix} = \begin{pmatrix} r^k + 1 - \delta & -\left(\frac{r^k}{J^h} - \delta\right) \\ \frac{\beta r^k (J^h-1)(r^k+1-\delta)}{\beta(r^k+1-\delta)} & \frac{1-\beta r^k (J^h-1)\left(\frac{r^k}{J^h}-\delta\right)}{\beta(r^k+1-\delta)} \end{pmatrix} \begin{pmatrix} \hat{K}_t \\ \hat{C}_t \end{pmatrix}.$$

This system features one forward looking, one backward looking variable. Hence, the condition for local determinacy is that one eigenvalue lies inside, one outside the unit circle. The determinant is given by

$$det = (r^k + 1 - \delta)\frac{1 - \beta r^k (J^h - 1)\left(\frac{r^k}{J^h} - \delta\right)}{\beta(r^k + 1 - \delta)} + \left(\frac{r^k}{J^h} - \delta\right)\frac{\beta r^k (J^h - 1)(r^k + 1 - \delta)}{\beta(r^k + 1 - \delta)} = \frac{1}{\beta}.$$

In turn, the trace is

$$tr = r^k + 1 - \delta + \frac{1 - \beta r^k (J^h - 1)\left(\frac{r^k}{J^h} - \delta\right)}{\beta(r^k + 1 - \delta)} > 0.$$

The trace is strictly positive, because of $J^h < 1$. Now recall that the eigenvalues can be obtained by using the formula

$$\frac{tr}{2} \pm \sqrt{\frac{tr^2}{4} - det}.$$

From this, we can infer that a sufficient condition for one eigenvalue within, one outside the unit circle is given by $det < tr - 1$.[50] Hence, the following condition must be satisfied

$$\frac{1}{\beta} < r^k + 1 - \delta + \frac{1 - \beta r^k (J^h - 1)\left(\frac{r^k}{J^h} - \delta\right)}{\beta(r^k + 1 - \delta)} - 1,$$

which can be written in the form of a quadratic equation in r^k

$$f(r^k) \equiv \frac{\beta}{J^h}(r^k)^2 + r^k(-\beta\delta + \beta(1-\delta) + \beta\delta(J^h - 1) - 1) + \delta(1 - \beta(1-\delta)) > 0.$$

Now recall again that $r^k = i + \delta$. This allows us to rewrite the previous equation in terms of i, as follows

$$\tilde{f}(i) \equiv i^2 + i\left[\delta(1 - J^h)(2 - J^h) - \frac{1-\beta}{\beta}J^h\right] + \delta^2(1 - J^h)^2 > 0.$$

But this is exactly condition (A.2) from Appendix A, stated there in terms of $g(i) > 0$. In Appendix A, we had also derived a sufficient condition for $g(i) > 0$ for all i in the permissible range. This sufficient condition was given by equation (12), Assumption 1. Recall that this condition implied that the steady state labor demand curve under high automation is strictly downward sloping. Therefore, as we have just shown, under the *same* condition the steady state associated with high automation is also locally determinate.

Partial automation. This case is much easier to analyze because output in this regime is

[50] To see this, we have to go through a few steps. Notice first that this condition implies that the discriminant is positive, hence that both eigenvalues are real. This is because

$$\frac{tr^2}{4} - det > \frac{tr^2}{4} - (tr - 1) = \frac{(tr-2)^2}{4} > 0.$$

Moreover, note that both eigenvalues must be strictly positive, because both tr and det are strictly positive.

Second, we infer that the larger of the two eigenvalues is larger than 1. This is because $\frac{tr}{2} + \sqrt{\frac{tr^2}{4} - det}$ is strictly increasing in tr (as long as this eigenvalue is positive, which we have argued before is the case). Hence we can write

$$\frac{tr}{2} + \sqrt{\frac{tr^2}{4} - det} > \frac{det+1}{2} + \sqrt{\frac{(det+1)^2}{4} - det} = \frac{det+1}{2} + \sqrt{\frac{(det-1)^2}{4}} = det > 1,$$

where we have used that $det = 1/\beta > 1$.

