PLAY FOR THE RICH AND WORK FOR THE POOR?
THE OPTIMAL DISTRIBUTION OF SAVING AND WORK IN THE HETEROGENEOUS AGENTS NEOCLASSICAL GROWTH MODEL

Akshay Shanker and Martin Wolf[†]

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In an economy with un-insurable idiosyncratic labor income risk, how should saving and work hours be distributed across income and wealth? To answer this question, we study a planner who cannot complete asset markets, but dictates how much individuals must work and save. With a U.S. calibration, the planner raises total savings but not hours, driving up aggregate wages which re-distributes income toward the consumption-poor. Across the distribution, the productive (high wage earners) should save more; the wealthy should work more; the poor should keep their work hours unchanged and take advantage of the indirect transfers from higher wages.

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[†]Shanker: School of Economics, University of New South Wales (UNSW), Sydney, CEPAR and CAMA, Australian National University, a.shanker@unsw.edu.au. Wolf: University of St. Gallen and CEPR, martin.wolf@unisg.ch. We thank Dan Cao, Warwick McKibbin, John Stachurski, David Stern, Martin Thompson and in particular, Per Krusell, for guidance on our project. The project was undertaken with the assistance of resources and services from the National Computational Infrastructure (NCI), which is supported by the Australian Government. We also acknowledge support from the UNSW Resource Allocation Scheme managed by Research Technology Services at UNSW Sydney. Code for the paper can be accessed at `https://github.com/akshayshanker/workandplay`.

1. INTRODUCTION

Keynes remarked how, as a result of improvements in productivity, future generations would be so rich that they would not have to work. Keynes was referring to the income effect implied by higher wages. However, we have seen in past decades how a substitution effect has countered the income effect; in the United States (U.S.), average work hours in past decades have remained stable (Boppart and Krusell, 2020).[1] While average hours worked have not declined, there have been important trends in the *distribution* of hours worked in the U.S. during the same time period. Hours worked by productive workers (high wage earners) have remained stable, but hours worked by low skilled individuals have declined, leading to higher earnings inequality (Boppart and Ngai, 2021). Moreover, the rise in earnings inequality has also been suggested to contribute to recent increases in wealth inequality (Benhabib et al., 2017). Motivated by these facts, in this paper we bring up some normative questions. What *should* be the distribution of savings and hours worked? For example, should the low wage earners work and save more? Should the high wage earners work and save less? How would the resulting general equilibrium effects shape welfare across the distribution? This paper begins to analyse such questions by studying constrained efficiency in the Aiyagari-Huggett model with endogenous work hours.

Our model economy is populated by a large number of households (agents) and firms. Firms produce a homogeneous consumption good by employing capital and labor, which households provide. Households receive un-insurable idiosyncratic labor productivity shocks, which generates a non-trivial distribution of income, consumption and wealth. As shown by Dávila et al. (2012), the competitive equilibrium in this type of model is *constrained inefficient*, implying aggregate utilitarian welfare could be improved if individual agents depart from purely self-interested optimization. The equilibrium is constrained inefficient due to the pecuniary externality of each agents' decisions on aggregate interest and wage rates, which leads to a breakdown of the first welfare theorem because insurance markets for idiosyncratic labor market risk are incomplete.[2] We extend the analysis of Dávila

[1]In contrast, outside of the U.S., work hours have trended slightly downwards as the income effect has slightly dominated the substitution effect (Boppart and Krusell, 2020).

[2]Constrained efficiency is an efficiency concept which goes back to previous papers by Stiglitz (1982) and Geanakoplos et al. (1990), but was only recently applied to the macroeconomic growth literature by Dávila et al. (2012), Nuño and Moll (2018) and Park (2018). In contrast to the first best planner who can complete markets / reallocate wealth directly between agents, a constrained planner faces the same budget constraints as the private agents; that is, the constrained planner faces the same incomplete insurance markets. While a constrained planner cannot complete markets, the

et al. (2012), who study an economy in which work hours are exogenous, to a setting in which work hours are endogenous. The extension to incorporate work effort allows us to study simultaneously optimal saving and work hours across the distribution.

Our first main result concerns macroeconomic aggregates. In an economy roughly matching U.S. income and wealth inequality, the constrained planner almost doubles the aggregate capital stock relative to the competitive equilibrium, a finding consistent with Dávila et al. (2012). However, the constrained planner only slightly increases aggregate work hours. Intuitively, in our calibrated economy the consumption-poor rely mostly on labor income. By raising the capital stock, the planner raises aggregate utilitarian welfare by increasing aggregate wage rates which re-distributes income toward the consumption-poor. In contrast, a rise in labor supply depresses wages, which reduces welfare by re-distributing income away from the consumption-poor. Because the capital stock is larger, interest rates are also lower in the constrained efficient optimum. However, the consumption-rich are the ones who rely mostly on interest income, and their loss in welfare is not fully offset by the gain in welfare of the consumption-poor due to the higher level of wages.

Our second main result concerns saving and work hours across the distribution. Starting with saving, we find that for a given level of wealth, the productive (high wage earners) should increase their saving while the unproductive should reduce their saving. Moreover, the rise in saving by the productive far offsets the reduction in saving by the unproductive which explains that on aggregate, savings (and therefore the capital stock) increase. Turning to work hours, for a given level of productivity, the wealthy should increase their work hours while the poor should keep their work hours unchanged. To see why, note how for a given level of productivity, the rich work less than the poor in the no-intervention economy because of a wealth effect. By making the rich work more, the planner re-alignes work hours across the asset distribution. Interestingly, the intervention thus has a flavor of the first best (complete markets), where work hours are completely independent of wealth. In summary, in the constrained efficient optimum, agents with the same level of productivity work about the same number of hours independent of their asset state (a result which answers the question posed by our title “play for the rich

constrained planner can dictate how much agents work and save. By internalizing the pecuniary externality of each agents’ saving and work hours on aggregate wage and interest rates, the planner can increase aggregate utilitarian welfare.

and work for the poor?" in the negative).[3,4]

In a final part of the paper, we show the planner increases aggregate utilitarian welfare not only through re-distribution (making the consumption-poor better off), but also through increasing average consumption insurance. To do so, we compute consumption insurance coefficients in our economy, along the lines of Kaplan and Violante (2010). We find agents are on average better insured against idiosyncratic income shocks in the constrained efficient optimum. Moreover, we also show the higher consumption insurance runs through precautionary savings, while changes in work hours are not used to cushion poor productivity shocks. In addition to generating a favorable re-distribution, a higher stock of aggregate saving thus also serves to increase agents' consumption insurance.

Related literature

The Aiyagari-Huggett model with labor-leisure choice has been studied before by Pijoan-Mas (2006) and Marcet et al. (2007). Our model draws directly from Pijoan-Mas, and also our baseline calibration is directly taken from this paper. The question asked by Pijoan-Mas is how the work hours distribution changes by moving from a complete markets to an incomplete markets economy. One result is that the correlation between work hours and productivity declines, because poor agents (who tend to be unproductive) work long hours due to a negative wealth effect. In contrast to both papers, who have only studied the competitive equilibrium, we study the constrained efficient optimum in this class of models.

The issue of work hours across the distribution has also been taken up by Boppart et al. (2017), who analyze how work hours will change as productivity grows, albeit in a complete markets setting. One of their main results is consistent with the analysis here. In the competitive equilibrium, the ex-ante asset rich work less and exit the work-force first due to strong wealth effects. By contrast, a utilitarian planner prefers these agents to keep working. The main difference between the current paper and Boppart et al. (2017) is that we consider an incomplete market setting

[3] Bear in mind the result is conditional on the level of productivity. Unconditionally, work hours of a rich agent may lie above those of a poor agent even in the competitive equilibrium. This happens because the rich tend to be the productive, and because the productive work more than the unproductive.

