The Scars of Supply Shocks:
Implications for Monetary Policy

Luca Fornaro and Martin Wolf[*]

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Abstract

We study the effects of supply disruptions - for instance due to energy price shocks or the emergence of a pandemic - in an economy with Keynesian unemployment and endogenous productivity growth. By temporarily disrupting investment, negative supply shocks generate permanent output losses - or scarring effects. By inducing a negative wealth effect, scarring effects depress aggregate demand, which may even fall below the exogenous fall in supply. However, that scarring effects depress aggregate demand does not necessarily translate into low rates of inflation. On the contrary, scarring effects may reinforce and prolong the inflationary impact of supply disruptions. A contractionary monetary policy response may end up deepening scarring effects and increasing inflation in the medium run. A successful disinflation may require a policy mix of monetary tightening and fiscal interventions aiming at supporting business investment and the economy's productive capacity.

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1 Introduction

In the last few years the global economy has been plagued by supply disruptions. First, the Covid-19 pandemic forced factories to shut down and disrupted global supply chains. Second, Russia's invasion of Ukraine has caused a sharp spike in energy and food prices. In addition to the short-run damage, many observers and policy institutions expect both shocks to leave deep scars, by inducing very persistent drops in potential output below pre-crisis trends (e.g., IMF, 2022).[1] In response to both shocks, moreover, inflation has surged in the global economy to levels not seen since the 1970s, and its persistence has been a surprise to many. These facts have renewed interest in the economic effects of supply disruptions. Can supply disruptions induce long-lasting damage to the economy? Can transitory negative supply shocks cause persistent rises of inflation? Which trade-offs are implied for monetary and fiscal policy? These questions are at the forefront of the current debate.

Much of the conventional thinking about supply shocks builds on the New Keynesian paradigm (Galí, 2009). In the New Keynesian model, following a negative supply shock demand contracts less than supply, and so the natural interest rate rises. Inflation is elevated during the period of the shock, but quickly falls back to trend once the shock abates. Monetary policy can single-handedly dampen the inflationary impact of supply disruptions, by slowing the economy and inducing a negative output gap. The New Keynesian framework, however, assumes that after the shock dissipates the economy quickly bounces back to its pre-shock trend, and so does not allow for the possibility that supply disruptions might have scarring, or hysteresis, effects.[2] This is in spite of a growing body of empirical evidence suggesting that deep recessions, including those triggered by negative supply shocks, are followed by extremely persistent output drops below pre-recession trends (Cerra and Saxena, 2008; Blanchard et al., 2015; Bluedorn and Leigh, 2018; Fatás and Summers, 2018; Aikman et al., 2022).

This paper provides a theory in which negative supply shocks may leave persistent scars on the economy, and shows that this effect might radically change the macroeconomic implications of supply disruptions relative to the traditional view. Our idea is that negative supply shocks - even if purely transitory - induce firms to reduce investment, and thus destroy the future productive capacity of the economy. The associated drop in wealth depresses consumers' demand, in fact so much that the natural interest rate may fall in response to a supply disruption. Moreover, hysteresis effects may amplify and prolong the rise in inflation triggered by negative supply shocks, as they entail a long-lasting drop in firms' productivity. Monetary tightenings may backfire by

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[1]For instance, in its Spring 2022 World Economic Outlook, the International Monetary Fund (IMF) writes (see IMF 2022): "Beyond short-term output losses, the pandemic and geopolitical conflict are likely to leave longer-lasting footprints. [...] Sanctions can induce permanent dismantling of trade and supply chain linkages, entailing productivity and efficiency losses along the way. [...] And scarring effects from the pandemic are likely to materialize through several other channels - including corporate bankruptcies, productivity losses, lower capital accumulation due to a drag on investment, slower labor force growth, and human capital losses from school closures." In response, the IMF has revised substantially downward its forecast for the growth rate of global real GDP per capita for the 2021-2027 period, relative to its pre-crisis projections.

[2]Throughout the paper we use interchangeably the terms scarring and hysteresis effects to refer to cases in which temporary shocks or policy interventions have a persistent impact on the economy's potential output.

inducing a drop in productivity and a rise in inflation in the medium run. A successful disinflation may thus require a policy mix of monetary tightening and fiscal interventions aiming at supporting business investment and the economy's productive capacity.

To formalize these insights, we provide a *Keynesian growth* framework with two key features. First, as in standard models of vertical innovation (Aghion and Howitt, 1992), firms invest in innovation in order to appropriate future monopoly rents. Second, as in the New Keynesian tradition, the presence of nominal wage rigidities implies that output may deviate from potential and that monetary policy has real effects. Our theory thus combines the Keynesian insight that unemployment may arise due to weak aggregate demand, with the notion, developed by the endogenous growth literature, that sustained productivity growth is the result of investment in innovation by profit-maximizing agents.

We study the response of the economy to supply disruptions, modeled as standard temporary negative productivity shocks.[3] In our framework, supply shocks drive down the return to investment, by reducing firms' market size and their profits, and by increasing firms' cost of funds. The result is lower investment and slower productivity growth. Once the shock dissipates, investment recovers and productivity growth returns to its pre-shock level, but output falls permanently below its pre-shock trend. Our model thus captures the notion that deep recessions can have hysteresis effects on productivity and potential output.

These scars of supply shocks matter critically for the response of aggregate demand. When productivity growth is exogenous, households' permanent income falls by little in response to transitory supply disruptions. In our endogenous growth model, instead, the drop in wealth caused by negative supply shocks is amplified by the associated decline in the trend component of productivity. In fact, we show that the fall in demand following a negative supply shock can even be larger than the exogenous fall in supply. In contrast with conventional wisdom, the natural interest rate may thus not rise sharply - and may even decline - during supply disruptions.

Turning to inflation, we show that scarring effects may reinforce and prolong the inflationary impact of negative supply shocks. The reason is that the endogenous drop in investment and trend productivity associated with supply disruptions raises firms' marginal costs, and so inflation. Moreover, since it takes time for investment to affect productivity, this effect arises with a delay. As a result, in our framework a temporary supply shock causes a persistent rise in inflation. Our model thus helps understand why large supply disruptions - such as the oil shocks of the 1970s or the Covid-19 pandemic - are accompanied by low real interest rates and highly persistent bursts of inflation.

What if the central bank hikes its policy rate, perhaps in an attempt to reduce inflation? First, we show that a monetary tightening may trigger a "supply-demand doom loop", which amplifies the direct impact of the supply shock on employment and productivity. To see why, start by considering that a monetary contraction leads to lower aggregate demand and employment. In

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[3]In Appendix B.1 we show that, under certain conditions, increases in energy prices have exactly the same effects as the productivity shocks considered in the paper.

turn, lower aggregate demand reduces firms' profits and incentives to invest. As firms cut back on investment, expected productivity growth declines. This causes a drop in households' wealth and another round of fall in aggregate demand, inducing a further decline in investment, and so forth. By triggering this vicious spiral, a tight monetary stance may thus depress both employment and productivity. This feature of the model is consistent with recent empirical evidence by Garga and Singh (2020), Moran and Queraltó (2018), Jordà et al. (2020) and Grimm et al. (2022), suggesting that monetary policy tightenings have a negative impact on investment in innovation and productivity growth.

We then show that monetary tightenings may be partially self-defeating in reducing inflation. The reason is that monetary contractions depress firms' investment and future productivity. In turn, lower productivity sustains firms' marginal costs and inflation. Because the inflation triggered by these scarring effects arises with a delay, moreover, a tight monetary stance may be successful at reducing inflation in the short run, but at the cost of higher inflation in the medium run. Hence, monetary tightenings may end up exacerbating the inflationary consequences of supply disruptions over the medium run.[4]

Central banks thus face a dilemma, as they may not be able to disinflate the economy without deepening the scarring effects. This suggests a potential role for fiscal interventions aiming at supporting business investment and the economy's productive capacity during disinflations. We show that a mix of monetary tightening and subsidies to investment lowers inflation both in the short and in the medium run, and improves the sacrifice ratio, that is the reduction in inflation associated with a given rise in unemployment. A successful disinflation can thus be seen as the outcome of both active supply and demand side management: while monetary policy slows inflation by reducing aggregate demand, fiscal policy slows inflation by supporting aggregate supply.[5] Interestingly, a similar policy mix characterized the 1980s disinflation in the United States, during which a sharp monetary tightening was accompanied by subsidies to business investment, especially in R&D (Blanchard, 1987; Modigliani, 1988).

This paper is related mainly to two strands of the literature. First, it is connected to the literature on supply disruptions and monetary policy. Compared to the standard New Keynesian approach (Blanchard and Galí, 2007a,b; Galí, 2009), our paper emphasizes the endogenous response of investment and productivity to supply disruptions and policy interventions.[6] While in the New Keynesian model the focus is on overall aggregate demand, in our framework its composition between consumption and business investment plays a crucial role. There is also recent literature, motivated by the Covid-19 epidemic, revisiting the macroeconomic implications of supply disruptions. Guerrieri et al. (2022) study an economy with multiple consumption goods. In

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[4]In a recent empirical contribution, Drechsler et al. (2022) argue that monetary tightenings contributed to the rise in US inflation during the 1970s, by hindering firms' productive capacity. Consistent with our framework, they find evidence of a strong negative impact of monetary tightenings on business investment.

[5]Akcigit et al. (2018) and Cloyne et al. (2022) provide empirical evidence in favor of a positive impact of subsidies to R&D on firms' future productivity.

[6]Our paper is thus connected to an older literature, e.g. Bruno and Sachs (1985), suggesting that the supply disruptions of the 1970s exerted a negative impact on firms' investment and productivity growth.

their model, a shock reducing the supply of some goods may induce consumers to cut spending also on those goods not directly affected by the shock. If this effect is strong enough, aggregate demand falls by more than supply. They dub supply shocks with this property *Keynesian supply shocks*. Baqaee and Farhi (2022) derive a similar result in an economy with production networks and multiple intermediate goods. Caballero and Simsek (2021) show that supply shocks can be Keynesian due to spillovers between asset prices and aggregate demand. In Bilbiie and Melitz (2020) supply disruptions depress demand by inducing firms' exit, while in L'Huillier et al. (2021) Keynesian supply shocks emerge due to the presence of diagnostic expectations.[7] Our paper studies a different - and complementary - channel through which supply shocks can become Keynesian, based on the endogenous response of investment and productivity growth.[8]

Second, this paper is related to the literature unifying the study of business cycles and endogenous growth. A central theme of this literature is the notion that deep recessions trigger hysteresis effects on productivity and potential output. Some examples of this literature are Fatas (2000), Comin and Gertler (2006), Reifschneider et al. (2015), Benigno and Fornaro (2018), Moran and Queraltó (2018), Anzoategui et al. (2019), Bianchi et al. (2019), Queraltó (2019), Garga and Singh (2020), Cozzi et al. (2021) and Queraltó (2022).[9] Our paper builds on the framework introduced by Benigno and Fornaro (2018), who study an endogenous growth model with vertical innovation and nominal wage rigidities. They show that in this Keynesian growth framework fluctuations can be driven by animal spirits, and derive the optimal monetary and fiscal policy. Garga and Singh (2020) and Queraltó (2022) derive, in similar Keynesian growth models, the optimal monetary policy response to fundamental demand and cost-push shocks. Our paper, instead, employs a Keynesian growth model to study supply shocks. To the best of our knowledge, we are the first to show that the scars of supply shocks may change dramatically the macroeconomic implications of supply disruptions.

The rest of the paper is composed of four sections. Section 2 describes the baseline model. Section 3 studies the macroeconomic implications of supply disruptions in presence of hysteresis effects. Section 4 considers the impact of monetary and fiscal policies aiming at disinflating the economy. Section 5 concludes. Appendix A provides the proofs of all propositions, and Appendixes B-E contain additional derivations as well as model extensions.

2   Baseline model

This section lays down our baseline *Keynesian growth* model. The economy has two key elements. First, the rate of productivity growth is endogenous, and it is the outcome of firms' investment. Second, the presence of nominal wage rigidities implies that output and employment can deviate

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[7]See Bilbiie (2008) for an early model in which supply shocks have Keynesian features.

[8]In our own earlier work (Fornaro and Wolf, 2020), we argued that permanent negative supply shocks can be Keynesian. In this paper, we move beyond the analysis in Fornaro and Wolf (2020) by providing a Keynesian growth model in which productivity growth is the result of firms' investment. We show that temporary negative supply shocks can be Keynesian by triggering endogenous drops in investment and productivity growth.

[9]See Cerra et al. (2020) for a recent survey of this literature.

from their potential levels. In order to illustrate transparently our key results, the framework in this section is kept voluntarily simple. Throughout the paper, however, we will extend this baseline model in several directions.

Consider an infinite-horizon closed economy. Time is discrete and indexed by $t \in \{0, 1, 2, ...\}$. The economy is inhabited by households, firms, and by a central bank that sets monetary policy. For simplicity, we focus on a perfect foresight economy.

2.1 Households

There is a continuum of measure one of identical households deriving utility from consumption of a homogeneous "final" good. The lifetime utility of the representative household is

$$\sum_{t=0}^{\infty} \beta^t \log C_t,$$

where C_t denotes consumption and $0 < \beta < 1$ is the subjective discount factor.

Each household is endowed with $\bar{L}$ units of labor and there is no disutility from working. However, due to the presence of nominal wage rigidities to be described below, a household might be able to sell only $L_t < \bar{L}$ units of labor on the market. Moreover, households can trade in one-period, non-state contingent bonds B_t. Bonds are denominated in units of currency and pay the nominal interest rate i_t. Finally, households own all the firms and each period they receive dividends D_t from them.

The problem of the representative household consists in choosing C_t and B_{t+1} to maximize expected utility, subject to a no-Ponzi constraint and the budget constraint

$$P_t C_t + \frac{B_{t+1}}{1 + i_t} = W_t L_t + B_t + D_t,$$

where P_t is the nominal price of the final good, B_{t+1} is the stock of bonds purchased by the household in period t, and B_t is the payment received from its past investment in bonds. W_t denotes the nominal wage, so that $W_t L_t$ is the household's labor income. The optimality conditions are the Euler equation

$$C_t = \frac{C_{t+1} \pi_{t+1}}{\beta(1 + i_t)}, \tag{1}$$

where we have defined the gross inflation rate as $\pi_t \equiv P_t / P_{t-1}$, and the standard transversality condition. In what follows, we denote by $1 + r_t \equiv (1 + i_t)/\pi_{t+1}$ the real interest rate.

2.2 Final good production

The final good is produced by competitive firms using labor and a continuum of measure one of intermediate inputs $x_{j,t}$, indexed by $j \in [0, 1]$. Denoting by Y_t the output of the final good, the production function is

$$Y_t = (Z_t L_t)^{1-\alpha} \int_0^1 A_{j,t}^{1-\alpha} x_{j,t}^{\alpha} dj, \tag{2}$$

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where $0 < \alpha < 1$, and $A_{j,t}$ is the productivity, or quality, of input j.[10] Z_t, instead, is an exogenous productivity shock, which we refer to as the "supply shock" in our model. This term captures all the transitory factors affecting labor productivity which are not directly linked to firms' investment. For instance, a reduction in Z_t could capture the fact that during a pandemic some firms cannot operate in order to preserve public health. Or, as we show in Appendix B.1, a fall in Z_t has a similar impact on the economy as an exogenous rise in the price of energy.

Profit maximization implies the demand functions

$$P_t(1-\alpha)Z_t^{1-\alpha}L_t^{-\alpha}\int_0^1 A_{j,t}^{1-\alpha}x_{j,t}^{\alpha}dj = W_t \tag{3}$$

$$P_t\alpha\left(Z_tL_t\right)^{1-\alpha}A_{j,t}^{1-\alpha}x_{j,t}^{\alpha-1} = P_{j,t}, \tag{4}$$

where $P_{j,t}$ is the nominal price of intermediate input j. Due to perfect competition, firms in the final good sector do not make any profit in equilibrium.

2.3 Intermediate goods production and profits

Every intermediate good is produced by a single monopolist. One unit of final output is needed to manufacture one unit of the intermediate good, regardless of quality. In order to maximize profits, each monopolist sets the price of its good according to

$$P_{j,t} = \frac{P_t}{\alpha}. \tag{5}$$

In words, each monopolist charges a constant markup $1/\alpha > 1$ over its marginal cost. Equations (4) and (5) imply that the quantity produced of a generic intermediate good j is

$$x_{j,t} = \alpha^{\frac{2}{1-\alpha}}A_{j,t}Z_tL_t. \tag{6}$$

Combining equations (2) and (6) gives

$$Y_t = \alpha^{\frac{2\alpha}{1-\alpha}}A_tZ_tL_t, \tag{7}$$

where $A_t \equiv \int_0^1 A_{j,t}dj$ is an index of average productivity of the intermediate inputs. Hence, production of the final good is increasing in the average productivity of intermediate goods, in the exogenous component of labor productivity, and in the amount of labor employed in production.