Third and last, we infer that the smaller of the two eigenvalues is smaller than 1. This is because $\frac{tr}{2} - \sqrt{\frac{tr^2}{4} - det}$ is strictly increasing in det. Hence we can write

$$\frac{tr}{2} - \sqrt{\frac{tr^2}{4} - det} < \frac{tr}{2} - \sqrt{\frac{tr^2}{4} - (tr-1)} = \frac{tr}{2} - \sqrt{\frac{(tr-2)^2}{4}} = 1.$$

The last equality follows because $tr > 2$, which follows from combining our assumption $tr > det+1$ and $det = 1/\beta > 1$.

linear, given by $Y_t = \gamma^k K_t + \gamma^l \bar{L}$, implying that $r_t^k = \gamma^k$, a constant. Hence we obtain the system

$$\begin{aligned} \hat{K}_{t+1} &= (\gamma^k + 1 - \delta)\hat{K}_t - (\gamma^k - \delta)\hat{C}_t \\ \beta(\gamma^k + 1 - \delta)\hat{C}_{t+1} &= \hat{C}_t. \end{aligned}$$

In matrix notation, this becomes

$$\begin{pmatrix} \hat{K}_{t+1} \\ \hat{C}_{t+1} \end{pmatrix} = \begin{pmatrix} \gamma^k + 1 - \delta & -(\gamma^k - \delta) \\ 0 & \frac{1}{\beta(\gamma^k+1-\delta)} \end{pmatrix} \begin{pmatrix} \hat{K}_t \\ \hat{C}_t \end{pmatrix}.$$

The eigenvalues are $\gamma^k + 1 - \delta$ and $\frac{1}{\beta(\gamma^k+1-\delta)}$. Because of $\gamma^k > \delta$ (see equation (12), Assumption 1), the first eigenvalue is explosive. Now recall that, in this steady state, $i = \bar{i} = \gamma^k - \delta$. Hence the second eigenvalue can be written as $\frac{1}{\beta(1+i)}$. Because of $i < 1/\beta - 1$ (see equation (11), Assumption 1), the second eigenvalue is also explosive. Because the model features one forward-looking variable, but two explosive eigenvalues, it follows that there is local instability.

Low automation. This case is symmetric as the case of high automation, with J^l replacing J^h. Hence the condition for local determinacy can be written as

$$\tilde{f}(i) \equiv i^2 + i\left[\delta(1 - J^l)(2 - J^l) - \frac{1-\beta}{\beta}J^l\right] + \delta^2(1 - J^l)^2 > 0.$$

As we have argued in Appendix A, condition (12) is sufficient for this to be positive for all i in the permissible range.

D Model with smooth technology

In this appendix, we consider a version of the model in which, as in Acemoglu and Restrepo (2018), labor productivity varies smoothly across production tasks. We will show that, even in this case, the labor demand curve may be non-monotonic.

D.1 Environment

The only change with respect to the baseline model concerns the production technology. As before, we denote γ_j^l the productivity of labor and γ_j^k the productivity of capital, where $j \in [0, 1]$ is the range of intermediate goods - or tasks. The production technology of firm j is given by

$$y_{j,t} = \gamma_j^k k_{j,t} + \gamma_j^l l_{j,t}.$$

Following Acemoglu and Restrepo (2018), we assume that γ_j^l/γ_j^k is strictly increasing in j, that is, we assume that labor has a comparative advantage at higher index tasks. Specifically, γ_j^l is given by

$$\gamma_j^l = \gamma^l e^{\lambda j},$$

where $\gamma^l > 0$ and $\lambda > 0$. Moreover, we again consider technological constraints on automation by assuming that tasks in the range $j > J^h$, where $J^h < 1$ is a positive constant, cannot be automated. In this region, therefore, $\gamma_j^k = 0$. Finally, we assume that for $j < J^h$, $\gamma_j^k = \gamma_k$, a constant.[51]

As in the baseline model, firms in industry j want to automate if and only if $r_t^k/\gamma_j^k < w_t/\gamma_j^l$. Denote by J_t^* the threshold such that all firms below J_t^* produce with capital and all firms above J_t^* produce with labor. When $r_t^k/\gamma_j^k < w_t/\gamma_j^l$ for all $j \leq J^h$, then $J_t^* = J^h$. Otherwise, J_t^* is determined by the equation

$$\frac{r_t^k}{\gamma^k} = \frac{w_t}{\gamma^l} e^{-\lambda J_t^*}. \tag{D.1}$$

As before, the aggregate production function is

$$\log Y_t = \int_0^1 \log y_{j,t} dj.$$

As a result, demand for intermediate good j obeys

$$p_{j,t} y_{j,t} = Y_t.$$

Assuming perfect competition in the production of intermediate goods, their prices are thus given by

$$p_{j,t} = \begin{cases} \frac{r_t^k}{\gamma^k} & \text{if } j \leq J_t^* \\ \frac{w_t}{\gamma_j^l} & \text{if } j > J_t^*. \end{cases}$$