[4] In the text, we also discuss the change in work hours along the productivity dimension. While the productive work more than the unproductive even in the no-intervention economy (for a given level of wealth), the planner in fact makes work hours even more dependent on productivity. Again, this has a flavor of the first best, where work hours are determined exclusively by the level of productivity.

where the planner corrects for price externalities. On the other hand, we do not consider long-run productivity growth implications on the distribution of hours worked.

At a conceptual level, this paper is linked to the literature studying price (or pecuniary) externalities and their implications for regulation. The externality in this paper is that households and firms take interest rates and wages as given, but both are determined by households' and firms' decisions in general equilibrium. When markets are complete and there are no other frictions, this type of externality has no implications for regulation, an implication of the first welfare theorem. Instead, in our incomplete market setting this externality makes the equilibrium (constrained) inefficient.[5] Other frictions that induce a breakdown of the first welfare theorem resulting from the same externality are nominal and financial frictions, among others. For example, Farhi and Werning (2016) and Wolf (2020) study the pecuniary externality in a setting with nominal rigidities, and Bianchi (2016) does the same in a setting with financial frictions.

Last, our paper relates to a growing literature on Ramsey taxation problems in incomplete market models such as Nuño and Thomas (2017), Chen et al. (2017), Bhandari et al. (2017), Le Grand and Ragot (2017) and Krueger et al. (2021). Unlike the planner we study, the Ramsey planner can re-distribute wealth across agents and thereby partially complete markets and also imposes simpler policy instruments (such as flat taxes). From an applied standpoint, we acknowledge our focus on zero net transfers across agents appears overly restrictive. However, we are not motivated here by questions of specific policy design *per se*; rather, the structure of the constrained planner's problem allows us to explain transparently one key channel through which policy interventions in incomplete markets models affect welfare — the pecuniary externality.

2. AIYAGARI-HUGGETT MODEL WITH LABOR-LEISURE CHOICE

This section characterises the competitive equilibrium and constrained efficient optimum of the Aiyagari-Huggett model with endogenous labor-leisure choice, following in particular, the characterisation by Dávila et al. (2012). In the interest of space, we remain deliberately brief and somewhat informal on mathematical notation, in particular, we characterise equilibria in terms of a recursive problem and

[5]See also Dávila et al. (2012), Nuño and Thomas (2017) and Park (2018), who also study constrained efficiency in an incomplete market setting. While Dávila et al. (2012) and Nuño and Thomas (2017) do not study the implications for work hours, Park (2018) develops a model of work hours dispersion by incorporating human capital investment.

do not build up the recursive structure from the underlying sequence problem. A complete mathematical characterization, including construction of the agent space, the relationship between sequential and recursive equilibrium and a proof of existence of constrained optima for the model is given by the earlier working paper version of our paper (Shanker, 2018).

2.1. *Competitive equilibrium*

Time periods are discrete and a continuum of infinitely lived households (agents) of measure one populate the economy. Agents receive an idiosyncratic labor productivity shock each period and are endowed with one unit of hours each period. After the realization of the labor productivity shock, agents decide the proportion of their hours to spend on work and leisure, respectively, and also how much to save. Households do not have access to state-contingent assets, but can accumulate non-negative amounts of real capital by saving.

The per-period utility function of an agent is given by

$$\nu(c,q) = \frac{c^{1-\gamma_c}}{1-\gamma_c} + A_l \frac{q^{1-\gamma_l}}{1-\gamma_l}, \qquad A_l > 0,\ \gamma_c > 1,\ \gamma_l > 0$$

where c is consumption q is leisure.

Let e represent the agents' idiosyncratic labor productivity shock; e takes on values in E, where $E \subset \mathbb{R}_+$. In the underlying sequence problem, e is drawn from a stationary Markov process with Markov kernel Q each period. Let x denote an agent's stock of assets; x takes on values in A, where $A = \mathbb{R}_+$. Let S, with $S = A \times E$ denote the agents' individual state space. The policy function instructing agents on next period assets will be denoted by h and the policy function instructing agents on leisure will be denoted by l. For a given aggregate state, which we describe below, both policy functions are defined on S.

We distinguish between the agents' individual states and the aggregate state of the economy, which is the joint distribution of agents' assets and shocks, denoted by μ. The aggregate capital stock, aggregate hours and aggregate labor supply are functions of the aggregate state, as follows

$$K(\mu) = \int\int x\mu(dx, de),$$

and

$$H(\mu, l) = \int\int (1 - l(x, e))\mu(dx, de)$$
$$L(\mu, l) = \int\int (1 - l(x, e))e\mu(dx, de)$$

Note how aggregate labor supply depends on how many hours are worked and by whom; an hour of work by someone who has received a good productivity shock increases effective labor supply more than an hour of work by someone with a poor labor productivity shock.

The aggregate state affects agents' decisions by determining prices agents face (wages and interest rates), through the firm problem. Specifically, the economy is populated by standard price taking firms with Cobb-Douglas production function $F(K, L) = AK^{\alpha}L^{1-\alpha}$, where $\alpha \in (0, 1)$. Hence prices in the economy are

$$r(\mu, l) = F_1(K(\mu), L(\mu, l)) - \delta, \qquad \delta \in (0, 1) \tag{1}$$

and

$$w(\mu, l) = F_2(K(\mu), L(\mu, l)) \tag{2}$$

where δ is depreciation.

We next describe the evolution of the aggregate state. Denote $\mathbb{Y}_h$ to be the space of asset policy functions and $\mathbb{Y}_l$ the space of leisure policy functions. Denote $\mathbb{M}$ as the space of joint distributions of assets and productivity shocks, μ. Define $\Phi \colon \mathbb{M} \times \mathbb{Y}_h \to \mathbb{M}$, as

$$\Phi(\mu, h)(B_A \times B_E) = \int\int \mathbb{1}_{B_A}\{h(x, e)\}Q(e, B_E)\mu(dx, de), \qquad B_A \times B_E \in \mathscr{B}(S)$$

where $\mathscr{B}(S)$ are the Borel sets of S. The evolution of the aggregate state—the joint distribution of assets and shocks— then follows:

$$\mu'(B_A \times B_E) = \Phi(\mu, h)(B_A \times B_E), \tag{3}$$

for each $B_A \times B_E \in \mathscr{B}(S)$, where we use recursive (prime) notation to define variables in the next period. In words, next period's joint distribution μ' will be the transition of μ jointly determined by the law of motion for endowment shocks e and individual agents' saving decisions through the policy function h.

Last, we define households' budget constraints. More precisely, we define the feasibility correspondence $\Theta\colon \mathbb{M} \times \mathbb{Y}_l \times S \twoheadrightarrow \mathbb{R}^2$, as

$$\Theta(\mu, l, s) = \left\{(x', q) \in A \times [0,1] \,|\, x' \leqslant (1 + r(\mu, l))x + w(\mu, l)(1 - q)e\right\},$$

where $s = (x, e)$. The feasibility correspondence implies non-negative consumption and the leisure choice falls between zero and one. Notice that agents' effective wage ew depends positively on agents' productivity shock. For this reason, we sometimes refer to productive agents as the high wage earners in this paper.

We are now in the position to state the definition of equilibrium.

DEFINITION 2.1 (**Recursive Competitive Equilibrium**)

An RCE is a value function W and policy functions $\{h, l\}$, where $W\colon \mathbb{M} \times S \to \mathbb{R}$, $h\colon \mathbb{M} \times S \to \mathbb{R}$ and $l\colon \mathbb{M} \times S \to \mathbb{R}$ such that:

1. the policy functions satisfy $h(\mu, s), l(\mu, s) \in \Theta(\mu, l(\mu, \cdot), s)$ for each $\mu \in \mathbb{M}$ and $s \in S$ (feasibility)
2. the policy and value functions satisfy (agent maximization)

   $$W(\mu, s) = \max_{(x',q) \in \Theta(\mu, l(\mu,\cdot), s)} v(c, q) + \beta \int W(\mu', (x', e'))\, Q(e, \mathrm{d}e'), \quad s \in S, \mu \in \mathbb{M}$$

   and

   $$h(\mu, s), l(\mu, s) = \operatorname*{argmax}_{(x',q) \in \Theta(\mu, l(\mu,\cdot), s)} v(c, q) + \beta \int W(\mu', (x', e'))\, Q(e, \mathrm{d}e'), \quad s \in S, \mu \in \mathbb{M}$$

   where $s = (x, e)$ and

   $$c = (1 + r(\mu, l(\mu, \cdot))x + w(\mu, l(\mu, \cdot))(1 - q)e - x'$$

   where $\beta \in (0, 1)$ is a discount factor, and where μ' and $\Theta(\mu, l(\mu, \cdot), s)$ are taken as given in the maximization
3. the conditions (1) and (2) hold (market clearing)
4. the joint distribution μ satisfies (3) (distribution consistency).