The profits earned by the monopolist in sector j are given by

$$(P_{j,t} - P_t)x_{j,t} = P_t\varpi A_{j,t}Z_tL_t,$$

where $\varpi \equiv (1/\alpha - 1)\alpha^{2/(1-\alpha)}$. According to this expression, the producer of an intermediate input

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[10]More precisely, for every good j, $A_{j,t}$ represents the highest quality available. In principle, firms could produce using a lower quality of good j. However, the structure of the economy is such that in equilibrium only the highest quality version of each good is used in production.

of higher quality earns higher profits. Moreover, profits are increasing in $Z_t L_t$ due to the presence of a market size effect. Intuitively, high production of the final good is associated with high demand for intermediate inputs, leading to high profits in the intermediate sector.

2.4 Investment and productivity growth

Firms operating in the intermediate sector can invest in innovation in order to improve the quality of their products. In particular, a firm that invests $I_{j,t}$ units of the final good sees its productivity evolve according to

$$A_{j,t+1} = A_{j,t} + \chi I_{j,t}, \tag{8}$$

where $\chi > 0$ determines the productivity of investment. Firms choose investment in innovation to maximize their discounted stream of real profits net of investment costs

$$\sum_{t=0}^{\infty} \frac{1}{\delta_t} \left(\varpi A_{j,t} Z_t L_t - I_{j,t} \right),$$

subject to (8) and given the initial condition $A_{j,0} > 0$.

The rate δ_t at which firms discount future profits is an important determinant of firms' investment decisions and thus of productivity dynamics. What discount rate should one use? The standard assumption is that firms discount future profits using the households' discount factor, which in our framework corresponds to the real interest rate $1 + r_t$. We deviate from this practice by assuming that firms discount profits at rate $1 + r_t + \eta$, where $\eta \geq 0$ captures a wedge between the firms' discount factor and the households' one. Formally, we assume

$$\frac{\delta_t}{\delta_{t-1}} = 1 + r_{t-1} + \eta,$$

which nests the standard framework when $\eta = 0$.[11]

We introduce this wedge for three reasons. First, for empirical realism. In fact, recent work by Gormsen and Huber (2022) shows that the discount rates used by firms to evaluate investment projects are substantially higher than the financial cost of capital, and only partly responsive to changes in market interest rates. Second, in innovation-based endogenous growth models the social return from investing in innovation is typically higher than the private one (Romer, 1990; Aghion and Howitt, 1992). For instance, this happens if knowledge is only partly excludable, and so inventors cannot prevent others from drawing on their ideas to innovate. The wedge η captures in reduced form these effects, because it leads firms to underestimate the positive impact of their investments on social welfare (see Appendix C).[12] Third, this wedge helps the model to generate

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[11] In fact, when $\eta = 0$ the discount factor used by firms can be written as

$$\frac{1}{\delta_t} = \prod_{j=1}^{t} \frac{1}{1 + r_{j-1}} = \prod_{j=1}^{t} \left(\beta \frac{C_{j-1}}{C_j} \right) = \beta^t \frac{C_0}{C_t},$$

where we have used the households' Euler equation (1).

[12] In fact, the wedge η could also be interpreted along the following lines. As in Benigno and Fornaro (2018), imagine

quantitatively relevant hysteresis effects, as we will argue below.

From now on, we assume that firms are symmetric and so $A_{j,t} = A_t$. Moreover, we focus on equilibria in which investment in innovation is always positive. As we show in Appendix B.2, optimal investment in innovation then requires

$$\frac{1}{\chi} = \frac{\beta C_t}{C_{t+1}} \left(\varpi Z_{t+1} L_{t+1} + \frac{1-\eta}{\chi} \right). \tag{9}$$

Intuitively, firms equalize the marginal investment cost $1/\chi$ to its discounted marginal benefit. The marginal benefit is given by the increase in next period profits $\varpi Z_{t+1} L_{t+1}$ plus the savings on future investment costs $1/\chi$. Naturally, a higher discount factor wedge η reduces firms' incentives to invest, because the returns from investment materialize in the future.

2.5   Aggregation and market clearing

Market clearing for the final good implies[13]

$$Y_t - \int_0^1 x_{j,t} dj = C_t + I_t, \tag{10}$$

where $I_t \equiv \int_0^1 I_{j,t} dj$. The left-hand side of this expression is the GDP of the economy, while the right-hand side captures the fact that all the GDP has to be either consumed or invested. Using equations (6) and (7) we can write GDP as

$$Y_t - \int_0^1 x_{j,t} dj = \Psi A_t Z_t L_t, \tag{11}$$

where $\Psi \equiv \alpha^{2\alpha/(1-\alpha)}(1-\alpha^2)$.

Turning to labor market clearing, the assumption about labor endowment implies that $L_t \leq \bar{L}$. Since labor is supplied inelastically by the households, $\bar{L} - L_t$ can be interpreted as the unemployment rate. For future reference, when $L_t = \bar{L}$ we say that the economy is operating at full employment, while when $L_t < \bar{L}$ the economy operates below capacity and there is a negative output gap.

Long-run growth in this economy takes place through increases in the quality of the intermediate goods, captured by increases in the productivity index A_t. We can thus think of $g_t \equiv A_t/A_{t-1}$ as

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that firms discount future profits using the households' discount factor, but that every period each incumbent firm faces a constant probability η of dying after production takes place. Once an incumbent dies, it is replaced by another firm that inherits its technology. Under this interpretation, η captures a variety of factors leading to the termination of the rents from innovation, including patent expiration and imitation by competitors. It turns out that this model is isomorphic to the one considered in the main text.

[13]The goods market clearing condition can be derived by combining the households' budget constraint with the expression for firms' profits

$$D_t = \underbrace{P_t Y_t - W_t L_t - P_t \frac{1}{\alpha} \int_0^1 x_{j,t} dj}_{\text{profits from final goods sector}} + \underbrace{P_t \int_0^1 \left(\frac{1}{\alpha} - 1 \right) x_{j,t} dj - P_t \int_0^1 I_{j,t} dj}_{\text{profits from intermediate goods sector}}.$$

We also use the equilibrium condition $B_{t+1} = 0$, which is implied by the assumption of identical households.

the trend component of productivity. Using this definition, we can write equation (8) as

$$g_{t+1} = 1 + \chi \frac{I_t}{A_t}.$$

This expression implies that higher investment in period t is associated with faster productivity growth between periods t and $t+1$.

2.6 Nominal rigidities and monetary policy

We consider an economy with frictions in the adjustment of nominal wages.[14] The presence of nominal wage rigidities plays two roles in the model. First, it creates the possibility of involuntary unemployment, by ensuring that nominal wages remain positive even when employment falls short of households' labor supply. Second, it implies that monetary interventions have real effects. Since prices inherit part of wage stickiness, in fact, by setting the nominal interest rate the central bank can affect the real interest rate.

Inspired by the empirical literature on wage Phillips curves (Galí and Gambetti, 2020), we assume that nominal wages evolve according to

$$\frac{W_t}{W_{t-1}} = \bar{g} \left(\frac{L_t}{\bar{\bar{L}}} \right)^\xi \pi_{t-1}^\lambda, \tag{12}$$

where $\xi > 0$, $0 \leq \lambda < 1$, and π_{t-1} denotes lagged gross price inflation. According to this equation, as in standard Phillips curves, an increase in employment puts upward pressure on wage growth. Moreover, when $\lambda > 0$ wages are partially indexed to past price inflation. While not crucial for our results, this feature is helpful to obtain reasonable inflation dynamics in response to supply shocks. Finally, we normalize wage inflation in the full employment steady state to be equal to productivity growth, denoted by $\bar{g}$.[15]

Using the wage-setting rule (12), as well as (3) and (6), we get an expression for price inflation

$$\pi_t = \frac{\bar{g}}{g_t} \frac{Z_{t-1}}{Z_t} \left(\frac{L_t}{\bar{\bar{L}}} \right)^\xi \pi_{t-1}^\lambda. \tag{13}$$

Intuitively, firms set prices equal to their marginal cost. Higher wage inflation puts upward pressure on marginal costs and leads to higher price inflation, while faster productivity growth reduces marginal costs and lowers price inflation. This explains why price inflation is increasing in employment, and decreasing in productivity growth. The term π_{t-1}^λ captures the inflation persistence arising from the indexation of wages. Absent this term, and since our model abstracts from price

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[14] A growing body of evidence emphasizes how nominal wage rigidities represent an important transmission channel through which monetary policy affects the real economy. For instance, this conclusion is reached by Olivei and Tenreyro (2007), who show that monetary policy shocks in the US have a bigger impact on output in the aftermath of the season in which wages are adjusted. Micro-level evidence on the importance of nominal wage rigidities is provided, for instance, by Fehr and Goette (2005), Gottschalk (2005), Barattieri et al. (2014) and Gertler et al. (2020).

[15] In Appendix D, we show that the main insights of the paper are preserved under a conventional New Keynesian wage Phillips curve, derived from the presence of wage adjustment costs à la Rotemberg (1982).

rigidities, inflation would be excessively volatile in response to supply shocks. Given our assumptions about wage inflation, inflation in the full employment steady state - which can be interpreted as the medium run central bank's target - is normalized to zero.

The central bank sets monetary policy by controlling the nominal interest rate i_t. Throughout the paper we will explore different monetary policy strategies.

2.7 Summary of equilibrium conditions

The equilibrium of the economy can be described by four equations. The first one is a standard IS equation, capturing consumers' behavior. It is obtained by rewriting the Euler equation (1) as

$$c_t = \frac{g_{t+1} c_{t+1} \pi_{t+1}}{\beta(1 + i_t)}, \tag{IS}$$

where we have defined $c_t \equiv C_t/A_t$ as consumption normalized by the trend component of productivity. As it is standard, this equation implies that current demand for consumption is increasing in future (normalized) consumption and decreasing in the real interest rate. It also implies a positive relationship between trend productivity growth and present demand for consumption. The reason is that faster productivity growth is associated with higher future incomes, which raises consumption demand through a positive wealth effect.

The second key relationship in our model is the *growth* equation, which is obtained by combining equation (1) with the optimality condition for investment (9)

$$g_{t+1} = \beta \frac{c_t}{c_{t+1}} \left(\chi \varpi Z_{t+1} L_{t+1} + 1 - \eta \right). \tag{GG}$$

This equation implies a positive relationship between growth and future market size. Intuitively, a rise in $Z_{t+1} L_{t+1}$ is associated with higher future monopoly profits. In turn, higher profits induce entrepreneurs to invest more, leading to a positive impact on the growth rate of the economy. This is the classic market size effect emphasized by the endogenous growth literature.[16] At the same time, growth depends inversely on the growth rate of normalized consumption c_{t+1}/c_t. This is a cost of funds effect: when today's consumption is low relative to consumption in the future, firms pay out dividends to households rather than invest.

The third equation combines the goods market clearing condition (10), the GDP equation (11) and the fact that $I_t/A_t = (g_{t+1} - 1)/\chi$

$$\Psi Z_t L_t = c_t + \frac{g_{t+1} - 1}{\chi}. \tag{MK}$$

Keeping GDP constant, this equation implies a negative relationship between consumption and growth, because to generate faster growth the economy has to devote a larger fraction of output

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[16]To be clear, what matters for our results is that productivity growth is increasing in employment relative to potential. This means that our key results would also apply to a setting in which scale effects related to population size were not present. For instance, in the spirit of Young (1998) and Howitt (1999), these scale effects could be removed by assuming that the number of intermediate inputs is proportional to population size.

to investment, reducing the resources available for consumption.

The fourth equation is just the price Phillips curve, which we rewrite here for convenience

$$\pi_t = \frac{\bar{g}}{g_t} \frac{Z_{t-1}}{Z_t} \left(\frac{L_t}{\bar{L}} \right)^\xi \pi_{t-1}^\lambda. \tag{PC}$$

We are now ready to define an equilibrium as a set of sequences $\{g_{t+1}, L_t, c_t, \pi_t\}_{t=0}^{+\infty}$ satisfying (IS), (GG), (MK) and (PC), as well as $0 < L_t \leq \bar{L}$, $g_{t+1} > 1$ and $c_t > 0$ for all $t \geq 0$, given paths for monetary policy $\{i_t\}_{t=0}^{+\infty}$ and the supply shock $\{Z_t\}_{t=0}^{+\infty}$, and the initial conditions $\{\pi_{-1}, Z_{-1}\}$.

2.8   The balanced growth path

Before studying the implications of the model, it is useful to spend a few words on the balanced growth path - or steady state - of the economy. A steady state is characterized by constant values for g_{t+1}, L_t, c_t, i_t, π_t and Z_t that satisfy the above equilibrium conditions. For most of the paper, we will be studying economies that fluctuate around a full employment steady state. We denote the value of a variable in this steady state with an upper bar, and normalize the exogenous component of productivity to $\bar{Z} = 1$. We now make some assumptions to ensure that a full employment steady state exists.

Proposition 1 *Suppose that the parameters satisfy*

$$\beta(\chi \varpi \bar{L} + 1 - \eta) > 1, \tag{14}$$

and that monetary policy is such that

$$\bar{i} = \chi \varpi \bar{L} - \eta. \tag{15}$$

Then there exists a unique full employment steady state. Moreover, this steady state is characterized by $\bar{g} > 1$.

Intuitively, condition (14) guarantees that in the full employment steady state the return to investment is sufficiently high so that growth is positive. Condition (15), instead, ensures that the central bank policy is consistent with the existence of a full employment steady state.

3   Macroeconomic implications of supply disruptions

We start by studying how supply disruptions affect the path of potential output and aggregate demand, and highlight the central role played by hysteresis effects. In particular, we show that the persistent scars left by supply disruptions depress aggregate demand and equilibrium interest rates. We then argue that hysteresis effects amplify the rise in inflation associated with negative supply shocks.

To make these points, throughout this section we assume that monetary policy maintains the economy at full employment ($L_t = \bar{L}$), and so output is equal to potential. In our simple model, this

corresponds to the allocation that would prevail under flexible wages, i.e. the natural allocation. Following standard practice, we define the natural interest rate as the equilibrium real interest rate in the natural allocation.

3.1 An exogenous growth benchmark

Let us first study a counterfactual economy in which there is no investment and trend productivity growth is exogenous. As is well known from the New Keynesian literature (Galí, 2009), in this case the natural interest rate rises after a negative supply shock, indicating that the supply disruption depresses aggregate supply by more than demand.[17] Intuitively, this happens because households' permanent income falls by little in response to a temporary supply disruption.

To illustrate this point, assume that firms don't undertake any investment, that trend growth is exogenous and equal to $g_t = \bar{g}$, and that Z_t follows the process

$$\log Z_t = \rho \log Z_{t-1}, \tag{16}$$

where $0 \leq \rho < 1$ determines the persistence of the productivity shock.

Now suppose that the economy is hit by a previously unexpected negative shock, which corresponds to the initial condition $Z_0 < 1$. The IS equation then implies that the natural interest rate evolves according to[18]

$$1 + \bar{r}_t = \frac{\bar{g} Z_{t+1}}{\beta Z_t} = \frac{\bar{g} Z_0^{\rho^t (\rho-1)}}{\beta},$$

where, from now on, we use upper bars to denote variables in the natural allocation. Since $Z_0 < 1$, this expression implies that the natural interest rate increases in response to a temporary supply disruption.

These results form part of the conventional wisdom on the macroeconomic implications of supply disruptions. As we will see, however, this conventional wisdom might fail once the impact of supply shocks on investment and productivity growth is taken into account.

3.2 Back to the Keynesian growth framework

We now revisit the conventional wisdom through the lens of our Keynesian growth model. Our first result is that supply disruptions trigger persistent drops in potential output and a negative wealth effect, which depresses aggregate demand. Indeed, this effect can be so strong that a negative supply shock might cause a demand shortage that is larger than the supply disruption itself. When this happens, the natural interest rate falls rather than rises in response to a negative supply shock.

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[17]The natural interest rate can be understood as a summary statistic of the balance between aggregate demand and supply in an economy. For instance, a high natural rate indicates an economy where aggregate demand is strong relative to supply. As we do, Guerrieri et al. (2022) use this insight to study the response of aggregate demand to supply disruptions.

[18]To derive this expression, notice that, since firms don't invest, all the output is consumed and so $c_t = \Psi Z_t \bar{L}$ for all t.