This reveals that firms producing with labor charge different prices, because they have different productivities (unlike in the baseline model, where $p_{j,t}$ is the same for all firms producing with labor in equilibrium). However, it still turns out that *labor demand* for all these firms is identical in equilibrium. This is because

$$Y_t = p_{j,t} y_{j,t} = \frac{w_t}{\gamma_j^l} \gamma_j^l l_{j,t} = w_t l_{j,t},$$

such that $l_{j,t} = l_t = Y_t/w_t$ for all $j > J_t^*$. Intuitively, more productive firms face a higher demand in exactly such a way that they employ the same amount of labor in equilibrium.

Inserting, we may now derive the aggregate production function

$$\log Y_t = J_t^* \log\left(\frac{\gamma^k K_t}{J_t^*}\right) + \int_{J_t^*}^1 \log \gamma_j^l dj + (1 - J_t^*) \log\left(\frac{L_t}{1 - J_t^*}\right),$$

[51] In the baseline model, instead, we had assumed that γ_j^l/γ_j^k is a step function which could take only three values, $[0, \gamma^l/\gamma^k, +\infty]$. The benefit of using this assumption was that we could obtain our main insights analytically. In the current model, instead, γ_j^l/γ_j^k is smooth in the region where $j < J^h$, before it jumps to plus infinity when $j > J^h$.

where we have used that $k_{j,t}$ and $l_{j,t}$ are the same for all firms j in the respective interval and replaced $J_t^* k_t = K_t$ and $(1 - J_t^*) l_t = L_t$. Rearranging we obtain

$$Y_t = e^{\int_{J_t^*}^1 \log \gamma_j^l dj} \left(\frac{\gamma^k K_t}{J_t^*} \right)^{J_t^*} \left(\frac{L_t}{1 - J_t^*} \right)^{1 - J_t^*}.$$

Inserting the functional form for γ_j^l, this becomes

$$Y_t = e^{\frac{\lambda}{2}(1 - (J_t^*)^2)} \left(\frac{\gamma^k K_t}{J_t^*} \right)^{J_t^*} \left(\frac{\gamma^l L_t}{1 - J_t^*} \right)^{1 - J_t^*}.$$

Finally, the factor shares are the same as in the baseline model

$$w_t L_t = (1 - J_t^*) Y_t$$

as well as

$$r_t^k K_t = J_t^* Y_t.$$

D.2 Monetary policy and labor demand

As in the baseline model, we now trace the labor demand curve against the interest rate in steady state.

First, as we have not changed the consumption block of the model, steady state consumption is still given by

$$C = \frac{1 - \beta(1 + i)}{\xi}.$$

Moreover, from the resource constraint, we still have the market clearing condition

$$Y = C + \delta K = \frac{1 - \beta(1 + i)}{\xi \left(1 - \frac{\delta J^*}{i + \delta} \right)},$$

where we used $r^k K = J^* Y$ and $i = r^k - \delta$. In turn, the production side implies

$$\begin{aligned} Y &= e^{\frac{\lambda}{2}(1 - (J^*)^2)} \left(\frac{\gamma^k K}{J^*} \right)^{J^*} \left(\frac{\gamma^l L}{1 - J^*} \right)^{1 - J^*} \\ &= e^{\frac{\lambda}{2}(1 + J^*)} \left(\frac{\gamma^k}{r^k} \right)^{\frac{J^*}{1 - J^*}} \frac{\gamma^l L}{1 - J^*} \end{aligned} \tag{D.2}$$

where we again used $r^k K = J^* Y$. Combining and solving for L, this implies

$$L = \frac{1 - J^*}{\gamma^l e^{\frac{\lambda}{2}(1 + J^*)}} \left(\frac{i + \delta}{\gamma^k} \right)^{\frac{J^*}{1 - J^*}} \frac{1 - \beta(1 + i)}{\xi \left(1 - \frac{\delta J^*}{i + \delta} \right)}, \tag{D.3}$$

where we have replaced $r^k = i + \delta$. This is almost the same expression as in the baseline model, with two differences.