2.2. *Constrained efficient optimum*

In the economy described above, what is the optimal distribution of saving and hours worked? We answer this question by studying a constrained planning problem, along the lines of Dávila et al. (2012). The constrained planner cannot complete asset markets, but must improve welfare through correcting pecuniary externalities of each agents' decisions on aggregate interest and wage rates.[6]

As in the description of the competitive equilibrium, we omit the underlying sequence problem and immediately switch to recursive notation. The recursive constrained planner's state space is the space of joint distributions $\mathbb{M}$ and the planner's action space is the space of policy functions $\mathbb{Y} = \mathbb{Y}_h \times \mathbb{Y}_l$. In turn, the planner's feasibility correspondence, denoted by $\Lambda\colon \mathbb{M} \twoheadrightarrow \mathbb{Y}$, is given by

$$\Lambda(\mu) = \left\{(h,l) \in \mathbb{Y} \;\middle|\; 0 \leqslant h(x,e) \leqslant \big(1 + r(\mu,l)\big)x + w(\mu,l)e\big(1 - l(x,e)\big)\,,\, 0 \leqslant l \leqslant 1\right\}.$$

Intuitively, the planner dictates directly agents' policy functions h and l, with the requirement that agents' budget constraints hold, that savings and consumption are positive and that leisure lies in between zero and one.

We define the constrained planner's per-period payoff, u, with $u\colon \operatorname{Gr}\Lambda \to \mathbb{R}_+ \cup \{-\infty\}$, as the integral of utility across the distribution of agents

$$u(\mu,h,l) = \int\int \nu\big((1 + r(\mu,l))x + w(\mu,l)e(1 - l(x,e)) - h(x,e), l(x,e)\big)\mu(dx,de)$$

In doing so, we focus on a utilitarian planner that puts equal weight on the utility of each agent.

Since we define the space of distributions $\mathbb{M}$ as the planner's state space and the space of *individual* policy functions $\mathbb{Y}$ as the planner's control space, the planner's optimal policy function for hours and next period assets will be a mapping from $\mathbb{M} \times S$ to $\mathbb{R}$. We make the dependence of the policy functions on $\mathbb{M}$ explicit in the definition of the constrained efficient optimum below.[7]

[6]In contrast, the unconstrained planner acts through completing asset markets. We say more on the relationship between the constrained planner and complete markets below, in Section 3.4.

[7]In the definition that follows, we take as given that an optimum exists. An existence proof is found in Shanker (2018).

DEFINITION 2.2 (**Recursive Constrained Efficient Optimum**)

A RCEO is a value function V and policy functions $\{h, l\}$, where $V \colon \mathbb{M} \to \mathbb{R} \cup \{-\infty, +\infty\}$, $h \colon \mathbb{M} \times S \to \mathbb{R}$ and $l \colon \mathbb{M} \times S \to \mathbb{R}$ such that the value function satisfies the Bellman equation

$$V(\mu) = \max_{(\tilde{h},\tilde{l}) \in \Lambda(\mu), \mu' = \Phi(\mu,\tilde{h})} u(\mu, \tilde{h}, \tilde{l}) + \beta V(\mu'), \quad \mu \in \mathbb{M}$$

and the policy functions achieve the maximum

$$h(\mu, \cdot), l(\mu, \cdot) = \operatorname*{argmax}_{(\tilde{h},\tilde{l}) \in \Lambda(\mu), \mu' = \Phi(\mu,\tilde{h})} u(\mu, \tilde{h}, \tilde{l}) + \beta V(\mu'), \quad \mu \in \mathbb{M}$$

To understand the salient features of a constrained efficient optimum, we now turn to the planner's necessary conditions.[8] The first is the Euler equation, summarizing each agents' trade off between consumption and savings

$$\begin{aligned} \nu_1\left(c, l(x,e)\right) \geqslant \beta(1 + r(\mu', l')) \int \nu_1\left(c', l'(h(x,e), e')\right) Q(e, de') \\ + \beta F_{KK}(K(\mu'), L(\mu', l'))\Delta(\mu', h', l') \end{aligned} \tag{4}$$

and the second is the labor supply curve, summarizing each agents' trade off between consumption and labor

$$\nu_2\left(c, l(x,e)\right) \geqslant \nu_1\left(c, l(x,e)\right) w(\mu, l)e - eF_{LL}(K(\mu'), L(\mu', l'))\Delta(\mu', h', l'), \tag{5}$$

where

$$\begin{aligned} c &= (1 + r(\mu, l))x + w(\mu, l)e(1 - l(x,e)) - h(x,e) \\ c' &= \big(1 + r(\mu', l')\big)h(x,e) + w(\mu', l')e'(1 - l'\big(h(x,e), e'\big)\big) - h'(h(x,e), e') \end{aligned}$$

Both the Euler equation and the labor supply curve look standard (the inequality in both conditions reflects that agents may not save in equilibrium or work zero hours), except for an additional term involving Δ that appears on the right hand side. The Δ term captures the essence of the constrained efficient optimum: unlike

[8] Again, a detailed derivation of the necessary conditions can be found in the working paper version of our paper (Shanker, 2018). For the sake of reducing notation, in the necessary conditions that follow, we will not make the dependence of the policy functions h, l and h', l' on the aggregate state explicit; however, they should be thought of as solutions that solve the planner's problem given the aggregate state μ.

the competitive equilibrium, where agents take prices in the economy as given, the planner internalizes the marginal effect of agents' saving and hours decisions on aggregate prices and hence on all other agents' utility.

The variable Δ can be shown to be a function of the state and planner's policy functions, defined by

$$\Delta(\mu, h, l) = \frac{\int\int \nu_1(c(x,e), l(x,e)) x \mu(dx, de)}{K(\mu)} - \frac{\int\int \nu_1(c(x,e), l(x,e)) e(1 - l(x,e)) \mu(dx, de)}{L(\mu, l)} \tag{6}$$

To gain some insights into the determinants of Δ, assume that μ has a density. Through a change of variables, we can write $\Delta(\mu, h, l) = \mathbf{R}_K - \mathbf{R}_L$, where

$$\mathbf{R}_K = \frac{\int \tilde{u} G_K(d\tilde{u})}{K(\mu)}, \mathbf{R}_L = \frac{\int \tilde{u} G_L(d\tilde{u})}{L(\mu, l)} \tag{7}$$

and $G_{K,t}$ and $G_{L,t}$ are measures on the Borel sets of $\mathbb{R}$ defined by

$$G_K(B) = \int\int \mathbb{1}_B\{\nu_1(c, l(x,e))\} x \, \mu(dx, de)$$
$$G_L(B) = \int\int \mathbb{1}_B\{\nu_1(c, l(x,e))\} (1 - l(x,e)) e \, \mu(dx, de)$$

for $B \in \mathscr{B}(\mathbb{R})$.

In words, $G_K(B)$ is total capital owned by agents whose marginal utility falls in B; $G_L(B)$ is total effective labor of agents whose marginal utility falls in B. Thinking of marginal utility as direction, with agents of high marginal utility being poor and agents with low marginal utility being rich, the terms $\mathbf{R}_K$ and $\mathbf{R}_L$ are *the center of mass* of the capital and labor distribution. Recall the center of mass is the point at which mass is similarly distributed on either side of the point. Thus $\mathbf{R}_K$ is the marginal utility (or a point on the rich-poor axis) at which there is equal amount of capital on either side. And $\mathbf{R}_L$ is the marginal utility at which there is equal amount of effective labor on either side.