Proposition 2 *Assume that Z_t is governed by the process* (16), *and that $Z_0 < 1$. If $\rho > 0$, the natural interest rate drops below its steady state value, $\bar{r}_t < \bar{r}$, for all $t \geq 0$. If $\rho = 0$, the natural rate remains equal to its steady state value, $\bar{r}_t = \bar{r}$, for all $t \geq 0$.*

To understand Proposition 2, let us start by studying the behavior of investment and productivity growth. According to the GG equation, in the natural allocation productivity growth evolves according to

$$\bar{g}_{t+1} = \beta \frac{\bar{c}_t}{\bar{c}_{t+1}} \left(\chi \varpi Z_{t+1} \bar{L} + 1 - \eta \right).$$

There are two channels through which a supply disruption reduces growth. First, a transitory drop in Z_t leads to a drop in $\bar{c}_t / \bar{c}_{t+1}$. This corresponds to an increase in the rate at which households discount future profits, reducing firms' incentives to invest. Second, if the shock is persistent, the fall in Z_{t+1} lowers the profits that firms appropriate by investing in innovation. Both effects point toward a negative impact of supply disruptions on investment and productivity growth.

What are the implications of the fall in investment for aggregate demand? Because investment is a component of aggregate demand, the fall in investment constitutes a drag on aggregate demand in itself. Quantitatively, this effect is stronger, the higher the share of investment in GDP. Following standard practice in the endogenous growth literature, we may interpret firms' investment in innovation as their expenditure in R&D. Given that spending in R&D represents a small fraction of GDP, the direct impact of fluctuations in investment on aggregate demand in our model will be small.[19]

There is, however, a second channel through which a fall in investment depresses aggregate demand. Lower investment drives down productivity growth and future output, and so agents' future incomes. This negative wealth effect causes a drop in consumption demand. This second effect, on its own, might be strong enough to reverse the response of the natural rate to a supply shock relative to the case in which productivity growth is exogenous. To see this point most clearly, consider the limit case where the investment share of GDP goes to zero, so that $\bar{c}_t \approx \Psi Z_t \bar{L}$. Then use the IS equation to obtain

$$1 + \bar{r}_t = \frac{\bar{g}_{t+1} Z_0^{\rho^t(\rho-1)}}{\beta}.$$

This expression shows how the endogenous drop in $\bar{g}_{t+1}$ puts downward pressure on the natural interest rate. As we show in Proposition 2, as long as the supply disturbance is persistent ($\rho > 0$), this effect is strong enough to induce a drop in the natural interest rate following a supply disruption. Hence, in our baseline model, supply disruptions trigger demand shortages that are larger than the supply disruption itself.

To further illustrate these results, we resort to a numerical simulation that shows how our model behaves under reasonable parameter values. We choose the length of a period to correspond to one year. We set χ, α and β by targeting three moments of the full employment steady state.

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[19]That said, Howitt and Aghion (1998) highlight how investment in innovation may be complementary to other forms of investment, including in physical capital. Hence, a drop in investment in innovation may depress aggregate demand by triggering a fall in other types of investment. We leave the study of this transmission channel to future research.

χ is set to 2.081 so that steady state productivity growth is equal to 2%. We set the labor share in gross output to $1 - \alpha = 0.837$, to match a ratio of spending on investment in innovation to GDP of 2%, close to the GDP share of business spending in R&D observed in the United States. This calibration choice implies that investment in innovation is a small component of aggregate demand. We choose $\beta = 0.995$ so that the real interest rate in steady state is equal to 2.5%. To calibrate the interest rate wedge η, we use the evidence provided by Gormsen and Huber (2022). In their sample of US firms the average nominal discount rate is 16% per year. Assuming an average inflation rate of 2%, this translates into a yearly real discount rate of 14%. Given that we target a real interest rate of 2.5%, to match this statistic we set $\eta = 0.14 - 0.025 = 0.115$. Turning to the supply shock, we set $\log(Z_0) = -0.03$ and $\rho = 0.5$, so that on impact potential output drops by 3% and the half life of the shock is one year.

Figure 1 illustrates the macroeconomic impact of a supply disruption, assuming that monetary policy replicates the full employment allocation. When productivity growth is exogenous, supply drops more than demand, and the real interest rate rises sharply. Moreover, the initial recession is followed by a period of fast recovery, so that output goes back to its pre-recession trend once the shock dissipates. In our Keynesian growth model, instead, the temporary supply disruption causes a drop in investment and in the trend component of productivity growth $\bar{g}_t$, and so a sizable decline in long-run output. The associated decline in households' wealth triggers a fall in consumers' demand, which explains why the real interest rate declines, rather than rises.

How do the predictions of the Keynesian growth model compare with the data? The presence of hysteresis effects is consistent with a growing body of empirical evidence showing that recessions, including those triggered by negative supply shocks such as oil shocks, are followed by extremely persistent deviations of output from its pre-recession trend (Cerra and Saxena, 2008; Blanchard et al., 2015; Bluedorn and Leigh, 2018; Fatás and Summers, 2018; Aikman et al., 2022).[20] While more research is needed to carefully quantify the empirical strength of hysteresis effects, the magnitudes implied by our model are in line with the estimates provided by Bluedorn and Leigh (2018). Using surveys of professional forecasters in 38 countries, they find that a 1% unexpected drop in current output is associated with a roughly 1% drop in output ten years forward. Interestingly, they also find that these effects are largely anticipated by professional forecasters, lending support to the notion that recessions lead economic agents to sharply revise downward their estimates of future potential output and wealth. Fatás and Summers (2018), who study the medium-run impact on ouput of fiscal consolidations, find hysteresis effects of similar magnitude. Finally, let us note that periods of large supply disruptions, such as the oil shocks of the 1970s or the recovery from the Covid-19 pandemic, tend to be associated with low real interest rates, in line with the predictions of our Keynesian growth model.

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[20]If interpreted literally, our model would imply that recessions may have a permanent impact on potential output. Of course, empirically it is not possible to distinguish between permanent or very persistent hysteresis effects. For this reason, we see our framework as capturing an economy in which temporary shocks may affect future output for longer than what traditional models imply.
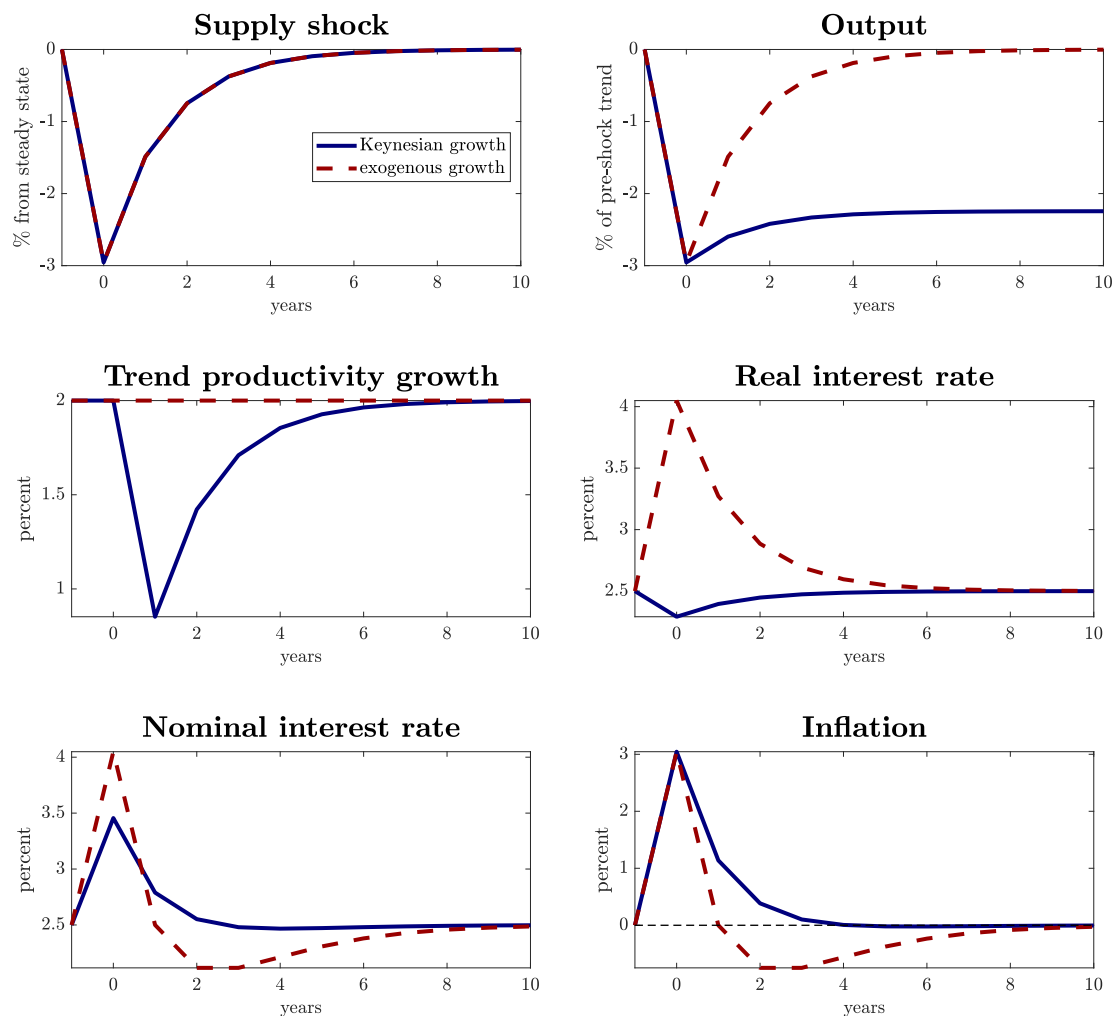
Figure 1: Macroeconomic impact of supply disruptions.

3.3 Can supply disruptions cause persistent rises in inflation?

Large supply disruptions typically trigger prolonged periods of high inflation. For instance, the 1970s oil shocks were accompanied by a full decade of high inflation. Similarly, the ongoing burst of inflation that is characterizing the recovery from the pandemic has been far more persistent than what many expected. We now show that scarring effects help explain this fact.

The bottom panels of Figure 1 illustrate the response of nominal variables to the same transitory negative supply shock considered in Section 3.2, assuming that monetary policy maintains the economy at full employment $(L_t = \bar{L})$.[21] Both in the Keynesian and exogenous growth models nominal interest rates and inflation rise in response to the supply disruption. What stands out, however, is that the Keynesian growth model exhibits much more inflation persistence than the exogenous growth one. Let's see why.

Start by considering the exogenous growth model $(g_t = \bar{g})$. To gain analytic insights, abstract

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[21]We set the parameter capturing inflation persistence in the Phillips curve to $\lambda = 0.5$, in line with the estimates provided by Barnichon and Mesters (2020).

for a second from wage indexation ($\lambda = 0$). Then equation PC implies

$$\bar{\pi}_t = \frac{Z_{t-1}}{Z_t} = \begin{cases} \frac{1}{Z_0} & \text{for } t = 0 \\ Z_0^{(1-\rho)\rho^{t-1}} & \text{for } t > 0. \end{cases}$$

Since $Z_0 < 1$, on impact the negative shock triggers a rise in inflation. Intuitively, lower productivity drives down the equilibrium real wages. Since nominal wages are rigid, the drop in real wages takes place through a burst of inflation. These dynamics capture the standard view that negative supply shocks are inflationary (Blanchard and Galí, 2007a,b). The rise in inflation is, however, extremely transitory. As productivity recovers, the reason is, firms' marginal costs decline over time, which results in a period of inflation below target.[22]

Now turn to the Keynesian growth model. To derive intuition, again assume that $\lambda = 0$ and that the ratio of innovation spending to GDP is close to zero, so that $\bar{c}_t \approx \Psi Z_t \bar{L}$. Equation PC then implies

$$\bar{\pi}_t = \frac{\bar{g}}{g_t} \frac{Z_{t-1}}{Z_t} = \begin{cases} \frac{1}{Z_0} & \text{for } t = 0 \\ \frac{\bar{g}/\beta}{\chi \varpi Z_0^{\rho^t} + 1 - \eta} & \text{for } t > 0. \end{cases}$$

Therefore, on impact the rise in inflation is exactly the same as in the exogenous growth case. Subsequently, however, the economy experiences a prolonged period of inflation above target. The associated rise in expected inflation explains why the nominal interest rate increases, even though weak demand depresses the equilibrium real interest rate.

The driver of inflation persistence is the drop in the endogenous component of productivity growth $\bar{g}_t$ caused by the supply disruption. Indeed, lower productivity growth sustains firms' marginal costs, leading to higher inflation. Thus, hysteresis effects reinforce the rise in inflation caused by negative supply shocks. Moreover, the additional inflation due to scarring arises with a delay. This happens because current investment decisions affect productivity with a one-period lag. Hence if a supply shock disrupts investment in period t, the inflation triggered by the decline in productivity will only be felt in period $t+1$.[23] This delay explains why, with scarring effects, a temporary supply disruption gives rise to a protracted period of stubbornly high inflation.[24]

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[22]Inflation falls below its steady state value for periods $t > 0$, as long as λ is not too large. For instance, this is the case in the example shown in Figure 1. With sufficient wage indexation to past inflation, the rise in inflation triggered by a transitory negative productivity shock could be persistent.

[23]Indeed, the endogenous component of productivity does not react to the shock in period 0, since it is determined by investment decisions taken before the shock was foreseen. This is the reason why the Keynesian growth economy experiences the same rise in inflation in $t = 0$ as the exogenous growth one.

[24]In our model, it takes one period for investment to increase productivity. While this assumption is standard in the literature, it is easy to imagine a scenario in which investment projects only pay off several periods in the future. For instance, Aghion et al. (2022) find that on average investments in innovation have a positive impact on productivity after 2 to 5 years. Comin and Gertler (2006) argue that, due to long diffusion lags, it takes on average about 10 years before new technologies are adopted by firms. By taking such additional lags into account, our model would predict even more endogenous persistence of the inflation spell triggered by supply disruptions.

3.4 Modulating hysteresis effects

What determines the strength of hysteresis effects, i.e. the impact of a temporary supply disruption on trend output? In Appendix E we show that, up to a first order approximation, in the natural allocation a one percent drop in Z_t leads to a percent drop in trend output below its pre-shock trend equal to

$$\gamma_h = \frac{1}{1 + \frac{\bar{g}}{\bar{g}-1}(1-\rho)\frac{s_i}{1-s_i}} \left(\frac{1}{1-s_i} + \frac{\rho}{1-\rho} \frac{(\bar{g} - \beta(1-\eta))}{\bar{g}} \right), \tag{17}$$

where s_i denotes the share of investment in GDP in steady state. As this equation shows, hysteresis effects are stronger if the rate at which firms discount future profits, i.e. η, is higher. The reason is that investment is more sensitive to temporary shocks when firms are more short sighted. Moreover, hysteresis effects are increasing in ρ due to two effects. First, a more persistent supply disruption triggers a larger drop in future profits, and so has a bigger negative impact on firms' incentives to invest. Second, a more persistent supply shock implies that growth is below its trend for longer, which amplifies hysteresis effects.[25]

Another determinant of the strength of hysteresis effects is the degree of diminishing returns to investment. In our baseline model there is a linear relationship between investment in innovation and productivity growth. This is a common assumption in the theoretical literature on endogenous growth, since it is consistent with free entry in the research sector (Aghion and Howitt, 1992). Quantitative analyses, however, often assume that investment in innovation is subject to diminishing returns, to capture the existence of congestion externalities or adjustment costs (Acemoglu and Akcigit, 2012).

We study this case in Appendix E, where we consider an economy in which the trend component of productivity grows according to

$$g_{t+1} = 1 + \chi \left(\frac{I_t}{A_t} \right)^\zeta,$$

where the parameter $0 \leq \zeta \leq 1$ captures the strength of diminishing returns in investment in innovation. This formulation is attractive, because it provides a simple way to modulate hysteresis effects. Intuitively, as ζ drops investment becomes less responsive to changes in macroeconomic conditions, and hysteresis effects become smaller. Our baseline model corresponds to $\zeta = 1$, while the polar opposite case $\zeta = 0$ captures an economy with exogenous growth. For intermediate values of ζ, the impulse responses of our model simply fall in between the solid and dashed lines in Figure 1.

It turns out that, to match hysteresis effects in the ballpark of those estimated by Bluedorn and Leigh (2018) and Fatás and Summers (2018), the model needs values of ζ very close to one.[26]

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[25]Under our parametrization $\gamma_h = 0.76$, i.e. the long-run drop in output is 76% of the initial output drop. This is in line with the simulations displayed in Figure 1.

[26]Interestingly, Garga and Singh (2020) reach a similar conclusion. In fact, they find that to match the empirical response of productivity to monetary shocks their model has to feature an innovation investment function close to linear.