First, there is an additional productivity term, given by $e^{\frac{\lambda}{2}(1+J^*)}$. Second, and more fundamentally, the level of automation J^*, using equation (D.1), is given by

$$J^* = \min\left(J^h, \frac{\log(w/\gamma^l) - \log(r^k/\gamma^k)}{\lambda}\right).$$

Using $r^k = i + \delta$, $wL = (1 - J^*)Y$ and equation (D.2) we can write

$$\begin{aligned}
\frac{\log(w/\gamma^l) - \log(r^k/\gamma^k)}{\lambda} &= \frac{\log\left((1-J^*)e^{\frac{\lambda}{2}(1+J^*)}\left(\frac{\gamma^k}{i+\delta}\right)^{\frac{J^*}{1-J^*}}\frac{1}{1-J^*}\right) - \log\left(\frac{i+\delta}{\gamma^k}\right)}{\lambda} \\
&= \frac{\frac{\lambda}{2}(1+J^*) + \frac{J^*}{1-J^*}\log\left(\frac{\gamma^k}{i+\delta}\right) - \log\left(\frac{i+\delta}{\gamma^k}\right)}{\lambda} \\
&= \frac{\frac{\lambda}{2}(1+J^*) + \frac{1}{1-J^*}\log\left(\frac{\gamma^k}{i+\delta}\right)}{\lambda}.
\end{aligned}$$

Rearranging for J^* yields the final expression

$$J^* = \min\left(J^h, 1 - \sqrt{\frac{2}{\lambda}\log\left(\frac{i+\delta}{\gamma^k}\right)}\right). \tag{D.4}$$

Equation (D.4) is a negative relationship between J^* and the policy rate i. That is, the degree of automation rises when the interest rate falls (up to the point where $J^* = J^h$, that is, firms fully exploit their automation potential).

Taking stock, for a given level of automation J^*, equation (D.3) is a strictly negative relationship between labor demand and the interest rate, under a similar assumption as the one that we state in the main text (see Assumption 1).[52] At the same time, as it can be verified, labor demand (D.3) is strictly decreasing in J^*, as more automation reduces the demand for labor.[53] Last, because J^* is strictly declining in the interest rate from equation (D.4), this implies that also in the current model, there is a force which raises labor demand when the interest rate is increased.

Figure 11 shows an example of the labor demand curve in this version of the model.[54] In the left panel, we see that the labor demand curve has a backward-bending part, due to the fact that firms start replacing workers with machines when the interest rate is low. This can also be seen in

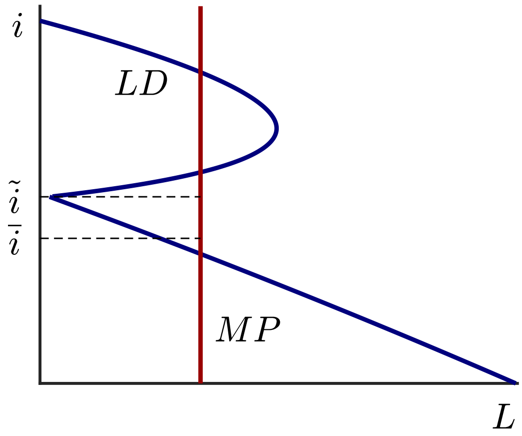
[52]In the baseline model, for given $J^* \leq J^h$, labor demand slopes downward provided that

$$\delta > \frac{1}{2}\frac{1-\beta}{\beta}\frac{J^h}{(1-J^h)(2-J^h)},$$

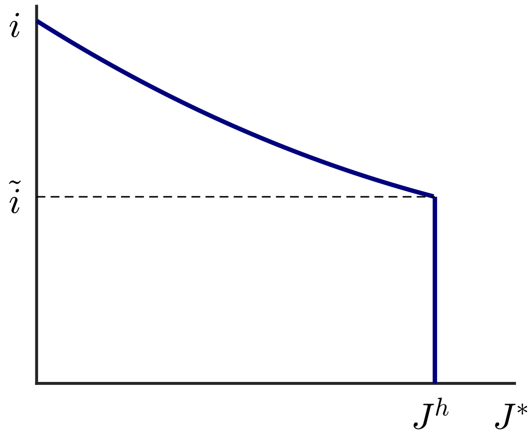
recall Appendix A. In the current model, the condition is slightly different because of the additional factor $e^{\frac{\lambda}{2}(1+J^*)}$ that appears in the labor demand curve.

[53]In fact, as in the baseline model, there is a parametric condition for this to true, see Section 5.3 and particularly condition (4). As explained in this section, a higher J^* may also raise labor demand due to the implied higher investment. However, for most (plausible) parameter values, labor demand is indeed declining in J^*.

[54]To be clear, in this version of the model the labor demand curve could also be monotonically downward sloped. It becomes backward bending when J^h is sufficiently large, so that the scope for automation is sufficiently high.