The center of mass interpretation tells us that the planner prefers agents to save more capital and to work less when $\mathbf{R}_L > \mathbf{R}_K$, that is, when the consumption-poor rely more on labor income than on capital. To see this, note that $\mathbf{R}_L > \mathbf{R}_K$ implies that $\Delta < 0$. Because $F_{KK} < 0$ and $F_{LL} < 0$, the wedge in the Euler equation (4) is therefore positive, and the one in the labor supply curve (5) is negative. We provide more intuition for this result in Section 3.2.

2.3. *Decentralization*

While the planner's problem is to select the policy functions directly of all agents, from the point of view of understanding constrained efficiency, it is instructive to examine how the competitive equilibrium responds to a planner that achieves constrained efficiency via a tax on agents' saving and leisure. This is the problem of decentralization, which we briefly discuss in this section.

Each agents' budget constraint in a competitive equilibrium with taxes is given by

$$\begin{aligned} c = \big(1 - \tau_K(\mu, x, e)\big)\big(1 + r\big(\mu, l(\mu, \cdot)\big)\big)x \\ + \big(1 - \tau_L(\mu, x, e)\big)w\big(\mu, l\big(\mu, \cdot\big)\big)\big(1 - q\big)e - x' + T(\mu, x, e), \end{aligned}$$

where $\tau_K(\mu, x, e)$ and $\tau_L(\mu, x, e)$ are the taxes on capital and labor, respectively, and where $T(\mu, x, e)$ is a lump-sum transfer. Note how the taxes depend on the aggregate state μ, but also on agents' individual states x and e. Second, the tax system must obey

$$\tau_K(\mu, x, e)\big(1 + r\big(\mu, l(\mu, \cdot)\big)\big)x + \tau_L(\mu, x, e)w\big(\mu, l\big(\mu, \cdot\big)\big)\big(1 - q\big)e = T(\mu, x, e),$$

that is, the planner's budget must be balanced at the level of individual agents. Combing both equations, one can see that in equilibrium the agents' budget constraints align with those in the no-intervention competitive equilibrium. Hence, assuming that the tax system must be balanced at the level of individual agents is another representation of the fact that the planner must respect each agents' budget constraints.

For agents with interior solutions under the constrained planner, we can then derive the capital tax wedge[9]

$$\tau_K(\mu, x, e) = -\frac{\Delta(\mu', h(\mu', \cdot), l(\mu', \cdot))F_{KK}(K(\mu'), L(\mu', l(\mu', \cdot)))}{(1 + r(\mu', l(\mu', \cdot)))\int v_1\,(c', l')\,Q(e, de')} \tag{8}$$

and the labor tax wedge

$$\tau_L(\mu, x, e) = \frac{\Delta(\mu, h(\mu, \cdot), l(\mu, \cdot))F_{LL}(K(\mu), L(\mu, l(\mu, \cdot)))}{v_1\,(c, l(\mu, x, e))\,w(\mu, l)} \tag{9}$$

[9]As Dávila et al. (2012) point out, for agents the planner wishes to work zero hours and/or take zero assets to the next period, there will be an infinite number of taxes the planner can use.

where

$$\begin{aligned}
c &= \big(1 + r\big(\mu, l(\mu,\cdot)\big)\big)x + w\big(\mu, l\big(\mu,\cdot\big)\big)e\big(1 - l(\mu,x,e)\big) - x' \\
c' &= \big(1 + r\big(\mu', l(\mu',\cdot)\big)\big)x' + w\big(\mu', l(\mu',\cdot)\big)e'(1 - l') - h(\mu',x',e') \\
\mu' &= \Phi(\mu,h), \qquad l' = l(\mu', h(\mu,x,e), e')
\end{aligned}$$

In equations (8)-(9), the policy functions $h(\mu,x,e)$ and $l(\mu,x,e)$ are those of the constrained efficient planner.

For completeness, we also write out the first order conditions for the agents facing a tax. For $\mu \in \mathbb{M}$ and $(x,e) \in S$

(10)
$$\nu_1\big(c, l(\mu,x,e)\big) \geqslant (1 - \tau_K(\mu,x,e))\beta\big(1 + r\big(\mu', l(\mu',\cdot)\big)\big) \int \nu_1\big(c', l(\mu',x',e')\big)\, Q(e,\mathrm{d}e')$$

and

$$\nu_2\big(c, l(\mu,x,e)\big) \geqslant (1 - \tau_L(\mu,x,e))\nu_1\big(c, l(\mu,x,e)\big)w\big(\mu, l(\mu,\cdot)\big)e \tag{11}$$

where c, c' and μ' have been defined after equation (9).

3. QUANTITATIVE ANALYSIS: OPTIMAL DISTRIBUTION OF SAVING AND WORK HOURS IN U.S. CALIBRATED ECONOMY

Our quantitative analysis computes the steady state of the competitive equilibrium and constrained efficient optimum numerically, using the calibration of Pijoan-Mas (2006). The section begins with discussion of aggregate statistics, then turns to a discussion of results across the distribution and also investigates the economic forces that underlie the interventions made by the planner. Details of the computational method to solve the model are given by Shanker (2018).

3.1. *Aggregate statistics*

We base our analysis on the calibration of Pijoan-Mas (2006). First, Pijoan-Mas uses individual-level data to estimate the agents' income process. He then calibrates the remaining parameters using moments in U.S. aggregate data and household survey data. Under this calibration, the model roughly matches observed U.S. income and wealth inequality. The parameters we use are shown in (the caption of) Table I. (We discuss an alternative calibration below, in Section 4.)

Before turning to the results, we briefly outline the stochastic structure underlying the labor market in this calibration. The labor endowment shocks are modeled as an AR1 process $z_{t+1} = \rho z_t + \eta_t$, where $(\eta_t)_{t=0}^{\infty}$ is a sequence of i.i.d. mean zero normal shocks and where $z_t = \ln e_t$. We use Tauchen's method to discretize the labor endowment shock (see Sargent and Stachurski (2017)), by using a seven-state Markov chain. The endowment shocks in the simulations are also normalized so the mean of their stationary distribution is equal to one. The endowment process generates a top income shock that is almost 30 times the low income shock. The shocks are highly persistent, although agents face a non-negligible probability of transitioning from a top shock to a middle and then lower shock. Moreover, periods of being in a low productivity state are relatively long.

The aggregate statistics are summarized in Table I. The first line shows results in the competitive equilibrium (called incomplete markets - IM). The second line shows results in the constrained efficient optimum (called CP). The third line is explained further below.

The aggregate statistics show the constrained planner accumulating a higher level of capital, close to twice the capital in the competitive equilibrium. However, the planner only slightly increases work hours, about 3 percent more than the competitive equilibrium.[10] Effective labor supply under the planner increases by 7 percent, pointing to a more efficient allocation of work hours across the distribution (see Section 3.3 for an explanation). Turning to prices, the combination of a much larger capital stock and only slightly increased labor supply implies wages are about 20 percent higher in the constrained efficient optimum, and interest rates decline drastically, from 3.7 to only 0.6 percentage points.

In Section 2.3, we argued the planner's outcome can be decentralized as a competitive equilibrium with a tax, given by equations (8) and (9). By taking this perspective, the constrained efficient optimum can be understood as the sum of two effects; first, the direct effect on agents' behavior implied by the tax; second, a general equilibrium effect, as agents' behavior adjusts in response to the level of interest rates and wages implied by the constrained efficient optimum. In the third line of Table I (IM with CP prices), we isolate the general equilibrium effect, by computing households' behavior in the competitive equilibrium once the interest rate and wages are fixed at their levels from the constrained efficient optimum.[11]

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[10] For better visibility, in the table we have normalized households' hours endowment to 100. Hence a value of 33 indicates that households spend about one third of their time working, corresponding to about 8 hours per day.