Besides analytical tractability, this is the reason why in our baseline model we have chosen to focus on a linear investment function. However, it is still interesting to explore how our results change with the degree of hysteresis effects.

For instance, in Appendix E we show that with diminishing returns to investment it is no longer true that the natural interest rate unambiguously declines following a supply disruption. In fact, a supply disruption triggers a drop in the natural rate only if it is sufficiently persistent. The intuition is that concavity in the investment function dampens the scarring effects of supply disruptions, implying a smaller drop in aggregate demand. That said, even in the case of short-lived supply disruptions, it is still true that the natural interest rate rises by less compared to an economy with exogenous productivity growth.[27]

4 Disinflation strategies

We now ask which policy interventions may contain the inflation burst associated with a negative supply shock.[28] We start by studying the impact of monetary tightenings, and show that they may backfire by triggering a supply-demand doom loop that amplifies hysteresis effects and the medium-run rise of inflation. We then argue that a successful disinflation can be the outcome of a monetary tightening coupled with fiscal interventions that support business investment.

4.1 Tight money and the supply-demand doom loop

So far we have assumed that the central bank targets full employment at all times. We now consider a tighter monetary policy, under which negative supply shocks cause involuntary unemployment and a negative output gap. In particular, the central bank now follows the monetary policy rule

$$1 + i_t = (1 + \bar{r}) \left(\frac{L_t}{\bar{L}} \right)^\phi \pi_{t+1}, \tag{18}$$

which satisfies

$$\phi > \frac{\chi \varpi \bar{L}}{\chi \varpi \bar{L} + 1 - \eta}. \tag{19}$$

These assumptions ensure that there is a unique steady state, characterized by zero inflation and full employment, and that this steady state is locally determinate.[29] This rule, however, allows for temporary deviations of the economy from full employment.

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[27]Even if one maintains the assumption of a linear investment function, there is a case in which the natural rate might rise after a negative supply disruption. This might happen if the shock is large enough to drive investment in innovation, and the endogenous component of productivity growth, to zero. The reason is simple. Once firms stop investing in innovation, further drops in Z_t no longer depress the endogenous component of productivity growth - and the associated drag on consumers' demand becomes muted. This means that the impact of supply disruptions on the natural rate might become non-linear. In particular, the natural rate might rise in response to a negative supply shock severe enough to drive investment in innovation to zero.

[28]To be clear, in this section we take a purely positive perspective by describing the macroeconomic impact of different policy interventions. We collect some insights about optimal policy in Appendix C.

[29]See Appendix B.3 for a proof.

4.1.1 An insightful case: a permanent supply disruption

While our focus is on temporary supply disruptions, it is useful to first study a case in which Z_t drops permanently to a lower level. The advantage is that, by focusing on a permanent shock, we can illustrate the key forces at the heart of the model using a simple graphical analysis.

Figure 2 shows how L and g are determined in steady state. The GG schedule corresponds to the growth equation evaluated in steady state

$$g = \beta(\chi \varpi ZL + 1 - \eta), \tag{GG}$$

and implies a positive relationship between g and L. Intuitively, an increase in employment - and so in market size - is associated with a rise in the return from investing in innovation. Firms respond by increasing investment and productivity growth accelerates.

The AD curve, instead, summarizes the aggregate demand side of the model. It is obtained by combining the IS equation and (18), evaluated in steady state

$$g = \beta(1 + \bar{r}) \left(\frac{L}{\bar{L}} \right)^\phi. \tag{AD}$$

This equation implies a positive relationship between g and L. To understand the intuition behind this equation, consider what happens after a rise in productivity growth. Due to the associated positive wealth effect, households respond by increasing their demand for borrowing and consumption. Higher consumption, in turn, puts upward pressure on employment. The central bank reacts to the rise in employment by increasing the interest rate, which cools down households' demand for borrowing and restores equilibrium on the credit market.

A steady state equilibrium corresponds to an intersection of the AD and GG curves. Under our assumption about ϕ, there is only one intersection between the two curves satisfying $L \leq \bar{L}$, meaning that the steady state exists and is unique.[30] The steady state shown in the left panel of Figure 2 corresponds to the full employment steady state.

Now imagine that we start from the full employment steady state, and a previously unexpected permanent fall in Z occurs. As shown in the right panel of Figure 2, the decline in Z induces a downward shift of the GG curve. As already explained, the exogenous fall in labor productivity depresses firms' profits and their incentives to invest. Firms react by reducing investment and so, holding constant L, productivity growth g drops. The fall in productivity growth, through its negative wealth effect, translates into lower aggregate demand. By our assumptions, the central bank does not impart enough stimulus to prevent unemployment from arising. The result is a drop in employment below the full employment level ($L < \bar{L}$). Consequently, the negative supply shock gives rise to a drop in aggregate demand and involuntary unemployment.[31]

This is not, however, the end of the story. Lower demand further reduces firms' profits and their

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[30]Moreover, under our assumption about ϕ, the AD curve is steeper than the GG curve at their intersection.

[31]This effect is well known from the literature on news shocks (e.g., Lorenzoni, 2009).
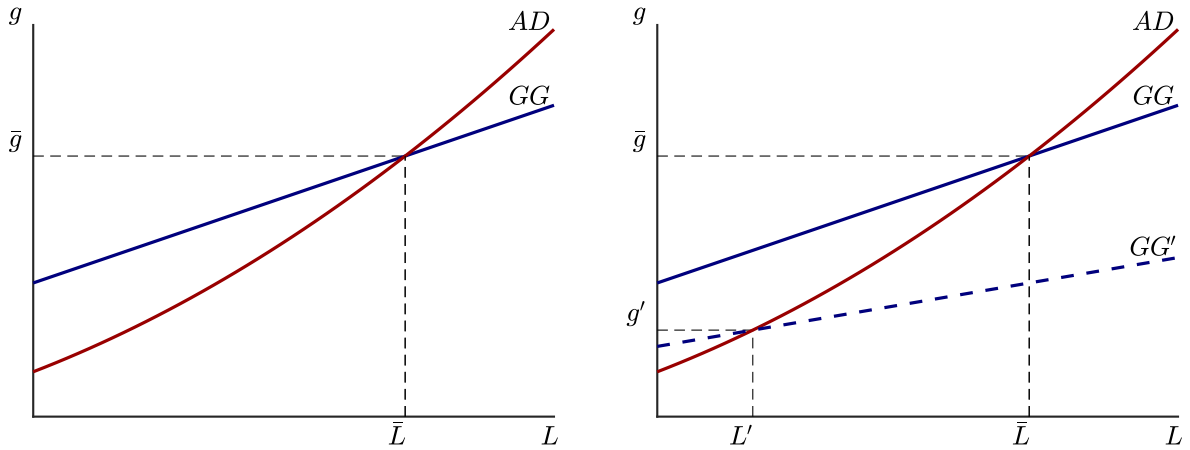
(a) The $L - g$ diagram.　　　　　　(b) A permanent supply disruption.

Figure 2: The $L - g$ diagram and permanent supply disruptions.

incentives to invest. This effect generates another round of drops in investment and productivity growth. Lower productivity growth, in turn, induces a further cut in demand, which again lowers investment and growth. This vicious spiral, or supply-demand doom loop, amplifies the impact of the initial supply shock on employment and labor productivity growth.

It is possible to derive an expression for the elasticity of the endogenous component of productivity growth with respect to the supply shock. Combining GG and AD and differentiating gives

$$\left(\frac{\partial g}{\partial Z}\right)\frac{Z}{g}\bigg|_{Z=1} = \frac{1 - (1-\eta)\beta/\bar{g}}{1 - \frac{1-(1-\eta)\beta/\bar{g}}{\phi}}.$$

In this expression, the numerator captures the direct impact of a change in Z on g. In line with our arguments in Section 3.4, in our model this direct effect is large when the interest rate wedge η is large. The denominator, instead, captures the multiplier effect associated with the supply-demand doom loop. This multiplier effect is decreasing in ϕ.[32] As it is intuitive, a smaller response of monetary policy to changes in employment amplifies the impact of supply shocks on productivity growth.

Therefore, in our framework monetary policy has an impact on the endogenous component of productivity growth. In particular, a tighter monetary stance leads to a slowdown in investment in innovation and productivity growth. This feature of the model is consistent with a growing body of empirical evidence (Garga and Singh, 2020; Moran and Queraltó, 2018; Jordà et al., 2020; Grimm et al., 2022), suggesting that monetary policy tightenings induce firms to cut investment

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[32]Notice that $\bar{g} = \beta(\chi\varpi\bar{L} + 1 - \eta)$. The denominator can thus be written as

$$1 - \frac{\chi\varpi\bar{L}}{\chi\varpi\bar{L} + 1 - \eta}\Big/\phi.$$

By assumption (19), the denominator is therefore positive for any permissible level of ϕ.

in innovation, such as R&D, and have a long lasting negative impact on productivity and potential output.

4.1.2 A temporary supply disruption

We turn back to the case of temporary shocks, by again assuming that Z_t evolves according to the process (16). In the case of temporary shocks, deriving analytic results is more challenging. However, some insights can be obtained by combining IS, GG and (18) to obtain

$$(1 + \bar{r}) \left(\frac{L_t}{\bar{L}} \right)^\phi = \chi \varpi Z_{t+1} L_{t+1} + 1 - \eta.$$

Since $L_{t+1} \leq \bar{L}$, this equation directly implies that a future supply disruption ($Z_{t+1} < 1$) causes involuntary unemployment in the present ($L_t < \bar{L}$). Of course, a lower L_t leads to a reduction in the incentives to invest in period $t - 1$, which lowers aggregate demand and employment in period $t - 1$, and so on. Thus, in the case of temporary shocks an intertemporal supply-demand doom loop emerges.

Figure 3 shows the response of the economy to a negative supply shock, contrasting the neutral monetary policy - which replicates the natural allocation - to a tighter monetary response which follows the rule (18). We set the strength of the monetary response to unemployment to $\phi = 0.17$, so that in the impact period of the shock employment falls by 2%. Given our calibration of the size of the shock, this implies that on impact the supply disruption causes a 5% decline in output. All other model parameters are kept as in Section 3.

As we can see in Figure 3, a tighter monetary response induces an additional decline of productivity growth, which deepens the permanent output loss. Because of the logic of the supply-demand doom loop, the effect of a temporary negative output gap on the permanent component of productivity can be quite large. Hence a tight monetary stance may greatly amplify the direct impact of negative supply shocks on employment and productivity growth.

4.2 Challenges to a purely monetary disinflation

What are the implications of a tight monetary stance for inflation? The conventional view is that a monetary contraction, by increasing unemployment, lowers inflation due to the Phillips curve logic. This force is present in our framework, but there is more. As we argued in the previous section, a monetary tightening causes a drop in firms' investment. The additional decline in productivity growth, in turn, tends to push inflation up. Moreover, this effect happens with a delay, since it takes time for investment to have an impact on productivity. A monetary tightening may thus be successful at reducing inflation initially, but at the cost of higher inflation in the medium run.

To illustrate this effect, we start by considering a purely transitory monetary tightening. That is, assume that the central bank raises the nominal interest rate in period 0, but brings it back to its steady state from $t = 1$ on. It is convenient to abstract from wage indexation ($\lambda = 0$), and to
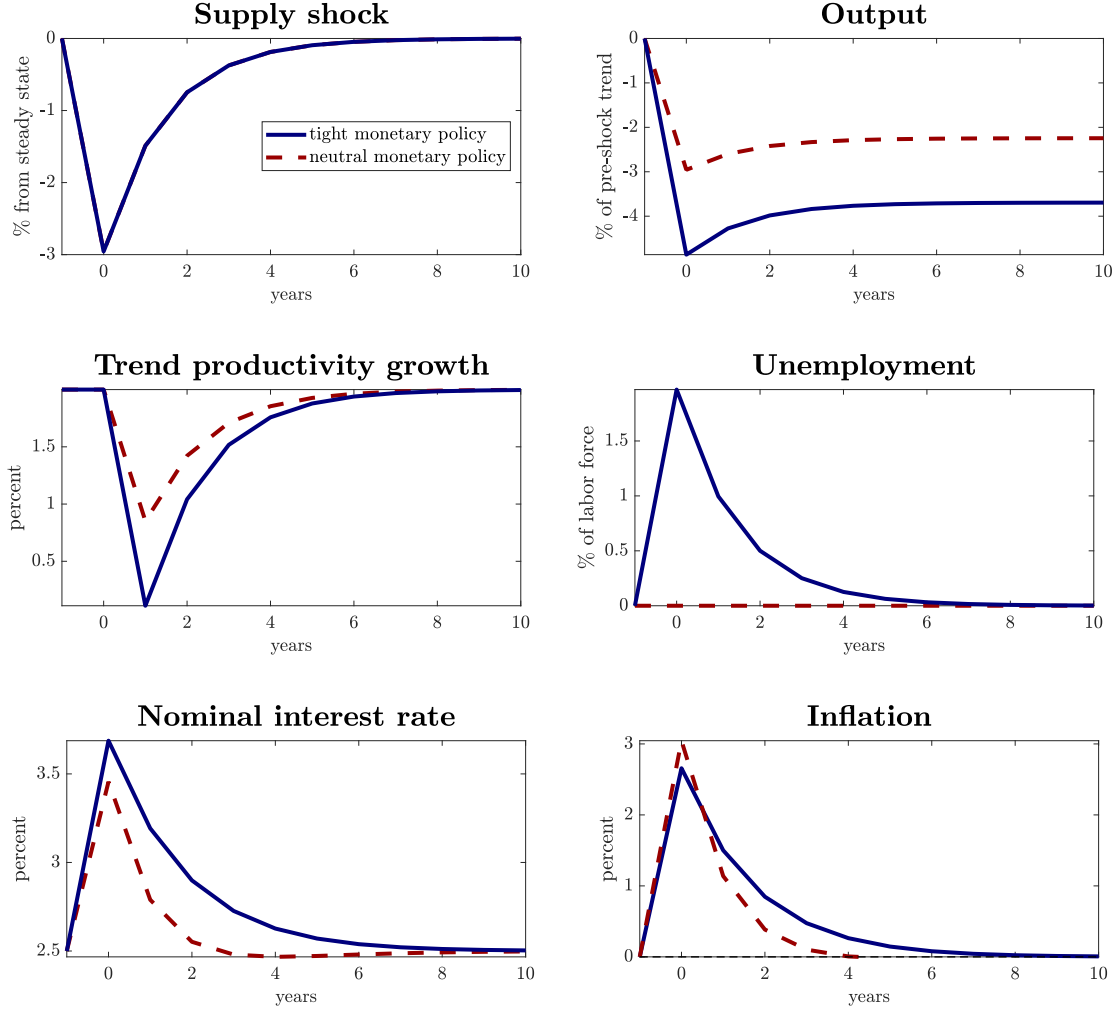
Figure 3: Tight monetary policy deepens hysteresis effects.

exploit once again the approximation $c_t \approx \Psi Z_t L_t$. With a bit of algebra, one can show that

$$L_0 = \bar{L}\frac{1+\bar{i}}{1+i_0}$$

$$g_1 = \bar{g}\frac{1+\bar{i}}{1+i_0}$$

where $1+\bar{i} = \bar{g}/\beta$ is the nominal interest rate in steady state. In line with the analysis in the previous section, the monetary tightening lowers employment, but it also depresses investment and so future productivity growth. Using (13), we can now back out the response of inflation

$$\pi_0 = \left(\frac{1+\bar{i}}{1+i_0}\right)^{\xi}$$

$$\pi_1 = \frac{1+i_0}{1+\bar{i}}.$$

Thus, the monetary tightening lowers inflation on impact, because it depresses employment and

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nominal wage growth. It does, however, increase inflation with a one-period delay, since lower productivity growth translates into higher future production costs.

These insights are more general, and extend to an arbitrary process for the nominal interest rate, as we summarize in the following proposition.

Proposition 3 *Assume that* $\lambda = 0$ *and that* $c_t \approx \Psi Z_t L_t$. *Consider the effects of a monetary policy shock,* $i_t \geq \bar{i}$ *for all* $t \geq 0$. *We have three results. First,* $L_t < \bar{L}$ *whenever* $i_t > \bar{i}$. *Second,* $i_0 > \bar{i}$ *implies that* $\pi_0 < 1$. *Third,* $\pi_{t+1} > 1$ *whenever* $i_t > \bar{i}$.

Proposition 3 makes two points. First, tight monetary policy depresses employment and thus wage inflation. Second, when wages are not indexed to past inflation, a monetary tightening reduces price inflation only on impact. Thereafter, inflation rises above its steady state value. Intuitively, this happens because the unemployment effect - driving inflation down - is dominated by the productivity effect - driving inflation up. This is true except in the impact period, when productivity is predetermined with respect to the monetary shock.