(a) Labor demand.



(b) Degree of automation.

Figure 11: Steady state labor demand curve and automation in the extended model.

the right panel, which plots the equilibrium degree of automation against the interest rate. As we can see, when the interest rate falls below a critical threshold $\tilde{i}$, then the economy is characterized by high automation.[55] This is the identical economic force that operates in the baseline model. The only difference is that in the current model, the degree of automation and the labor demand curve are both changing smoothly in the interest rate, at least in the region where $J^* < J^h$.

As in Figure 2, the economy may or may not be characterized by steady state multiplicity. In the left panel of Figure 11, we show an example where the labor demand curve intersects three times with the level of full employment, implying that the economy has again three full employment steady states.

References

Acemoglu, Daron and Pascual Restrepo (2018) "The race between man and machine: Implications of technology for growth, factor shares, and employment," *American Economic Review*, Vol. 108, No. 6, pp. 1488–1542.

——— (2019) "Automation and new tasks: How technology displaces and reinstates labor," *Journal of Economic Perspectives*, Vol. 33, No. 2, pp. 3–30.

Acharya, S., J. Bengui, K. Dogra, and S. L. Wee (2021) "Slow Recoveries and Unemployment Traps: Monetary Policy in a Time of Hysteresis," *Economic Journal*.

Aghion, P., B. Jones, and F. Jones (2017) "Artificial Intelligence and Economic Growth," NBER working paper.

[55] For comparison, we also draw the level of interest rates which triggered the move to high automation in the baseline model, given by $\bar{i} \equiv \gamma^k - \delta$ (see equation (17)). In the current model, this happens at the level of interest rates $\tilde{i} = \gamma^k e^{\frac{\lambda}{2}(1-J^h)^2} - \delta > \bar{i}$.

Aiyagari, S Rao (1994) "Uninsured idiosyncratic risk and aggregate saving," *The Quarterly Journal of Economics*, Vol. 109, No. 3, pp. 659–684.

Barattieri, Alessandro, Susanto Basu, and Peter Gottschalk (2014) "Some Evidence on the Importance of Sticky Wages," *American Economic Journal: Macroeconomics*, Vol. 6, No. 1, pp. 70–101.

Benhabib, Jess, Stephanie Schmitt-Grohé, and Martın Uribe (2001) "The Perils of Taylor Rules," *Journal of Economic Theory*, Vol. 96, No. 1, pp. 40–69.

Benigno, Gianluca and Luca Fornaro (2018) "Stagnation traps," *Review of Economic Studies*, Vol. 85, No. 3, pp. 1425–1470.

Bertolotti, F., A. Gavazza, and A. Lanteri (2021) "Dynamics of Expenditures on Durable Goods: the Role of New-Product Quality," Duke University working paper.

Caballero, R. and E. Farhi (2018) "The Safety Trap," *Review of Economic Studies*, Vol. 85, No. 1, pp. 223–274.

Caballero, R., E. Farhi, and M. Hammour (2006) "Speculative Growth: Hints from the U.S. Economy," *American Economic Review*, Vol. 96, No. 4, pp. 1159–1192.

Cerra, V., A. Fatas, and S. Saxena (2021) "Hysteresis and Business Cycles," *Journal of Economic Literature.*

Christiano, Lawrence J and Martin Eichenbaum (1992) "Liquidity effects and the monetary transmission mechanism," *The American Economic Review*, Vol. 82, No. 2, pp. 346–353.

Del Negro, Marco, Gauti Eggertsson, Andrea Ferrero, and Nobuhiro Kiyotaki (2017) "The great escape? A quantitative evaluation of the Fed's liquidity facilities," *American Economic Review*, Vol. 107, No. 3, pp. 824–57.

Eggertsson, Gauti B, Neil R Mehrotra, and Jacob A Robbins (2019) "A model of secular stagnation: Theory and quantitative evaluation," *American Economic Journal: Macroeconomics*, Vol. 11, No. 1, pp. 1–48.

Fehr, E. and L. Goette (2005) "Robustness and Real Consequences of Nominal Wage Rigidity," *Journal of Monetary Economics*, Vol. 52, No. 4, pp. 779–804.

Fisher, Jonas DM (2015) "On the structural interpretation of the Smets–Wouters "risk premium" shock," *Journal of Money, Credit and Banking*, Vol. 47, No. 2-3, pp. 511–516.