[11] Mathematically, the problem corresponds to computing the supply of capital and labor holding

TABLE I

AGGREGATE STATISTICS.

	Interest rate	Wage	Capital	Output	Hours	Labor
IM	3.70	2.24	3.44	1.15	32.77	32.79
CP	0.62	2.64	5.88	1.46	34.03	35.29
IM with CP prices	0.62	2.64	0.70	–	35.34	34.21

Calibrated parameters: $\beta = 0.945$, $\alpha = 0.36$, $\delta = 0.083$, $\gamma_c = 1.458$, $\gamma_l = 2.883$, $A_L = 0.856$, $A = 1.5$, $\rho = 0.95$ and $\sigma(\eta_t) = 0.3$.

In the IM economy with CP prices, savings are dramatically lower than in the constrained planner's allocation and even in the competitive equilibrium. Hours increase slightly relative to the constrained planner's allocation, whereas effective labor supply falls slightly. Thus, the combination of low interest rates and high wages affects household behaviour in the following way: on average, households save much less and work slightly more. The low saving and slightly higher hours in the IM economy with CP prices implies that, in order to increase the capital stock and to reduce work hours to the levels of the constrained efficient optimum, the planner actively *subsidizes* saving and *taxes* hours worked. We offer an interpretation of this result in the following section.[12]

3.2. *A center-of-mass interpretation of the optimal tax*

In this section, we offer an interpretation why the planner finds it optimal to subsidize saving and to tax hours worked that is based on the center of mass of the distribution of effective labor and capital in the competitive equilibrium.

Begin with the effect of Δ in the first order conditions (4) and (5), which affects the sign of the constrained planner's tax (8) and (9). When Δ is negative (positive), then the planner implements a saving subsidy (tax), giving a greater incentive for agents to save more (less). And when Δ is negative (positive), the planner implements a labor tax (subsidy), giving a greater incentive for agents to take more (less) leisure.[13]

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the wage and interest rate fixed at the constrained planner's level. Note that this is a counterfactual economy, because at this level of interest rate and wages, firms would not be willing to absorb households' capital and labor supply.

[12]In addition, there is an interaction between the two taxes, as the saving subsidy itself acts to reduce work hours. We study this interaction in Section 3.3.

[13]Recall that the tax/subsidy is rebated lump-sum to agents in equilibrium. This implies that only the substitution effect of the tax/subsidy is at work, not an income effect. Also note that, while the

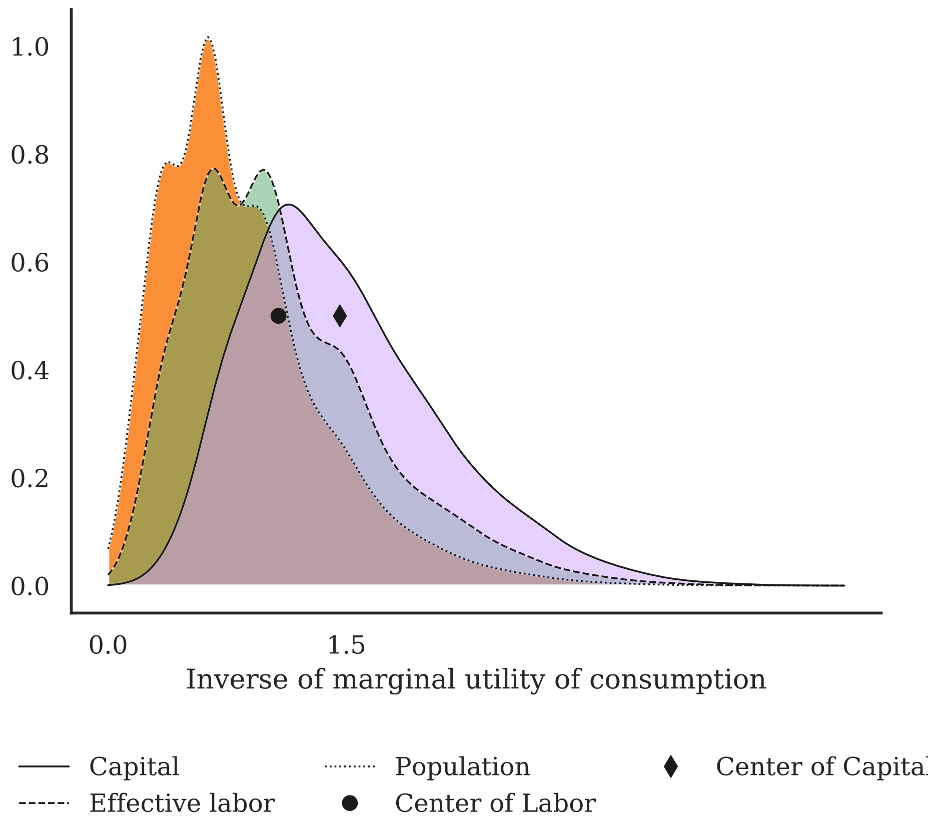

Figure 1: Density of capital, labor and population over inverse of marginal utility.

As we explained before, the sign of Δ depends on the center of mass of capital $\mathbf{R}_K$ and the center of mass of effective labor $\mathbf{R}_L$, see equations (6) and (7). Thinking of marginal utility as direction, recall that $\mathbf{R}_K$ is the level of marginal utility at which there is equal amount of capital on either side (and equivalently for $\mathbf{R}_L$). For example, if $\mathbf{R}_L > \mathbf{R}_K$, then capital tends to be concentrated among the consumption-rich and work effort among the consumption-poor. In this case, Δ is negative, hence the planner finds it optimal to subsidize saving and to tax hours worked.

The intuitive reason for why the planner subsidizes saving and taxes hours is that the resulting rise in wages and decline in interest rates re-distributes income toward those agents with the high marginal utility of consumption. This is the case

tax depends on each agents' individual state, its *sign* (which is dictated by Δ) is in fact identical for all agents.

in the calibrated model, as can be seen in Figure 1. The figure shows the density of capital, population and effective labor. The densities are shown on the *inverse* of the marginal utility of consumption, implying that on the left of the horizontal axis, one can find the consumption-poor.[14] As we can see, the mass of capital is concentrated towards the consumption-rich, whereas it is the consumption-poor who are dependent more on labor income. Hence, the constrained planner wishes to improve wages and to depress capital returns, as this re-distributes income toward those agents with the high marginal utility of consumption.

3.3. *Saving and work hours across the distribution*

Having investigated aggregate statistics, we now zoom in and study results at the level of individual agents.

We start by looking at policy functions for saving and work hours, again distinguishing the three economies IM, CP and IM with CP prices (as in Table I). Policy functions for next period assets are shown in Figure 2. The upper panel shows results for agents who have a low stock of assets (poor households), the middle one for agents which have an intermediate stock of assets, and the bottom panel shows results for wealthy agents. Within each panel, the policy functions are plotted against the level of labor productivity e. In turn, Figure 3 does the same for hours worked.

As we discussed before, aggregate savings are much larger in the constrained efficient optimum compared to the competitive equilibrium. However, as Figure 2 reveals, not all agents save more in the constrained efficient optimum. In fact, only agents who are productive enough increase their saving, while the unproductive save less. Turning to work hours policy functions in Figure 3, we see work hours increase substantially for wealthy agents (who are productive enough) under the constrained planner, while work hours stay unchanged (even somewhat decline) for poor agents.

In summary, productive agents should save more compared to the no-intervention economy, while unproductive agents should save less. In turn, the wealthy should work more compared to the no-intervention economy, but the poor should keep their work hours roughly unchanged.

We will next explain these results by using again our earlier interpretation of the

[14]For reasons of better visualization, we have chosen to plot results against the inverse of marginal utility, rather than on marginal utility itself, so the consumption-poor are to the left of Figure 1 and the consumption-rich are to the right.