There are two caveats to Proposition 3 worth mentioning. First, the proposition abstracts from wages indexation to past inflation. With indexation, a monetary tightening could lead to a persistent drop in inflation. Second, our baseline model features a linear investment function. With decreasing returns to investment the hysteresis effects are milder, and one can think of scenarios in which a monetary tightening persistently lowers inflation even when wages are not indexed to past inflation. That said, as we show in Appendix E, even in this case the impact of a monetary tightening on inflation would be smaller than in an economy with exogenous growth.

To close this section, we go back to our numerical example.[33] The bottom-right panel in Figure 3 shows how inflation reacts to a monetary tightening implemented during a supply disruption. As expected, the monetary tightening causes a reduction in inflation on impact. In the medium run, however, the monetary tightening becomes self-defeating. In fact, the productivity scars associated with tight money eventually push inflation above its value under the neutral policy. These results sound a note of caution on the macroeconomic impact of blunt monetary tightenings in response to supply disruptions. Not only a monetary tightening is likely to lead to lower investment and productivity growth, but it may also fail to mitigate significantly inflation in the medium run.[34]

4.3 Protecting productive capacity during a disinflation

Hysteresis effects appear to put central banks in front of a dilemma: reducing inflation in the present comes at the cost of lower productivity growth which raises inflation in the future. This dynamic trade-off arises because monetary tightenings damage the future productive capacity of

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[33]We set the slope of the Phillips curve to $\xi = 0.19$. As we show in Appendix D, this slope is consistent with a Calvo model of wage adjustment with a 25% quarterly probability that a wage is reset, which is in line with the estimates of Beraja et al. (2019). Moreover, a value of $\xi = 0.19$ falls into the range of the empirical Phillips curves estimates for the United States. For instance, Barnichon and Mesters (2020) estimate a slope of 0.4, while Hazell et al. (2022) estimate a flatter Phillips curve with a slope close to 0.1.

[34]Drechsler et al. (2022) provide some evidence suggesting that monetary tightenings - by disrupting the economy's productive capacity - may have contributed to the rise in US inflation during the 1970s.

the economy through their negative effect on investment. But this line of reasoning also suggests a possible way out of this dilemma. A successful disinflation can be the outcome of a monetary tightening, coupled with fiscal interventions aiming at protecting business investment.

Imagine that the government subsidizes investment in innovation at rate s_t, financing the subsidy with lump-sum taxes.[35] With this subsidy in place, firms' investment maximizes

$$\sum_{t=0}^{\infty} \frac{1}{\delta_t} \left(\varpi A_{j,t} Z_t L_t - (1 - s_t) I_{j,t} \right),$$

subject to the law of motion (8). Optimal investment in innovation now implies that

$$\frac{1 - s_t}{\chi} = \beta \frac{c_t}{c_{t+1} g_{t+1}} \left(\varpi Z_{t+1} L_{t+1} + \frac{1 - s_{t+1} - \eta(1 - s_t)}{\chi} \right). \tag{20}$$

Naturally, a rise in the subsidy to innovation (i.e. an increase in s_t) leads firms to invest more which generates faster productivity growth.

To build up intuition, consider a case in which the government grants a subsidy in period 0 ($s_0 > 0$), but not in any subsequent period. Assume that the central bank sets monetary policy so that employment is not affected by this fiscal intervention, so that $L_t = \bar{L}$ throughout, and that the exogenous component of productivity is constant and equal to its steady state value ($Z_t = 1$). Using a bit of algebra, one can show that

$$g_1 = \frac{(\chi \Psi \bar{L} + 1)(\bar{g} + \beta \eta s_0)}{\chi \Psi \bar{L} + 1 - s_0[\chi \bar{L}(\Psi - \beta \varpi) + 1 - \beta]}.$$

Unsurprisingly, investment in innovation and productivity growth are both increasing in the subsidy.[36] This happens because, given our assumptions about monetary policy, the subsidy increases the fraction of output devoted to investment. Indeed, to prevent the economy from overheating, the central bank has to hike the nominal rate in period 0 according to

$$1 + i_0 = \frac{1 + \bar{i}}{1 - \frac{g_1 - \bar{g}}{\bar{c}\chi}}.$$

A higher subsidy is thus associated with a higher policy rate, to ensure that consumption declines so as to offset the impact of higher investment on aggregate demand.

What about inflation? Since monetary policy counteracts the impact of the subsidy on aggregate demand in period 0, on impact inflation is not affected. Higher productivity growth, however, translates into lower inflation in period 1. More precisely, period 1 inflation is equal to

$$\pi_1 = \frac{\bar{g}}{g_1} = \frac{\bar{g}(\chi \Psi \bar{L} + 1 - s_0[\chi \bar{L}(\Psi - \beta \varpi) + 1 - \beta])}{(\chi \Psi \bar{L} + 1)(\bar{g} + \beta \eta s_0)}$$

and it is therefore decreasing in the subsidy s_0. The message is that subsidizing investment dampens

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[35]Since there are no financial frictions, it doesn't matter whether lump-sum taxes are levied on firms or households.
[36]Notice that $\varpi/\Psi = \alpha/(1 + \alpha) < 1$, and hence the factor multiplying s_0 in the denominator is strictly positive.
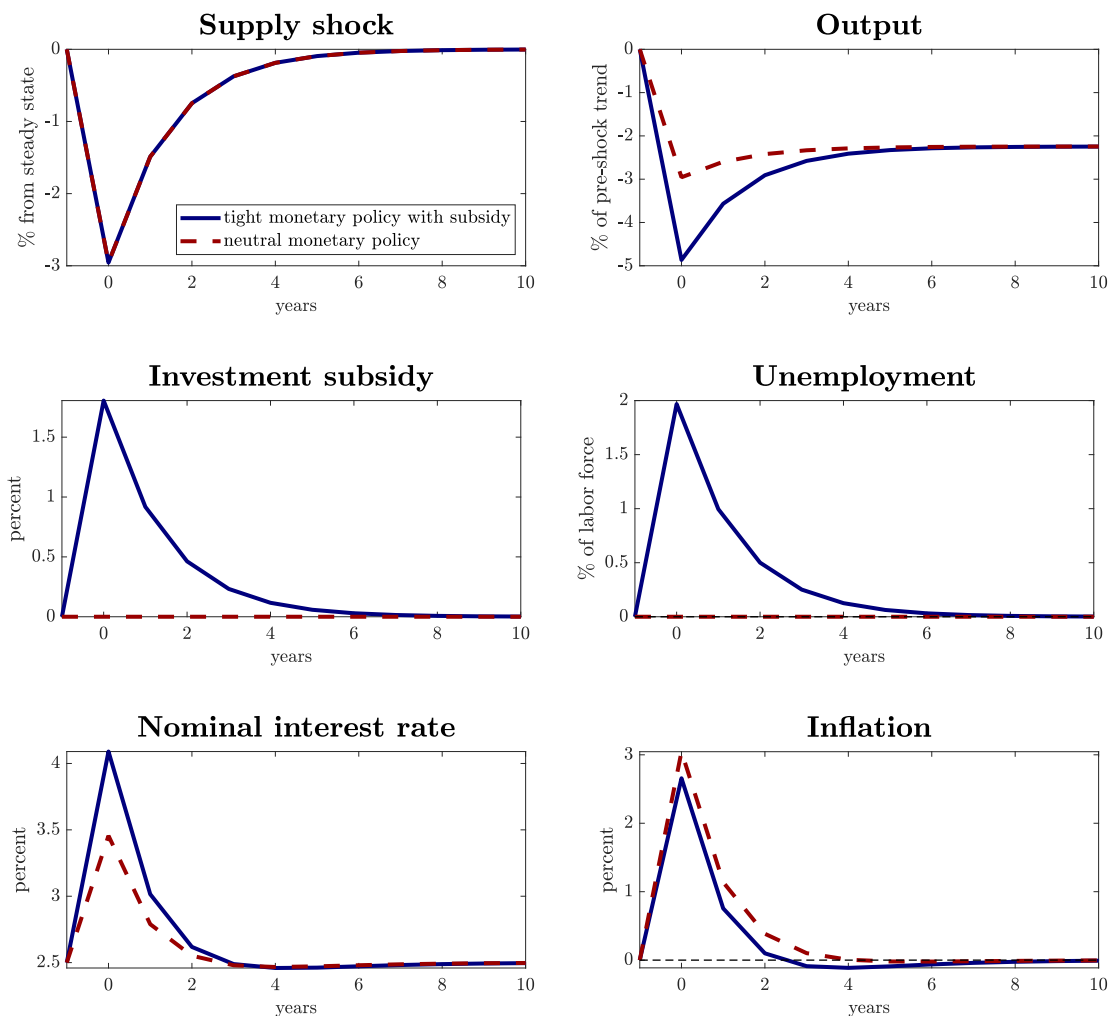
Figure 4: Monetary tightening with subsidies to investment.

inflation in the medium run.

We now illustrate how supporting firms' investment during a supply disruption can help a tight monetary policy in combating inflation. Let's use our, by now familiar, numerical example. As in the previous section, we consider a monetary tightening that produces the same unemployment path considered in Figure 3. But now we assume that the government shields firms' investment from the disinflation, by adjusting the subsidy so as to maintain the same growth rate of productivity as in the natural allocation ($g_{t+1} = \bar{g}_{t+1}$). This policy mix ensures that the monetary tightening does not leave any permanent scar on potential output.[37]

Figure 4 shows the economy's response to this policy mix, by contrasting it with an economy in which monetary policy is neutral ($L_t = \bar{L}$) and in which there is no subsidy. The key result is that the monetary tightening, coupled with investment subsidies, successfully reduces inflation throughout the whole duration of the supply disruption. This contrasts with the solitary monetary

---

[37]Notice, however, that now the central bank needs to hike the policy rate by more to obtain the same unemployment path as in Figure 3. The reason is that, as we explained above, the subsidy to investment sustains aggregate demand. The higher policy rate offsets the impact of the subsidy on employment.

tightening considered before, which lead to higher inflation in the medium run. Hence, subsidizing business investment during a disinflation improves the sacrifice ratio, that is the amount of inflation reduction brought about by a given rise in unemployment.[38]

Taking stock, during a disinflation not only overall aggregate demand matters, but also its composition between consumption and business investment. A monetary tightening can have undesirable consequences for inflation, if it damages heavily the economy's productive capacity by depressing firms' investment. As we just showed, this problem can be mitigated using appropriately designed fiscal interventions. Interestingly, there are historical antecedents to our proposed policy mix. In fact, fiscal incentives for firms to invest, especially in R&D, were part of the Volcker-Reagan 1980s disinflation package (Blanchard, 1987; Modigliani, 1988). Recent evidence suggests that these fiscal incentives boosted innovation activities and productivity growth in the United States (Akcigit et al., 2018).[39] It would be interesting to evaluate their impact on inflation in future research.

5 Conclusion

In this paper, we have revisited the macroeconomic implications of supply disruptions through the lens of a Keynesian growth framework. In our model, negative supply shocks generate very persistent - or even permanent - drops in GDP below its pre-shock trend. These scars of supply shocks depress aggregate demand, and therefore the natural interest rate. Scarring effects also tend to amplify the inflationary impact of supply disruptions and make it more persistent. Monetary tightenings may backfire by depressing productivity growth and increasing inflation in the medium run. A successful disinflation may require a policy mix of monetary tightening coupled with subsidies to business investment.

We conclude by pointing out an insight from the Keynesian growth framework on which future research can leverage. Traditional New Keynesian analyses focus on the relationship between monetary policy, overall aggregate demand and inflation. In the Keynesian growth framework, in contrast, the composition of aggregate demand between consumption and investment takes a central role. For instance, we have just seen that to predict the impact of a monetary tightening on inflation, it is important to understand how consumption and business investment will react to it. We believe that studying monetary policy in frameworks in which the composition of demand matters can be an exciting area for future research. In recent work, Fornaro and Wolf (2021), we take another step in this direction, but much more is yet to be done.

---

[38]In Appendix C we derive the optimal monetary-fiscal policy mix that maximizes welfare, and show that this entails a rise in the subsidy to investment in response to a negative supply shock. The implication is that supply disruptions exacerbate firms' tendency to underinvest in innovation. This is another reason why a government may want to use fiscal interventions to sustain investment in reaction to negative supply shocks.

[39]More broadly, Cloyne et al. (2022) show empirically that tax cuts on firms tend to boost investment in R&D and future productivity.

Appendix

A Proofs

A.1 Proof of Proposition 1

Proposition 1 *Suppose that the parameters satisfy*

$$\beta(\chi\varpi\bar{L} + 1 - \eta) > 1,$$

and that monetary policy is such that

$$\bar{i} = \chi\varpi\bar{L} - \eta.$$

Then there exists a unique full employment steady state. Moreover, this steady state is characterized by $\bar{g} > 1$.

Proof. The fact that growth is positive follows directly from the first condition in the proposition, because growth in the full employment steady state is given by $\bar{g} = \beta(\chi\varpi\bar{L} + 1 - \eta)$. Equation (PC) implies that in the full employment steady state $\bar{\pi} = 1$. Then the assumption $1 + \bar{i} = \bar{g}/\beta$ ensures that (IS) is consistent with the full employment steady state. Note next that, by using (MK),

$$\bar{c} = \Psi\bar{L} - \frac{\bar{g} - 1}{\chi} = \Psi\bar{L} - \frac{\beta(\chi\varpi\bar{L} + 1 - \eta) - 1}{\chi} = \bar{L}(\Psi - \beta\varpi) + \frac{1 - \beta(1 - \eta)}{\chi}.$$

Due to $\beta < 1$, $1 - \eta \le 1$ and $\varpi < \Psi$, this expression implies that steady state consumption is positive. To see that $\varpi < \Psi$, note that $\varpi/\Psi = \alpha/(1 + \alpha) < 1$. These arguments also directly imply that the full employment steady state is unique. ∎

A.2 Proof of Proposition 2

Proposition 2 *Assume that Z_t is governed by the process* (16)*, and that $Z_0 < 1$. If $\rho > 0$, the natural interest rate drops below its steady state value, $\bar{r}_t < \bar{r}$, for all $t \ge 0$. If $\rho = 0$, the natural rate remains equal to its steady state value, $\bar{r}_t = \bar{r}$, for all $t \ge 0$.*

Proof. In the natural allocation it holds that $L_t = \bar{L}$ at all times. The system of equations (IS)-(MK) thus collapses to

$$\bar{c}_t = \frac{\bar{g}_{t+1}\bar{c}_{t+1}}{\beta(1 + \bar{r}_t)},$$

$$\bar{g}_{t+1} = \beta\frac{\bar{c}_t}{\bar{c}_{t+1}}\left(\chi\varpi Z_{t+1}\bar{L} + 1 - \eta\right).$$

$$\Psi Z_t\bar{L} = \bar{c}_t + \frac{\bar{g}_{t+1} - 1}{\chi},$$

where, as in the main text, we use bars to denote variables in the natural allocation.

Combining the first two equations reveals that

$$1 + \bar{r}_t = \chi \varpi Z_{t+1} \bar{L} + 1 - \eta.$$

Using the process (16) and $Z_0 < 1$, we can write

$$1 + \bar{r}_t = \chi \varpi Z_0^{\rho^{t+1}} \bar{L} + 1 - \eta. \tag{A.1}$$

For $\rho > 0$ ($\rho = 0$), this expression implies that $\bar{r}_t < \bar{r}$ ($\bar{r}_t = \bar{r}$) for all $t \geq 0$. ∎

A.3 Proof of Proposition 3

Proposition 3 *Assume that $\lambda = 0$ and that $c_t \approx \Psi Z_t L_t$. Consider the effects of a monetary policy shock, $i_t \geq \bar{i}$ for all $t \geq 0$. We have three results. First, $L_t < \bar{L}$ whenever $i_t > \bar{i}$. Second, $i_0 > \bar{i}$ implies that $\pi_0 < 1$. Third, $\pi_{t+1} > 1$ whenever $i_t > \bar{i}$.*

Proof. Since we study a monetary policy shock, we abstract from the supply shock and set $Z_t = 1$ for all t. Hence the model reduces to

$$c_t = \beta \frac{c_{t+1} g_{t+1} \pi_{t+1}}{1 + i_t}$$

$$g_{t+1} = \beta \frac{c_t}{c_{t+1}} (\chi \varpi L_{t+1} + 1 - \eta)$$

$$\Psi L_t = c_t$$

$$\pi_t = \frac{\bar{g}}{g_t} \left(\frac{L_t}{\bar{\bar{L}}} \right)^\xi .$$

These are the Euler equation, the growth equation, the resource constraint and the inflation equation, respectively. In the resource constraint, we used our approximation $(g_{t+1} - 1)/\chi \approx 0$. Notice that g_0 is predetermined from the point of view of period 0.