Fornaro, L. and M. Wolf (2020) "The Scars of Supply Shocks," CREI working paper.

Furlanetto, F., A. Lepetit, K. Haugeland, O. Robstad, and P. Ulvedal (2021) "Estimating Hysteresis Effects," Norges Bank working paper.

Galí, Jordi (2009) *Monetary Policy, Inflation, and the Business Cycle: An Introduction to the New Keynesian Framework*: Princeton University Press.

Garga, Vaishali and Sanjay R. Singh (2020) "Output hysteresis and optimal monetary policy," *Journal of Monetary Economics*.

Gertler, Mark, Christopher Huckfeldt, and Antonella Trigari (2020) "Unemployment fluctuations, match quality, and the wage cyclicality of new hires," *The Review of Economic Studies*, Vol. 87, No. 4, pp. 1876–1914.

Gertler, Mark and Peter Karadi (2011) "A model of unconventional monetary policy," *Journal of monetary Economics*, Vol. 58, No. 1, pp. 17–34.

Gottschalk, P. (2005) "Downward Nominal-Wage Flexibility: Real or Measurement Error?" *Review of Economics and Statistics*, Vol. 87, No. 3, pp. 556–568.

Ilzetzki, E. (2021) "Learning by Necessity: Government Demand, Capacity Constraints, and Productivity Growth," LSE working paper.

Jaimovich, N., I. Saporta-Eksten, H. Siu, and Y. Yedid-Levi (2020) "The Macroeconomics of Automation: Data, Theory and Policy Analysis," University of Zurich working paper.

Jordà, Òscar, Sanjay R Singh, and Alan M Taylor (2020) "The Long-Run Effects of Monetary Policy," Working Paper 26666, National Bureau of Economic Research.

Konczal, M. and J.W. Mason (2021) "How to Have a Roaring 2020s (Without Wild Inflation)," New York Times, June 15.

Korinek, A. and P. Trammel (2020) "Economic Growth under Transformative AI," NBER working paper.

Kurz, Mordecai (1968) "Optimal economic growth and wealth effects," *International Economic Review*, Vol. 9, No. 3, pp. 348–357.

Mian, Atif R, Ludwig Straub, and Amir Sufi (2021) "Indebted demand," *Quarterly Journal of Economics*.

Michaillat, Pascal and Emmanuel Saez (2021) "Resolving New Keynesian anomalies with wealth in the utility function," *Review of Economics and Statistics*, Vol. 103, No. 2, pp. 197–215.

Michau, Jean-Baptiste (2018) "Secular stagnation: Theory and remedies," *Journal of Economic Theory*, Vol. 176, pp. 552–618.

Moll, Benjamin, Lukasz Rachel, and Pascual Restrepo (2021) "Uneven Growth: Automation's Impact on Income and Wealth Inequality," LSE working paper.

Moran, Patrick and Albert Queralto (2018) "Innovation, productivity, and monetary policy," *Journal of Monetary Economics*, Vol. 93, pp. 24 – 41.

Nakamura, Emi and Jón Steinsson (2018) "High-frequency identification of monetary non-neutrality: the information effect," *The Quarterly Journal of Economics*, Vol. 133, No. 3, pp. 1283–1330.

Olivei, G. and S. Tenreyro (2007) "The Timing of Monetary Policy Shocks," *The American Economic Review*, Vol. 97, No. 3, pp. 636–663.

Pessoa, João Paulo and John Van Reenen (2014) "The UK productivity and jobs puzzle: does the answer lie in wage flexibility?" *The Economic Journal*, Vol. 124, No. 576, pp. 433–452.

Restrepo, Pascual (2015) "Skill mismatch and structural unemployment," Boston University working paper.

Sandbu, Martin (2020) *The economics of belonging: A radical plan to win back the left behind and achieve prosperity for all*: Princeton University Press.

——— (2021) "Beware the ketchup-bottle economy."

Schmitt-Grohé, Stephanie and Martín Uribe (2017) "The Making of a Great Contraction with a Liquidity Trap and a Jobless Recovery," *American Economic Journal: Macroeconomics*, Vol. 9, No. 1, pp. 165–204.

Summers, L. (2021) "The Biden stimulus is admirably ambitious. But it brings some big risks, too.," Washington Post, February 4.

Summers, Lawrence (2016) "The Age of Secular Stagnation," Foreign Affairs, February 15.

Zeira, J. (1998) "Workers, Machines and Economic Growth," *Quarterly Journal of Economics*, Vol. 113, pp. 1091–1117.