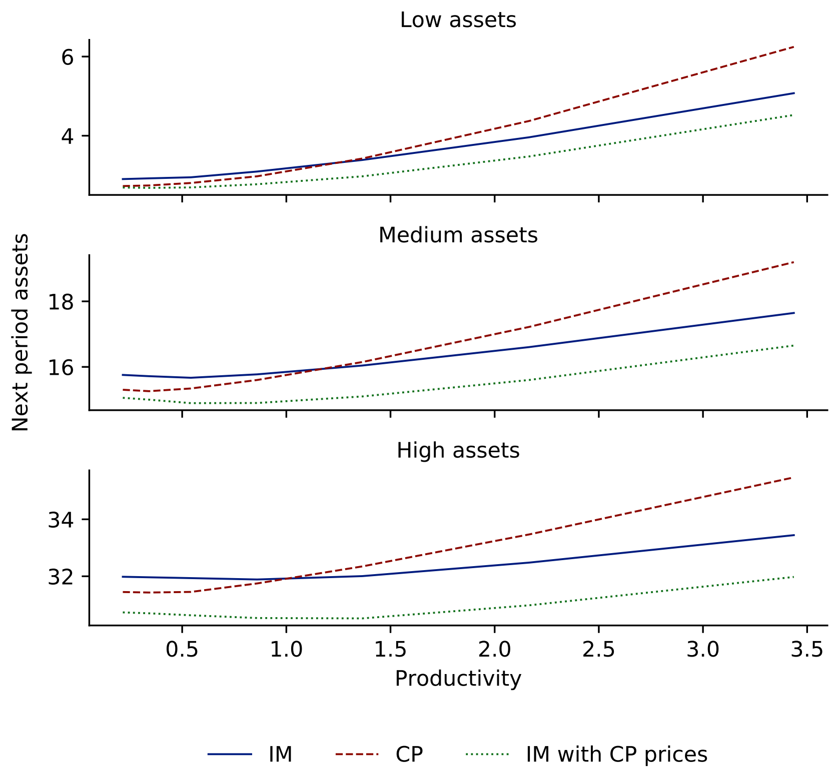

Figure 2: Policy functions for next period assets.

constrained efficient optimum as being equivalent to a competitive equilibrium in which a direct tax on saving and hours worked and a general equilibrium effect working through prices interact. To this end, have a look at Figure 4, which shows policy functions for the tax on saving and hours worked in the calibrated economy.

Consider first the saving tax. It is negative because, as we have argued before, the planner finds it optimal to subsidize saving. Moreover, the subsidy is increasing in the level of productivity. This explains why the productive agents in Figure 2 increase their saving in the constrained efficient optimum relative to the competitive equilibrium. Turning to the unproductive agents in Figure 2, how can they reduce their savings, despite receiving a (smaller, but still positive) saving subsidy? This can be understood by considering the general equilibrium effect. Recall that in the constrained efficient optimum, the interest rate is lower compared to the competitive equilibrium. The lower interest rate itself tends to reduce saving by all agents (the green line in Figure 2). In fact, only for the productive agents is the saving

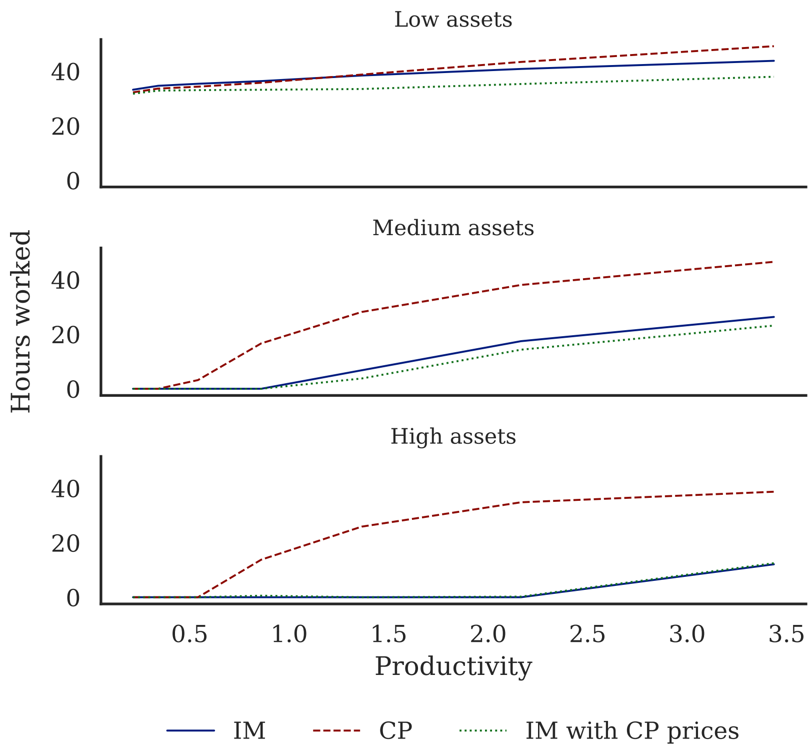

Figure 3: Policy functions for hours worked.

subsidy large enough to offset the general equilibrium effect, such that on net they increase their level of saving.[15]

Consider next the labor income tax. It is positive because, as we saw before, the planner finds it optimal to tax hours worked. The tax on labor tends to reduce work hours by all agents. Turning to the general equilibrium effect, note that the combination of lower interest rates and higher wages has an ambiguous effect on work hours across the distribution. For example, the general equilibrium effect depresses work hours for the poor and productive agents, while it slightly increases work hours for agents with medium wealth and medium productivity (see Figure 3). On net, as we saw in Table I, work hours slightly increase due to the general

[15]Moreover, as already shown in Dávila et al. (2012), in the case of a U.S. calibrated economy the magnitude of the saving subsidy is larger for the wealthy than for the poor.

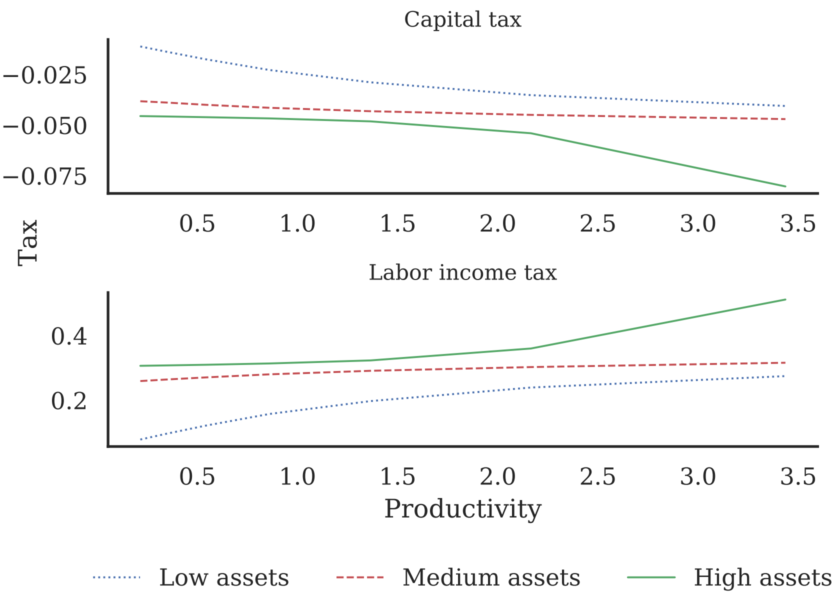

Figure 4: Capital and labor income taxes.

equilibrium effect.[16]

The fact that work hours increase for some agents in the constrained efficient optimum, while both the general equilibrium effect and the labor tax point to lower work hours for these agents (see for instance the poor but productive), represents a bit of a puzzle. To resolve the puzzle, recall labor supply is given by equation (11), which for agents with strictly positive work hours holds with equality

$$\nu_2\big(c, l(\mu, x, e)\big) = (1 - \tau_L(\mu, x, e))\nu_1\big(c, l(\mu, x, e)\big) w\big(\mu, l(\mu, \cdot)\big) e$$

The policy function for leisure l is influenced by the tax τ_L and by the general equilibrium effect working through wages w, but in addition, by a *wealth* effect which works through the marginal utility of consumption ν_1. In turn, the wealth effect is influenced by the saving behavior of agents, which is different in the constrained efficient optimum compared to the competitive equilibrium. Notably, the saving behavior is influenced by the saving subsidy. The higher the saving subsidy, the lower current consumption which by the wealth effect encourages agents

[16]Since the mass of agents with medium wealth and medium productivity is large, the number of agents working less under the constrained planner is substantial.