In steady state, inflation is equal to zero hence the nominal rate is given by $1 + \bar{i} = \bar{g}/\beta$ (see the Euler equation), which from the growth equation can also be written as $1 + \bar{i} = \chi \varpi \bar{L} + 1 - \eta$.

Combining the Euler equation and the growth equation reveals that

$$\frac{1 + i_t}{\pi_{t+1}} = \chi \varpi L_{t+1} + 1 - \eta.$$

Comparing this with the steady state expression for $1 + \bar{i}$, and recognizing that $L_{t+1} \leq \bar{L}$, we can see that $i_t > \bar{i}$ implies $\pi_{t+1} > 1$. This proves the third part of the proposition.

Next, we replace π_{t+1} in the previous expression by the inflation equation

$$
\begin{aligned}
1 + i_t &= \pi_{t+1}(\chi\varpi L_{t+1} + 1 - \eta) \\
&= \frac{\bar{g}}{g_{t+1}}\left(\frac{L_{t+1}}{\bar{L}}\right)^{\xi}(\chi\varpi L_{t+1} + 1 - \eta) \\
&= (1 + \bar{i})\frac{L_{t+1}}{L_t}\left(\frac{L_{t+1}}{\bar{L}}\right)^{\xi},
\end{aligned}
\tag{A.2}
$$

where in the third line we replace g_{t+1} with the growth equation to cancel the term $\chi\varpi L_{t+1} + 1 - \eta$, we replace the consumption ratio c_{t+1}/c_t by L_{t+1}/L_t using the resource constraint, and we use $\bar{g}/\beta = 1 + \bar{i}$.

Because $L_{t+1} \leq \bar{L}$, equation (A.2) shows that $i_t > \bar{i}$ entails $L_t < \bar{L}$. This proves the first part of the proposition.

Finally, to prove the second part of the proposition, note that $i_0 > \bar{i}$ implies that $L_0 < \bar{L}$, from previous arguments. But in the impact period of the shock, g_0 is predetermined and inflation is hence given by

$$
\pi_0 = \left(\frac{L_0}{\bar{L}}\right)^{\xi}.
$$

This shows that $\pi_0 < 1$ whenever $i_0 > \bar{i}$. $\blacksquare$

B   Additional derivations

B.1   Energy price shocks

Let's consider a country that uses energy in production. For concreteness, all the energy is produced with oil, which is fully imported from the rest of the world. The production function is now given by

$$
Y_t = o_t^{\gamma}\left((Z_t L_t)^{1-\alpha}\int_0^1 A_{j,t}^{1-\alpha}x_{j,t}^{\alpha}dj\right)^{1-\gamma},
\tag{B.1}
$$

where o_t denotes the quantity of oil used in production and $0 \leq \gamma < 1$. We denote the price of oil in real terms (i.e. normalized by P_t) as p_t^o. The price of oil is set exogenously on the global oil markets. The optimal demand for oil by firms implies

$$
p_t^o o_t = \gamma Y_t.
\tag{B.2}
$$

We can then rewrite the production function as

$$
Y_t = \left(\frac{\gamma}{p_t^o}\right)^{\frac{\gamma}{1-\gamma}}(Z_t L_t)^{1-\alpha}\int_0^1 A_{j,t}^{1-\alpha}x_{j,t}^{\alpha}dj.
\tag{B.3}
$$

Following the derivations in the main text, we can further write GDP as

$$Y_t - p_t^o o_t - \int_0^1 x_{j,t} dj = \left(\frac{\gamma}{p_t^o}\right)^{\frac{\gamma}{(1-\gamma)(1-\alpha)}} \tilde{\Psi} Z_t A_t L_t, \tag{B.4}$$

where we define $\tilde{\Psi} \equiv (1 - \alpha^2 - \gamma)\alpha^{\frac{2\alpha}{1-\alpha}}$.

To close the model, we assume that the country is in financial autarky, so that trade is balanced every period. In this case, the market clearing condition for the final good is

$$\left(\frac{\gamma}{p_t^o}\right)^{\frac{\gamma}{(1-\gamma)(1-\alpha)}} \tilde{\Psi} Z_t A_t L_t = C_t + I_t. \tag{B.5}$$

With these results, one can see that the response of the economy to an increase in the price of oil is qualitatively isomorphic to its response to a negative productivity shock.

B.2 Optimal investment by firms

In this appendix we show that optimal investment by firms satisfies condition (9). Firms producing intermediate goods choose investment in innovation to maximize

$$\sum_{t=0}^{\infty} \left(\prod_{j=1}^t \frac{1}{1 + r_{j-1} + \eta}\right) (\varpi A_{j,t} Z_t L_t - I_{j,t}),$$

subject to

$$A_{j,t+1} = A_{j,t} + \chi I_{j,t}$$

$$I_{j,t} \geq 0,$$

given the initial condition $A_{j,0}$. The last constraint takes into account the fact that investment cannot be negative.

We define the following Lagrangian

$$\mathcal{L} = \sum_{t=0}^{\infty} \left(\prod_{j=1}^t \frac{1}{1 + r_{j-1} + \eta}\right) (\varpi A_{j,t} Z_t L_t - I_{j,t}(1 - \chi v_{j,t})) + \gamma_t (A_{j,t+1} - A_{j,t} - \chi I_{j,t}).$$

Optimal investment satisfies

$$\frac{1}{\chi} - v_{j,t} = \frac{1}{1 + r_t + \eta} \left(\varpi Z_{t+1} L_{t+1} + \frac{1}{\chi} - v_{j,t+1}\right)$$

$$v_{j,t} I_{j,t} = 0,$$

where $v_{j,t} \geq 0$ is the Lagrange multiplier on investment's non-negativity constraint.

If investment is always positive ($v_{j,t} = 0$ for all $t \geq 0$) the optimality condition reduces to equation (9). To see this, simply bring $1 + r_t + \eta$ on the other side, subtract η/χ, and replace

$1 + r_t = g_{t+1} c_{t+1}/(\beta c_t)$. However, investing might not be profitable for firms. This happens if the marginal increase in profits obtained from investing is lower than its marginal cost. For instance, this might happen following very large negative supply shocks, as we mention in footnote 27. In this case, $I_{j,t} = 0$ and the previous equation pins down the multiplier $\nu_{j,t}$, which becomes strictly positive.

B.3   Determinacy under the interest rate rule

In this appendix we establish that condition (19) ensures local determinacy around the full employment steady state under the monetary policy rule (18).

Start by noticing that, given our assumptions about wage and price setting, once real variables are determined the Phillips curve can be used to back out the path of inflation. Moreover, remember that we are considering a monetary policy that targets a path for the real interest rate. For these reasons, we can abstract from nominal variables when deriving the determinacy conditions.

The model can then be summarized by the following equations

$$c_t = \frac{g_{t+1} c_{t+1}}{\beta (1 + \bar{r}) \left(\frac{L_t}{L}\right)^{\phi}}$$

$$g_{t+1} = \beta \frac{c_t}{c_{t+1}} \left(\chi \varpi Z_{t+1} L_{t+1} + 1 - \eta\right)$$

$$\Psi Z_t L_t = c_t + \frac{g_{t+1} - 1}{\chi},$$

where the first equation is the combination of (IS) and (18).

Now consider a first-order approximation of the model around the full employment steady state[40]

$$\hat{g}_{t+1} = \phi \hat{L}_t + \hat{c}_t - \hat{c}_{t+1}$$

$$\hat{g}_{t+1} = \hat{c}_t - \hat{c}_{t+1} + \frac{\bar{g} - \beta(1 - \eta)}{\bar{g}} \hat{L}_{t+1}$$

$$\Psi \bar{L} \hat{L}_t = \bar{c} \hat{c}_t + \frac{\bar{g}}{\chi} \hat{g}_{t+1},$$

where $\hat{x}_t \equiv \log(x_t) - \log(\bar{x})$ for a given variable x_t. This system can be written as

$$\hat{L}_t = \xi_1 \hat{L}_{t+1} + \xi_2 \hat{g}_{t+2}$$

$$\hat{g}_{t+1} = \xi_3 \hat{L}_{t+1} + \xi_4 \hat{g}_{t+2},$$

where

$$\xi_1 \equiv \frac{1}{\phi} \frac{\bar{g} - \beta(1 - \eta)}{\bar{g}}$$

$$\xi_2 \equiv 0$$

---

[40]In the approximation, we keep Z_t fixed at its steady state value $Z_t = 1$ as variations in this variable are irrelevant for the determinacy properties of the dynamic system.

$$\xi_3 \equiv \frac{\bar{g} - 1}{\bar{g}\frac{\Psi\bar{L}}{\bar{c}} - 1}\left(\phi\xi_1 + \frac{\Psi\bar{L}}{\bar{c}}(\xi_1 - 1)\right)$$

$$\xi_4 \equiv \frac{\bar{g}\left(1 - \frac{\bar{c}}{\Psi\bar{L}}\right)}{\bar{g} - \frac{\bar{c}}{\Psi\bar{L}}}.$$

The system is determinate if and only if:[41]

$$|\xi_1\xi_4 - \xi_2\xi_3| < 1 \tag{B.6}$$

$$|\xi_1 + \xi_4| < 1 + \xi_1\xi_4 - \xi_2\xi_3. \tag{B.7}$$

Condition (B.6) holds if

$$\phi > \frac{(\bar{g} - \beta(1 - \eta))\left(1 - \frac{\bar{c}}{\Psi\bar{L}}\right)}{\bar{g} - \frac{\bar{c}}{\Psi\bar{L}}},$$

while condition (B.7) holds if

$$\phi > \frac{\bar{g} - \beta(1 - \eta)}{\bar{g}} > \frac{(\bar{g} - \beta(1 - \eta))\left(1 - \frac{\bar{c}}{\Psi\bar{L}}\right)}{\bar{g} - \frac{\bar{c}}{\Psi\bar{L}}}, \tag{B.8}$$

where the last inequality follows from $\bar{g} > 1$. Inserting $\bar{g} = \beta(\chi\varpi\bar{L} + 1 - \eta)$ in (B.8) shows that the steady state is locally determinate if and only if condition (19) holds.

C Optimal monetary and fiscal policy

In this appendix we use our framework to derive some insights about the optimal monetary-fiscal policy mix.

C.1 First best allocation

Consider a social planner maximizing households' welfare, subject to the economy's technological constraints. First, every period the planner solves a static maximization problem by setting employment and production of intermediate inputs to maximize GDP. Naturally, the planner chooses $L_t = \bar{L}$, since letting the economy operate below full employment would be wasteful. Moreover, by symmetry, the planner ensures that every intermediate input is produced in the same quantity x_t to maximize

$$\max_{x_t}\left\{\left(A_t Z_t \bar{L}\right)^{1-\alpha} x_t^\alpha - x_t\right\} = (1 - \alpha)\alpha^{\frac{\alpha}{1-\alpha}} A_t Z_t \bar{L} \equiv \Omega A_t Z_t \bar{L}.$$

We are left to determine the planner's optimal investment behavior. As in the main text, we focus on interior equilibria in which investment is always positive. Again by symmetry, it is optimal to set the same level of investment for each firm in the intermediate sector. Using the

---

[41]See Bullard and Mitra (2002).

results above, the problem of the planner can then be written as

$$\max_{C_t, I_t, A_{t+1}} \sum_{t=0}^{\infty} \beta^t \log(C_t)$$

subject to

$$A_{t+1} = A_t + \chi I_t$$

$$\Omega A_t Z_t \bar{L} = C_t + I_t.$$

The optimality condition for investment is

$$g_{t+1} = \beta(\chi \Omega Z_t \bar{L} + 1), \tag{C.1}$$

which defines the growth rate of trend productivity in the first best allocation.

C.2 Implementing the first best through monetary and fiscal policy

We next show that the first best can be implemented with three policies i) production subsidies in the intermediate sector to offset the static distortions due to monopoly power ii) investment subsidies to correct the externalities in the innovation process iii) monetary policy to offset the distortions due to nominal wage rigidities and maintain full employment.

Assume that monetary policy sets the interest rate to offset the distortions due to nominal wage rigidities. That is, i_t is set to satisfy the IS equation with $L_t = \bar{L}$ for all t. Next assume that the government provides a subsidy τ - financed with lump-sum taxes on households - to firms in the intermediate sector proportional to their production costs, so that their profits are given by

$$(P_{j,t} - (1 - \tau)P_t)\, x_{j,t}.$$

Setting $\tau = 1 - \alpha$ ensures that firms produce the first-best amount of intermediate inputs. These two policies imply that, given A_t, GDP is at its efficient level $\Omega A_t Z_t \bar{L}$.

Lastly, as in Section 4.3, assume that investment is subsidized at rate s_t, so that firms' optimal investment strategy is defined by[42]

$$\frac{1 - s_t}{\chi} = \beta \frac{c_t}{c_{t+1} g_{t+1}} \left(\alpha \Omega Z_{t+1} \bar{L} + \frac{1 - s_{t+1} - \eta(1 - s_t)}{\chi} \right). \tag{C.2}$$

To implement the first-best allocation, the government needs to set s_t to equate (C.1) and (C.2) for all t. Using this result and the fact that in the first best allocation

$$c_t = (1 - \beta) \left(\Omega Z_t \bar{L} + \frac{1}{\chi} \right),$$

---

[42]Notice that the term ϖ is replaced by $\alpha \Omega$, to take into account the impact of the production subsidy on firms' profits.

implies that the law of motion for the optimal investment subsidy is

$$s_t = \frac{(1-\alpha)\chi\Omega Z_{t+1}\bar{L} + \eta + s_{t+1}}{\chi\Omega Z_{t+1}\bar{L} + 1 + \eta}. \tag{C.3}$$

With these policies in place all the equations describing a competitive equilibrium are satisfied, and the allocation coincides with the first-best one.

C.3   Optimal policy response to a supply disruption

We now characterize the policy reaction to a negative supply shock. Since the production subsidy τ is constant, we focus on i_t and s_t.

In steady state, since inflation is zero, the equilibrium policy rate that satisfies the Euler equation is

$$i = \chi\Omega\bar{L}.$$

Moreover, since $Z_t = 1$, the optimal investment subsidy is

$$s = \frac{(1-\alpha)\chi\Omega\bar{L} + \eta}{\chi\Omega\bar{L} + \eta} > 0.$$

A positive subsidy is needed because firms do not fully internalize the social benefits of investment. This is due to two effects. First, firms do not internalize the positive impact of productivity improvements on wages. This effect is decreasing in α, which captures the share of firms' profits in GDP, and vanishes as $\alpha \to 1$. Second, the discount rate wedge effectively renders firms short sighted from a social welfare perspective. Indeed, the subsidy is increasing in the discount rate wedge η. This second effect is similar to the classic intertemporal knowledge spillovers commonly present in innovation-based endogenous growth frameworks.

Now consider the response of s_t and i_t to the kind of persistent drops in Z_t that we studied in the main text. First, expression (C.3) implies that under the optimal policy s_t rises after a persistent negative supply shock.[43] In words, fiscal policy should respond to a persistent supply disturbance by subsidizing more heavily investment. This is an interesting results, because it signals that supply disruptions exacerbate the externalities associated with investment in innovation.

What about the response of the interest rate? Under the optimal monetary-fiscal policy mix, the real interest rate evolves according to[44]

$$r_t = \chi\Omega Z_{t+1}\bar{L}.$$

---

[43]Specifically, the derivative of (C.3) with respect to Z_{t+1} is negative if $s_{t+1} > 1 - \alpha(1+\eta)$, and this condition is satisfied in steady state. It follows that a persistent decline in Z_{t+1} must be associated with a rise in s_t.

[44]To derive this expression, we have used the IS equation

$$1 + r_t = \frac{g_{t+1}c_{t+1}}{\beta c_t},$$

evaluated at the first best allocation.

Hence, following a persistent negative supply shock the real interest rate falls. Turning to the nominal rate, using the Phillips curve (PC) gives

$$1 + i_t = (1 + r_t)\pi_{t+1} = (1 + r_t)\frac{\bar{g}}{g_{t+1}}\frac{Z_t}{Z_{t+1}}\pi_t^{\lambda}.$$

So a negative supply shock exerts two opposing forces on the nominal rate. On the one had, the fact that the real rate drops points toward a lower nominal rate. On the other hand, the shock itself as well as the decline in trend productivity growth increase expected inflation, which points toward a higher nominal rate. The response of the nominal rate to a supply disruption is therefore ambiguous.

D   Model with New Keynesian wage Phillips curve

In this appendix we study a version of the model featuring a New Keynesian wage Phillips curve. To do so, we assume that households are monopolistically competitive suppliers of labor, and that wage changes entail Rotemberg (1982)-type adjustment costs.

D.1   Model

There is a unit mass of households indexed by $k \in [0, 1]$. Each household k has utility

$$\sum_{t=0}^{\infty} \beta^t \left(\log(C_t(k)) - G(L_t(k)) \right),$$

where $G(L_t(k))$ is a function capturing the disutility from working, satisfying $G'(\cdot) > 0$ and $G''(\cdot) > 0$. The budget constraint is given by

$$P_t C_t(k) + \frac{B_{t+1}(k)}{1 + i_t} = W_t(k)L_t(k) + B_t(k) + D_t. \tag{D.1}$$

Households' Euler equation is as in the baseline model. We discuss households' labor supply choice below.

Final goods firms' production technology is still given by

$$Y_t = (Z_t L_t)^{1-\alpha} \int_0^1 A_{j,t}^{1-\alpha} x_{j,t}^{\alpha} dj.$$

However, labor demand L_t is now a composite of the labor supplied by the different households $k \in [0, 1]$

$$L_t = \left(\int_0^1 L_t(k)^{\frac{\varepsilon-1}{\varepsilon}} dk \right)^{\frac{\varepsilon}{\varepsilon-1}}, \tag{D.2}$$

where $\varepsilon > 1$ denotes the elasticity of substitution among different labor varieties. Firms' optimal labor demand solves

$$\min_{\{L_t(k)\}_{k\in[0,1]}} \int_0^1 W_t(k)L_t(k)dk \quad \text{subject to (D.2)},$$

by taking as given $\{W_t(k)\}_{k \in [0,1]}$. The first order condition is

$$L_t(k) = \left(\frac{W_t(k)}{W_t}\right)^{-\varepsilon} L_t. \tag{D.3}$$

We now turn to labor supply. Each household sets its wage to maximize utility subject to firms' demand for its labor (D.3), its budget constraint (D.1) and a wage-adjustment cost. The wage-adjustment cost in nominal terms is given by

$$\frac{\theta}{2} A_t P_t \left(\frac{1}{\bar{g}} \frac{W_t(k)}{W_{t-1}(k)} - 1\right)^2, \tag{D.4}$$

where $\theta \geq 0$ is the adjustment cost parameter. The adjustment cost is multiplied with $A_t P_t$ to ensure the existence of a balanced growth path. Moreover, we assume that wage inflation is indexed to the growth rate of productivity in the full employment steady state, $\bar{g}$. This ensures that households do not pay any wage adjustment cost in steady state. Note that, for simplicity, there is no indexation of wages to past inflation.

Optimal wage setting satisfies the condition

$$\frac{1}{P_t C_t(k)} \left((1-\varepsilon)L_t(k) - \theta A_t P_t \left(\frac{1}{\bar{g}} \frac{W_t(k)}{W_{t-1}(k)} - 1\right) \frac{1}{\bar{g}} \frac{1}{W_{t-1}(k)}\right) - G'(L_t(k))(-\varepsilon)\frac{L_t(k)}{W_t(k)}$$
$$+ \beta \frac{1}{P_{t+1} C_{t+1}(k)} \theta A_{t+1} P_{t+1} \left(\frac{1}{\bar{g}} \frac{W_{t+1}(k)}{W_t(k)} - 1\right) \frac{1}{\bar{g}} \frac{W_{t+1}(k)}{W_t(k)^2} = 0.$$

We now assume that $W_{-1}(k) = W_{-1}$, that is, all households face identical initial conditions. Because households face identical problems, this implies that in equilibrium households make identical decisions. From now on, we thus omit the household index k.

Denoting gross nominal wage inflation by $\pi_t^W \equiv W_t/W_{t-1}$, the optimality condition for wage setting can be written as

$$\frac{A_t}{C_t} \theta \left(\frac{\pi_t^W}{\bar{g}} - 1\right) \frac{\pi_t^W}{\bar{g}} - \beta \frac{A_{t+1}}{C_{t+1}} \theta \left(\frac{\pi_{t+1}^W}{\bar{g}} - 1\right) \frac{\pi_{t+1}^W}{\bar{g}} = \varepsilon L_t \left(G'(L_t) - \frac{\varepsilon - 1}{\varepsilon} \frac{W_t}{P_t} \frac{1}{C_t}\right). \tag{D.5}$$

This New Keynesian wage Phillips curve replaces equation (12) in the main text.

The rest of the model is as in the main text. In particular, we assume that the wage-adjustment cost is rebated in a lump-sum manner to households, ensuring that the resource constraint of the economy is the same as in our baseline model.

D.2 Equilibrium

Given a path for the supply shock $\{Z_t\}_{t=0}^{+\infty}$ and a path for monetary policy $\{i_t\}_{t=0}^{+\infty}$, an equilibrium is a set of processes $\{c_t, L_t, g_{t+1}, \pi_t, \pi_t^W\}_{t=0}^{+\infty}$ satisfying $g_{t+1} > 1$, $L_t > 0$, $c_t > 0$ as well as the following equations for all $t \geq 0$

$$\frac{g_{t+1}}{c_t} = \frac{\beta(1 + i_t)}{\pi_{t+1} c_{t+1}} \tag{D.6}$$

$$g_{t+1} = \beta \frac{c_t}{c_{t+1}} \left(\chi \varpi Z_{t+1} L_{t+1} + 1 - \eta \right) \tag{D.7}$$

$$\Psi Z_t L_t = c_t + \frac{g_{t+1} - 1}{\chi} \tag{D.8}$$

$$\left(\frac{\pi_t^W}{\bar{g}} - 1 \right) \frac{\pi_t^W}{\bar{g}} - \beta \frac{c_t}{c_{t+1}} \left(\frac{\pi_{t+1}^W}{\bar{g}} - 1 \right) \frac{\pi_{t+1}^W}{\bar{g}} = \frac{\varepsilon}{\theta} L_t \left(\frac{G'(L_t)}{c_t^{-1}} - \frac{\varepsilon - 1}{\varepsilon} (1 - \alpha) \alpha^{\frac{2\alpha}{1-\alpha}} Z_t \right) \tag{D.9}$$

$$\pi_t = \frac{Z_{t-1}}{Z_t} \frac{\pi_t^w}{g_t}. \tag{D.10}$$

The flexible-wage allocation, i.e. the natural allocation, is nested for $\theta \to 0$. In contrast with the baseline model, the natural level of employment $\bar{L}_t$ is now time varying.

D.3 Steady state

In steady state, $Z = 1$, $L_t = \bar{L}$, $\pi_t^w = \bar{g}$ and $\pi_t = 1$. The system of equations collapses to

$$\bar{g} = \beta(1 + \bar{i})$$
$$\bar{g} = \beta \left(\chi \varpi \bar{L} + 1 - \eta \right),$$
$$\Psi \bar{L} = \bar{c} + \frac{\bar{g} - 1}{\chi}$$
$$\frac{G'(\bar{L})}{\bar{c}^{-1}} = \frac{\varepsilon - 1}{\varepsilon} (1 - \alpha) \alpha^{\frac{2\alpha}{1-\alpha}}.$$

D.4 Calibration

We illustrate the properties of this version of the model using numerical simulations. We calibrate the model so that its steady state coincides with the one of our baseline model with an ad-hoc wage Phillips curve. We assume that $G(\cdot)$ takes the conventional functional form

$$G(L_t) = \iota \frac{L_t^{1+\varphi}}{1 + \varphi},$$

where $\varphi > 0$ is the inverse Frisch elasticity of labor supply, and where $\iota > 0$ is a parameter. We set ι such that steady state employment is $\bar{L} = 1$, just as in the baseline model. Because the real side of this model coincides with the one of the baseline model, we set the parameters α, β, χ and η as described in Section 3, so that $\chi = 2.081$, $\beta = 0.995$, $1 - \alpha = 0.837$ and $\eta = 0.115$. The implied value for ι is 0.790.

Next, we choose the three parameters of the wage Phillips curve. We set the inverse of the Frisch elasticity of labor supply to $\varphi = 3$, and the elasticity of substitution between labor varieties to $\varepsilon = 10$. These values are in the range of those commonly used by the literature (e.g., Galí, 2011).

To calibrate the wage adjustment cost parameter, we draw a parallel between the wage Phillips curve implied by our model and the one that would emerge under wage adjustment with Calvo

frictions. Up to a first-order approximation, the wage Phillips curve (D.9) is given by

$$\hat{\pi}_t^w = \beta\hat{\pi}_{t+1}^w + \frac{\varepsilon - 1}{\theta}(1 - \alpha)\alpha^{\frac{2\alpha}{1-\alpha}}(\varphi\hat{L}_t + \hat{c}_t - \hat{Z}_t), \tag{D.11}$$

where a hat above a variable denotes its log-deviation from steady state. Under a Calvo-type of wage adjustment friction, the log-linearized wage Phillips curve would instead be[45]

$$\hat{\pi}_t^w = \beta\hat{\pi}_{t+1}^w + \frac{(1 - \theta_c)(1 - \beta\theta_c)}{\theta_c(1 + \varepsilon\varphi)}(\varphi\hat{L}_t + \hat{c}_t - \hat{Z}_t),$$

where θ_c denotes the probability that a wage is not reset in a given period.

In the New Keynesian literature, it is standard to assume values of θ_c close to 0.75 per quarter. At yearly frequency, this translates into a probability that a wage is not reset in a given period of $\theta_c = 0.75^4 = 0.31$. The implied average duration of a wage is $1/(1-0.31) = 1.45$ years, which is in line with the empirical evidence provided by Beraja et al. (2019). Given this value of θ_c, as well as the values for β, ε and φ specified above, we can compute the slope of the Phillips curve implied by the Calvo model. We then set $\theta = 77.894$, to match the same slope of the Phillips curve in the Rotemberg model as in the Calvo model.

How to map this wage Phillips curve with the ad-hoc one considered in our baseline model? To answer this question, let's rewrite (D.11) in terms of deviations of employment from its steady state value. To do so, consider that when investment in innovation is small $\hat{c}_t \approx \hat{Z}_t + \hat{L}_t$, and so (D.11) becomes

$$\hat{\pi}_t^w = \beta\hat{\pi}_{t+1}^w + \frac{\varepsilon - 1}{\theta}(1 - \alpha)\alpha^{\frac{2\alpha}{1-\alpha}}(1 + \varphi)\hat{L}_t \equiv \beta\hat{\pi}_{t+1}^w + \kappa\hat{L}_t.$$

Our calibration implies $\kappa = 0.191$, which is the value that we use for ξ in our baseline model.

D.5 Supply shocks, monetary policy and inflation in the Rotemberg model

We next revisit the impact of supply disruptions and monetary tightenings on inflation when wage-adjustment frictions take the Rotemberg form. Let us start from considering the inflation response to a supply disruption in the natural allocation. The real side of the model behaves exactly as the baseline one, and so real variables respond to the supply disruption as described in Section 3.

The inflation response is shown in Figure 5. Of course, since now we are abstracting from indexation of wages to past inflation, inflation falls rather quickly after the initial burst triggered by the supply disruption. However, as in our baseline model, the figure shows that hysteresis effects reinforce the rise in inflation associated with a supply disruption.

We next discuss the impact of the same monetary tightening considered in Section 4. Also in this case the path of real variables, including the unemployment rate, is identical to the one of the baseline model. As shown in Figure 6, the inflation response is qualitatively in line with the one of the baseline model. While the monetary tightening reduces inflation on impact, it increases

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[45]The derivation of the two log-linearized wage Phillips curves is available on request.
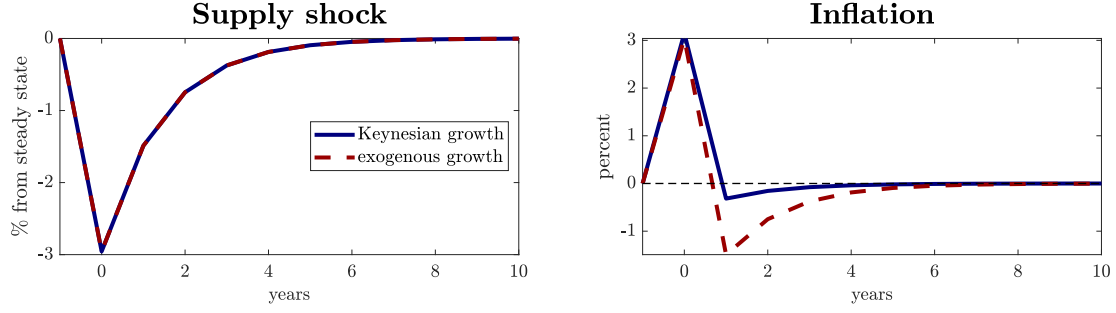
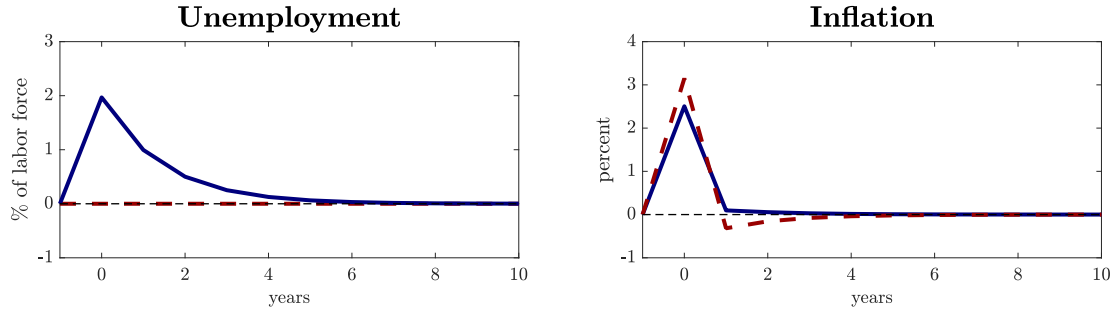**Figure 5: Inflation response to a supply disruption.**

Figure 6: Disinflation attempts by the central bank.

inflation in the medium run, because of the endogenous drop in investment and productivity.

E Decreasing returns to investment

In this Appendix, we introduce a version of the model in which investment features decreasing returns, for instance because of congestion effects or adjustment costs in the innovation process.

Firms' productivity now evolves according to

$$A_{j,t+1} = A_{j,t} + \chi I_{j,t}^\zeta A_t^{1-\zeta}, \tag{E.1}$$

where $0 < \zeta \leq 1$ is a parameter capturing the curvature in the investment function. Under this formulation, firms combine investment and the aggregate stock of knowledge to increase their future productivity.[46]

Firms choose investment in innovation to maximize their expected profits

$$\sum_{t=0}^{\infty} \left(\prod_{j=1}^{t} \frac{1}{1 + r_{j-1} + \eta} \right) (\varpi A_{j,t} Z_t L_t - I_{j,t}),$$

subject to (E.1). The non-negativity constraint on investment $I_{j,t} \geq 0$ never binds in equilibrium,

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[46]This formulation guarantees the existence of a balanced growth path with positive growth. Assuming that firms combine investment with their individual stock of knowledge would not change any of the results that follow.

since the return to investment becomes infinity as $I_{j,t}$ approaches zero. The Lagrangian becomes

$$\mathcal{L} = \sum_{t=0}^{\infty} \left(\prod_{j=1}^{t} \frac{1}{1 + r_{j-1} + \eta} \right) (\varpi A_{j,t} Z_t L_t - I_{j,t}) + \gamma_t (A_{j,t+1} - A_{j,t} - \chi I_{j,t}^{\zeta} A_t^{1-\zeta}).$$

The optimality condition for investment is therefore

$$\frac{1}{\chi \zeta} \left(\frac{I_t}{A_t} \right)^{1-\zeta} = \frac{1}{1 + r_t + \eta} \left(\varpi Z_{t+1} L_{t+1} + \frac{1}{\chi \zeta} \left(\frac{I_{t+1}}{A_{t+1}} \right)^{1-\zeta} \right), \tag{E.2}$$

where we have already imposed $I_{j,t} = I_t$ in equilibrium.[47]

E.1 Summary of equilibrium conditions

As in the baseline model, the equilibrium can be summarized by four equations

$$c_t = \frac{g_{t+1} c_{t+1} \pi_{t+1}}{\beta(1 + i_t)} \tag{E.3}$$

$$\left(\frac{g_{t+1} - 1}{\chi} \right)^{\frac{1-\zeta}{\zeta}} g_{t+1} = \beta \frac{c_t}{c_{t+1}} \left(\chi \zeta \varpi Z_{t+1} L_{t+1} + \left(\frac{g_{t+2} - 1}{\chi} \right)^{\frac{1-\zeta}{\zeta}} - \eta \left(\frac{g_{t+1} - 1}{\chi} \right)^{\frac{1-\zeta}{\zeta}} \right) \tag{E.4}$$

$$\Psi Z_t L_t = c_t + \left(\frac{g_{t+1} - 1}{\chi} \right)^{\frac{1}{\xi}} \tag{E.5}$$

$$\pi_t = \frac{\bar{g}}{g_t} \frac{Z_{t-1}}{Z_t} \left(\frac{L_t}{\bar{L}} \right)^{\xi} \pi_{t-1}^{\lambda}. \tag{E.6}$$

These expressions generalize the ones from the baseline model to the case $0 < \zeta < 1$. For given monetary policy $\{i_t\}_{t=0}^{+\infty}$ and the supply shock $\{Z_t\}_{t=0}^{+\infty}$, they pin down the paths for all endogenous variables $\{L_t, g_{t+1}, c_t, \pi_t\}_{t=0}^{+\infty}$ for given initial conditions $\{\pi_{-1}, Z_{-1}\}$.