TABLE II

UNCONDITIONAL CORRELATIONS.

	productivity, assets	productivity, hours	hours, assets
IM	0.51	0.02	-0.69
CP	0.41	0.30	-0.43

to work more. For some agents, this effect is strong enough to raise work hours, even though the labor tax and the general equilibrium effect both encourage these agents to work less.

3.4. *Comparing the constrained planner to complete markets*

One interpretation of the constrained optimal distribution of work hours involves viewing the constrained planner's actions as reducing the gap between work hours in the competitive equilibrium and work hours under first best — when asset markets are complete. Under complete markets, work hours are independent of asset levels but strongly positively correlated with labor productivity. From inspecting Figure 3, one can guess the constrained planner does indeed raise the correlation between work hours and productivity. Moreover, for a given level of productivity, work hours in the constrained efficient optimum appear indeed more independent of asset levels (except for the poor agents, see again Figure 3).

We confirm the interpretation by examining unconditional correlations in Table II. The first column shows the correlation between productivity and assets, the second one between work hours and productivity and the last column shows the correlation between work hours and assets. In the competitive equilibrium, the correlation between productivity and assets is strongly positive, reflecting the tendency of the rich to be the productive and the poor to be unproductive (since shocks are highly persistent). In turn, the correlation between productivity and hours is close to zero, while the correlation between hours and assets is strongly negative.

As shown in Pijoan-Mas (2006), when markets are *complete*, the productivity-hours correlation is strongly positive (it is 0.74), due to households taking advantage of "opportunities to work". Moreover, the hours-assets correlation is equal to zero. As Pijoan-Mas (2006) explains, under incomplete markets, the low correlation between hours and productivity arises because poor households - who tend to be the unproductive - work long hours because they have a high marginal utility of consumption (a negative wealth effect). This also explains the negative correlation between hours and assets under incomplete markets. The poor work long hours due to a negative wealth effect, while the rich work few hours, due to a positive wealth effect. As we can see, the constrained planner pushes these correlations towards complete markets; the planner makes the productivity-hours correlation more positive, and takes the hours-asset correlation closer to zero.

The more positive correlation between work hours and productivity in the constrained efficient optimum also explains why effective labor supply increases by an aggregate 7 percent relative to the competitive equilibrium, while work hours increase only by 3 percent, a fact that we had highlighted in Section 3.1. Moreover, this fact reveals that a productivity-hours correlation that is close to zero is not an implication of lacking insurance markets *per se*. In our model, also the constrained planner acts under incomplete markets, yet the correlation between work hours and productivity is relatively high.[17]

3.5. *Consumption insurance or re-distribution?*

While we, and the literature, have used terms such as re-distribution to describe the actions of the constrained planner, we remind readers the constrained planner does not optimise the steady-state welfare distribution. Rather we are studying features of a steady state emerging from the constrained planner who optimises over agents' stream of consumption. Conceptually, by comparing steady states, we are limited in ability to discuss the trade offs between insurance and re-distribution faced by the planner. Moreover, behind the veil of ignorance, any re-distribution is also a form of insurance, since all inequality, wealth and income, derives from a sequence of ex-post transitory income shocks. Notwithstanding the limitations, we may still ask the question of whether consumption is more or less "equal" in the constrained planner's steady state and whether agents are more or less insured against risk.

3.5.1. *Gini coefficients*

We start by examining re-distribution by studying inequality in the competitive equilibrium and the constrained efficient optimum. In particular, we compute Gini coefficients in the steady state. The results are shown in Table III. The first two columns show the income and wealth Gini coefficients, while the last column shows the Gini coefficient for consumption. Recall a higher Gini coefficient implies

[17]Notice that, while unconditional correlations become closer to their complete-markets counterparts under the constrained planner, the same is not true for aggregate statistics. For instance, the constrained efficient optimum is characterized by a larger capital stock than the competitive equilibrium, even though (in this calibration) the capital stock under complete markets is smaller.

TABLE III
GINI COEFFICIENTS.

	Income gini	Wealth gini	Consumption gini
IM	0.33	0.65	0.23
CP	0.36	0.74	0.21

a higher degree of inequality.[18]

Both the income and wealth Gini coefficients are higher under the planner. The income Gini coefficient becomes higher because agents under the constrained planner respond more to incentives to work from a good productivity shock. In turn, the wealth Gini coefficient becomes higher because the productive (who tend to be the rich) are saving more and the unproductive (who tend to be the poor) are saving less.

In sharp contrast, the consumption Gini coefficient becomes lower under the constrained planner; the higher income and wealth inequality do not imply a higher inequality of consumption. The consumption Gini falls because wages are higher and interest rates are lower in the constrained efficient optimum, re-distributing income toward the consumption-poor, consistent with our analysis in the previous sections.

3.5.2. *Insurance coefficients*

We now examine how the constrained planner affects insurance by asking how much agents' consumption paths are impacted by idiosyncratic uncertainty. In contrast to inequality/re-distribution which are concepts related to the cross section, insurance relates to the *time series* path of consumption. To make the time-series nature of our discussion on insurance transparent, in this section, we switch to sequence notation; we denote time with a subscript t and an individual agent with a subscript i.

To define consumption insurance, we follow Kaplan and Violante (2010) and com-

[18]Recall the definition of the Gini coefficient of a real-valued random variable is given by

$$G = \frac{1}{\mathbb{E}x} \int_0^\infty F(x)(1 - F(x))\ dx$$

where F is the cumulative distribution function of the random variable and where $\mathbb{E}x$ is its expectation. A value of zero indicates zero inequality, and a value of one expresses maximum inequality.

pute the co-variance of consumption growth with productivity shocks over the variance of income shocks. Formally, an insurance coefficient ϕ will be

$$\phi = 1 - \frac{\operatorname{cov}(\Delta \log(c_{i,t}), \eta_{i,t})}{\operatorname{var}(\eta_{i,t})}. \tag{12}$$

When agents' consumption responds strongly to income shocks, ϕ will be close to zero or even negative, reflecting a small degree of consumption insurance. In contrast, when ϕ is closer to one, the degree of consumption insurance is high.

In our model, agents can insure against fluctuations in consumption via accumulating savings and dis-saving if a bad income shock hits or relying on the ability to work longer hours if a bad income shock hits. In order to separate the two sources of insurance, we introduce a second consumption insurance coefficient, which controls for a potential change in hours worked. We call this the fixed-hours coefficient, denoted $\tilde{\phi}$. Define

$$\tilde{\eta}_{i,t} = \eta_{i,t} \frac{1 - l_{i,t}}{H_t}$$

as a productivity shock faced by the agents once they have adjusted work hours. Then $\tilde{\phi}$ is defined as

$$\tilde{\phi} = 1 - \frac{\operatorname{cov}(\Delta \log(c_{i,t}), \tilde{\eta}_{i,t})}{\operatorname{var}(\tilde{\eta}_{i,t})}. \tag{13}$$

Intuitively, imagine productivity shocks are partly insured through changes in work hours. In this case, the variance of $\tilde{\eta}_{i,t}$ is going to be smaller than the variance of $\eta_{i,t}$, because drops in $\eta_{i,t}$ are partly offset by rises in $1 - l_{i,t}$. The co-variance between consumption growth and $\tilde{\eta}_{i,t}$ will therefore be larger than the co-variance between consumption growth and $\eta_{i,t}$. Hence $\tilde{\phi} < \phi$ will hold if households insured themselves through changing their work hours. This also means that we can view $\tilde{\phi}$ as the part of consumption insurance that is achieved through *saving alone*.[19]

Table IV shows the consumption insurance coefficients computed for agents in the steady state of the model. The first column shows that the baseline coefficient ϕ is larger in the constrained efficient optimum compared to the competitive equilibrium, that is, agents' consumption paths are more insulated from idiosyncratic

[19]For example, in a model with exogenous work hours, ϕ would equal $\tilde{\phi}$ at all times, reflecting that all consumption insurance comes from saving and dis-saving.