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[47]Another possibility, sometime used by the literature, is to assume that returns to investment are linear at the firm level, but decreasing at the aggregate level. The underlying idea is that congestion externalities reduce individual firms' efficiency of research χ when aggregate investment is high. For instance, Kung and Schmid (2015) assume the following law of motion for technology

$$A_{j,t+1} = A_{j,t} + \chi_t I_{j,t},$$

where χ_t is given by

$$\chi_t = \frac{\chi A_t}{I_t^{1-\zeta} A_t^{\zeta}},$$

where $A_t \equiv \int_0^1 A_{j,t} dj$ and $I_t \equiv \int_0^1 I_{j,t} dj$ are taken as given by the individual firm. In this case, the first order condition for investment is given by (imposing $I_{j,t} = I_t$ in equilibrium)

$$\frac{1}{\chi} \left(\frac{I_t}{A_t} \right)^{1-\zeta} = \frac{1}{1 + r_t + \eta} \left(\varpi Z_{t+1} L_{t+1} + \frac{1}{\chi} \left(\frac{I_{t+1}}{A_{t+1}} \right)^{1-\zeta} \right).$$

As one can see, this expression is equal to (E.2), up to the parameter χ. Since we calibrate χ by targeting a certain growth rate of productivity in steady state, this model version produces identical outcomes as the model with decreasing returns at the firm level.

E.2 Macroeconomic impact of supply disruptions

We now explore the macroeconomic impact of supply disruptions in the model with decreasing returns to investment. As in the main text, we focus on the natural allocation ($L_t = \bar{L}$), and denote variables in the natural allocation with upper bars. In this case, (E.4)-(E.5) jointly pin down the equilibrium for $\{\bar{g}_{t+1}, \bar{c}_t\}_{t=0}^{+\infty}$. In turn, (E.3) pins down the implied equilibrium for the real natural rate $\{\bar{r}_t\}_{t=0}^{+\infty}$, while (E.6) pins down the implied equilibrium for inflation $\{\bar{\pi}_t\}_{t=0}^{+\infty}$.

To derive analytic insights, we consider a first-order approximation of the model. A first order approximation of equations (E.4)-(E.5) around the full employment steady state gives

$$\bar{g}\left(\left[\frac{1-\zeta}{\zeta}\frac{\bar{g}}{\bar{g}-1}+1\right]\hat{g}_{t+1} - \hat{c}_t + \hat{c}_{t+1}\right) = (\bar{g}-\beta(1-\eta))\hat{Z}_{t+1} + \beta\frac{1-\zeta}{\zeta}\frac{\bar{g}}{\bar{g}-1}(\hat{g}_{t+2} - \eta\hat{g}_{t+1}).$$

$$\hat{Z}_t = s_c\hat{c}_t + (1-s_c)\frac{1}{\zeta}\frac{\bar{g}}{\bar{g}-1}\hat{g}_{t+1},$$

where $s_c \equiv \bar{c}/\Psi\bar{L}$ is the consumption share of GDP in steady state. Here, $\hat{x}_t \equiv \log(\bar{x}_t) - \log(\bar{x})$ for every variable $\bar{x}_t$.

Using $\hat{Z}_{t+1} = \rho\hat{Z}_t$, we can guess and verify that the solution to the model can be written as

$$\hat{c}_t = \gamma_c\hat{Z}_t$$

$$\hat{g}_{t+1} = \gamma_g\hat{Z}_t,$$

where the two parameters γ_c and γ_g are equal to

$$\gamma_c = \frac{1}{s_c}\frac{\bar{g} + \frac{\bar{g}}{\bar{g}-1}\left(\frac{1-\zeta}{\zeta}(\bar{g}-\beta(\rho-\eta)) - \frac{1-s_c}{\zeta}(\bar{g}-\beta(1-\eta))\rho\right)}{\bar{g} + \frac{\bar{g}}{\bar{g}-1}\left[\frac{1-\zeta}{\zeta}(\bar{g}-\beta(\rho-\eta)) + \frac{1}{\zeta}\bar{g}(1-\rho)\frac{1-s_c}{s_c}\right]}$$

$$\gamma_g = \frac{(\bar{g}-\beta(1-\eta))\rho + \frac{\bar{g}(1-\rho)}{s_c}}{\bar{g} + \frac{\bar{g}}{\bar{g}-1}\left[\frac{1-\zeta}{\zeta}(\bar{g}-\beta(\rho-\eta)) + \frac{1}{\zeta}\bar{g}(1-\rho)\frac{1-s_c}{s_c}\right]}.$$

E.2.1 Modulating hysteresis effects

We start the analysis by discussing the determinants of hysteresis effects. Assume the economy is initially in steady state, when a supply shock hits in period t. We define hysteresis effects as

$$h_t \equiv \lim_{j\to\infty}\left(\log(\Psi\bar{A}_{t+j}Z_{t+j}\bar{L}) - \log(\Psi\tilde{A}_{t+j}Z_{t+j}\bar{L})\right),$$

where $\Psi\bar{A}_{t+j}Z_{t+j}\bar{L}$ is real GDP in period $t+j$ and $\Psi\tilde{A}_{t+j}Z_{t+j}\bar{L}$ is a counterfactual level of real GDP, assuming productivity keeps growing at its trend level in response to the shock, $\tilde{A}_t = \bar{g}\tilde{A}_{t-1}$.

Rewriting we get

$$h_t = \lim_{j \to \infty} \log\left(\frac{\bar{A}_{t+j}}{\tilde{A}_{t+j}}\right) = \lim_{j \to \infty} \sum_{k=1}^{j} (\log(\bar{g}_{t+k}) - \log(\bar{g})) = \sum_{k=1}^{\infty} \hat{g}_{t+k},$$

where we used $\bar{A}_{t+j} = \bar{g}_{t+1}...\bar{g}_{t+j}\bar{A}_t$ and $\tilde{A}_{t+j} = \bar{g}^j \bar{A}_t$. Hysteresis effects are therefore the cumulative sum of growth differentials. Inserting our solution for $\hat{g}_{t+k}$, we finally obtain

$$h_t = \sum_{k=1}^{\infty} \gamma_g \hat{Z}_{t+k} = \frac{\gamma_g}{1-\rho} \hat{Z}_t \equiv \gamma_h \hat{Z}_t.$$

Hysteresis effects depend on the size of the shock $\hat{Z}_t$, on its persistence ρ, and on the responsiveness of growth with respect to the shock γ_g.

In the main text we discussed the determinants of γ_h for our baseline model with $\zeta = 1$. Here we note that hysteresis effects are increasing in ζ. Intuitively, more curvature in the investment function (i.e. a lower ζ) reduces firms' investment response to changes in macroeconomic conditions. The parameter ζ can thus be used to calibrate the strength of hysteresis effects. In fact, rearranging the expression for γ_h gives

$$\zeta = \frac{\bar{g} - \beta(\rho - \eta) + \bar{g}(1-\rho)\frac{1-s_c}{s_c}}{\bar{g} - 1} \left(\frac{\gamma_h}{\frac{1}{s_c} + \frac{\rho}{1-\rho}\frac{\bar{g}-\beta(1-\eta)}{\bar{g}}} - 1 + \frac{\bar{g} - \beta(\rho - \eta)}{\bar{g} - 1} \right)^{-1}.$$

So consider a negative supply shock that on impact causes a recession of size $\hat{Z}_0$. Then the expression above defines the value of ζ that approximately matches a long-run drop in output equal to a fraction γ_h of the initial recession.

E.2.2 Natural rate response to supply disruptions

We next study the response of the natural real interest rate to a supply disruption. Log-linearizing (E.3) yields an equation for the natural rate

$$\hat{c}_t = \hat{g}_{t+1} - \hat{r}_t + \hat{c}_{t+1}.$$

Inserting the solution for $\hat{c}_t$ and $\hat{g}_{t+1}$, and rearranging we then obtain

$$\hat{r}_t = (\gamma_g - (1-\rho)\gamma_c)\hat{Z}_t$$
$$= \frac{(\bar{g} - \beta(1-\eta))\rho + \frac{\bar{g}(1-\rho)}{s_c} - (1-\rho)\frac{1}{s_c}\left(\bar{g} + \frac{\bar{g}}{\bar{g}-1}\left(\frac{1-\zeta}{\zeta}(\bar{g} - \beta(\rho - \eta)) - \frac{1-s_c}{\zeta}(\bar{g} - \beta(1-\eta))\rho\right)\right)}{\bar{g} + \frac{\bar{g}}{\bar{g}-1}\left[\frac{1-\zeta}{\zeta}(\bar{g} - \beta(\rho - \eta)) + \frac{1}{\zeta}\bar{g}(1-\rho)\frac{1-s_c}{s_c}\right]}\hat{Z}_t.$$

Manipulating the expression above shows that the natural rate falls following a negative supply shock if and only if

$$\zeta > \frac{(\bar{g} - \beta(1 - \eta))\rho + (\bar{g} + \beta\eta)(1 - \rho)\frac{1}{s_c}}{\frac{1}{s_c}(\bar{g} - \beta(\rho - \eta)) + \frac{\rho}{1-\rho}\frac{\bar{g}-1}{\bar{g}}(\bar{g} - \beta(1 - \eta))} \equiv \bar{\zeta}. \tag{E.7}$$

Note that $\bar{\zeta}$ is decreasing in ρ. Intuitively, the stronger the diminishing returns to investment the larger the shock persistence must be for the natural rate to fall following a negative supply shock.[48]

Let us now consider a version in which growth is exogenous and there is no investment. Since productivity growth is exogenous, then $\gamma_g = 0$, while since all the output is consumed $\gamma_c = 1$. Now consider that the natural rate evolves according to $\hat{r}_t = (\gamma_g - (1 - \rho)\gamma_c)\hat{Z}_t$, and that under endogenous growth $\gamma_c < 1$ and $\gamma_g > 0$. The implication is that, after a negative supply shock, the natural rate necessarily rises by less when trend productivity growth is endogenous compared to an economy with fully exogenous productivity growth.

E.2.3 Numerical illustration

We conclude this appendix with a numerical simulation that shows how the model behaves with milder hysteresis effects than the ones considered in the main text. To calibrate χ, α and β, we target a growth rate in steady state of $\bar{g} = 1.02$, an investment share of GDP of $1 - s_c = 0.02$, and a real interest rate of $\bar{r} = 0.025$. This yields $\chi = 1.889$, $1 - \alpha = 0.833$ and $\beta = 0.995$. We set $\eta = 0.115$, $\lambda = 0.5$, $\log(Z_0) = -0.03$ and $\rho = 0.5$ for the reasons spelled out in the main text. The new parameter to calibrate is ζ, the degree of decreasing returns to innovation. We set $\zeta = 0.976$ by targeting a hysteresis effect of 50% ($\gamma_g/(1 - \rho) = \gamma_h = 0.5$).

Figure 7 shows the economy's response to a negative supply shock in the natural allocation. As we can see, the impulse responses under mild hysteresis effects fall in between those of the models considered in the main text. For instance, under mild hysteresis effects the negative supply shock causes a slight rise in the natural real interest rate.[49]

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[48]It is instructive to look at two limiting cases. First, assume that the shock is purely transitory ($\rho = 0$). In this case, (E.7) reduces to $\bar{\zeta} = 1$. From (E.1), this implies that the natural rate unambiguously rises for any degree of curvature $\zeta < 1$. The second limit case is the one of permanent shocks, in which case $\rho = 1$. In this case, (E.7) implies that $\bar{\zeta} = 0$. Therefore, for permanent shocks the natural rate unambiguously declines for any degree of curvature $\zeta < 1$.

[49]From Figure 7, one might be tempted to conclude that our model cannot account for hysteresis effects which are larger than about 75%, which are those in our calibrated baseline model. This is not correct. As we showed in equation (17), hysteresis effects are increasing in both η and ρ. Thus to account for larger hysteresis effects, possibly above 100% as reported in Bluedorn and Leigh (2018) and others, we would simply need to assume a higher discount factor wedge (higher η), or a more persistent supply disruption (higher ρ).
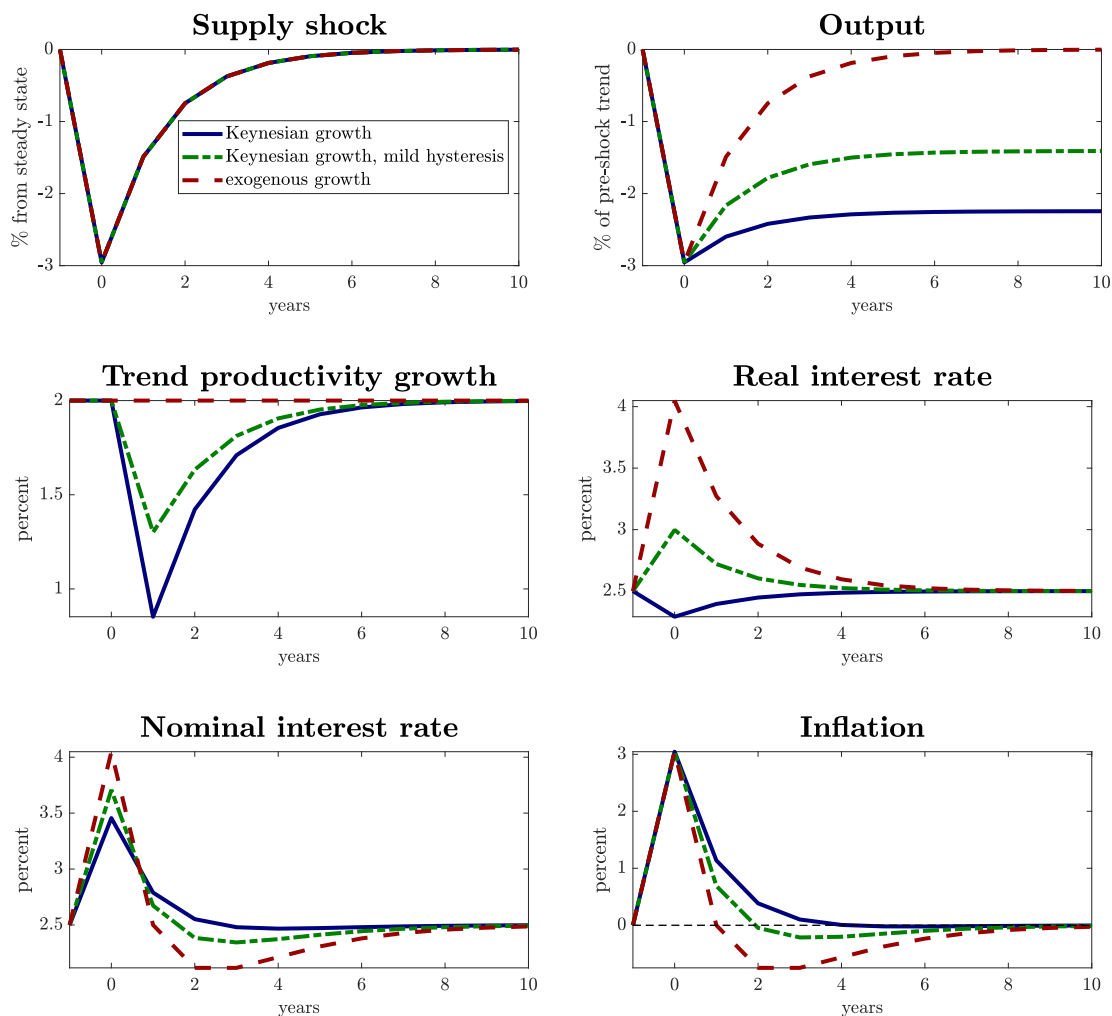
Figure 7: Macroeconomic impact of supply disruptions.

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