TABLE IV
INSURANCE COEFFICIENTS.

	Baseline coefficient, ϕ	Fixed-hours coefficient, $\tilde{\phi}$	$\frac{\text{cov}(\Delta \log(1-l_{i,t}),\eta_{i,t})}{\text{var}(\eta_{i,t})}$
IM	0.64	0.75	1.92
CP	0.70	0.84	2.63

uncertainty under the planner. As in Section 3.4, the planner thus pushes the equilibrium distribution closer to the one under complete markets (which features perfect consumption insurance, $\phi = 1$). Above we had argued that the planner raises aggregate utilitarian welfare through re-distribution that results from a change in prices. What we now add is that the rise in welfare also reflects a higher degree of average consumption insurance.

Turn now to the second column, which shows the fixed-hours coefficient $\tilde{\phi}$. Focus first on the competitive equilibrium. We find $\tilde{\phi} > \phi$, implying, perhaps surprisingly, households are *not* using hours to insure consumption from period to period fluctuations in productivity. In fact, $\tilde{\phi} > \phi$ implies the way agents vary work hours in response to shocks *increases* their exposure to idiosyncratic uncertainty. To understand the finding further, inspect the last column of Table IV which shows directly the period to period co-movement of hours and productivity; the co-movement is positive. Following negative productivity shocks, households' income therefore falls first because of a lower productivity, but additionally because of a decline in work hours.

Our findings on the behavior of hours in response to shocks shed new light on the result by Pijoan-Mas (2006) that households are using hours as a consumption smoothing mechanism. True, poor agents work more hours due to a negative wealth effect as Pijoan-Mas (2006) discovered, which insulates their consumption from the low stock of assets (recall Section 3.4). However, households do not work longer hours in response to negative productivity shocks period by period.

What explains the behavior of agents causing a period by period positive correlation between hours and productivity shocks? Recall productive agents are the high wage earners, while the unproductive ones earn a low (effective) wage ew. As a result, transitory productivity shocks act like transitory changes in agents' effective wages. Due to a standard substitution effect which outweighs the income effect, positive productivity shocks induce agents to work more.

We last discuss the fixed-hours coefficient under the constrained planner. As in the competitive equilibrium, it is larger than the baseline coefficient, implying the planner does not use hours to self-insure agents. The difference between ϕ and $\tilde{\phi}$ is even larger under the planner, reflecting that the planner increases the correlation between hours and productivity (as we had argued before). In sum, the overall increase in insurance we documented for the planner derives purely from the higher saving the planner asks agents to undertake, not from a (qualitative) change in the behavior of work hours.

4. AN UNEMPLOYMENT ECONOMY

We now show how incentives faced by the constrained planner are quite different when productivity dispersion becomes so large that unproductive agents are effectively unemployed. Our calibration for the unemployment economy is along the lines of Marcet et al. (2007). Other robustness exercises can be found in the earlier working paper version of our paper (Shanker, 2018).

We have documented so far how the wealthy work fewer hours than the poor in the competitive equilibrium because of a positive wealth effect (recall for instance Figure 3). Marcet et al. (2007) show how the wealth effect causing low hours can be strong enough to lead to lower capital accumulation under incomplete markets compared to complete markets, despite the precautionary saving effect. Stated differently, Marcet et al. (2007) showed how the result by Aiyagari (1994) that capital accumulation is higher under incomplete markets than under complete markets could be overturned when work hours are endogenous and positive wealth effects are strong.

To demonstrate their argument, Marcet et al. (2007) used an economy where agents face only two realizations of the productivity shock, a close-to-zero level of productivity and a high level of productivity. When faced with close-to-zero productivity, the agent becomes effectively unemployed, because the effective wage ew falls very close to zero. In contrast to our baseline calibration, this implies that work hours by the unproductive/poor are not driven up by a negative wealth effect. Overall work hours (and therefore income and capital accumulation) then fall under incomplete markets compared to complete markets — the productive/wealthy work less because of a positive wealth effect, and the unproductive/poor don't work at all in both market arrangements because they are effectively excluded from the labor market.

Table V presents aggregate statistics for an economy with parameters similar to the ones used by Marcet et al. (2007). Focus first on the first two rows, which compare results under complete markets (first best) and the competitive equilibrium with

TABLE V

AGGREGATE STATISTICS FOR UNEMPLOYMENT ECONOMY.

	Interest rate	Wage	Capital	Output	Hours	Labor
Complete markets	3.52	2.32	4.74	1.56	41.05	43.68
IM	3.38	2.30	4.11	1.33	34.94	37.18
CP	3.36	2.30	4.12	1.33	34.94	37.17

Model parameters are $\beta = 0.966$, $\alpha = 0.36$, $\delta = 0.083$, $\gamma_c = 5$, $\gamma_l = 0.5$, $A_L = 2$, $A = 1.5$. Probability matrix: [[0.09 0.91] [0.06 0.94]]. Shocks: [0.04361 1.63934426].

incomplete markets. Under this calibration, indeed we find that the capital stock stock, output, total hours and labor supply are *lower* in the competitive equilibrium under incomplete markets compared to the complete markets economy.

We now ask how the incentives by the planner are different compared to our baseline calibration. Inspect the last row in Table V which shows the constrained efficient optimum. We can see how the planner almost does not intervene, that is, the planner cannot improve (much) over outcomes in the competitive equilibrium.

To understand why the planner does not intervene, recall the planner in our baseline calibration was driven by a motive for re-distribution. By raising the capital stock, the planner raised the aggregate level of wages which re-distributed income to the consumption-poor. In the current calibration, however, the consumption-poor cannot be helped by a higher level of wages, because they are effectively unemployed. Indeed a higher level of wages benefits only those agents who are currently employed, who are the productive ones. The unemployed also do not own much capital, as spells of unemployment are long due to the high shock persistence, thus the planner does not have an incentive to increase interest rates either.

In summary, the (extreme) insider-outsider structure of the labor market in the calibration here means welfare cannot be improved without completing the asset markets. Unlucky agents have neither capital income nor income from labor. As a result, the planner cannot do anything to help these agents by adjusting prices.

5. CONCLUSION

In an economy with un-insurable idiosyncratic labor income risk and endogenous work hours, how should saving and work hours be distributed across income and wealth? We answer this question by studying the problem of a social planner who cannot complete asset markets, but can dictate how much individual agents must work and save. The planner improves aggregate utilitarian welfare by internalizing the pecuniary externality of each agents' saving and work hours decisions on aggregate wage and interest rates.

The main findings of the paper are that, in an economy roughly matching U.S. income and wealth inequality, the planner raises aggregate savings but not aggregate hours (by much). Across the distribution, the *productive* agents (high wage earners) should increase their saving, while the *wealthy* agents should increase their work hours. The resulting general equilibrium effects re-distribute income toward the consumption-poor. Moreover, the degree of consumption insurance against idiosyncratic uncertainty increases under the constrained planner.

While our motivation for this paper has been the policy concerns about income and wealth inequality and how both are shaped by heterogeneity in work hours, we have only taken a first step at studying the optimal distribution of saving and hours worked. As we mentioned in the related literature section in the introduction, a more applied account would also allow for net transfers across agents, as in the literature studying Ramsey taxation. We hope our analysis and insights regarding the pecuniary externalities of work hours and saving will be useful in designing implementable tax systems in environments involving heterogeneity in work hours.

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