Fear of Hiking?
Rising Interest Rates in Times of High Public Debt

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Abstract

We build a sovereign default model to understand the implications of rising safe interest rates for countries with high public debt. When debt levels are below a critical threshold, countries respond to higher interest rates by reducing their debt due to a dominant substitution effect. For high debt levels, in contrast, the same rate rise triggers even more debt - and possibly a slow moving debt spiral - due to a dominant income effect. The seeds for a debt spiral are laid by a long phase of low interest rates: they imply that debt levels rise over time, making a future interest rate normalization more difficult. A successful interest rate normalization involves a credible path of rising interest rates, the speed of which must be intermediary: a too fast normalization leads to debt spirals, but a too slow one undermines incentives by the government to repay.

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1 Introduction

In this millennium, safe real interest rates on government debt have steadily declined to historically low levels, but lately they have been on the rise.[1] To fight the recent spike in inflation, central banks are tightening monetary policy, raising expected real rates in the medium term. At the same time, the structural forces that pushed down interest rates may have gone into reverse, implying safe interest rates could be permanently higher in the future.[2] The prospect of rising interest rates comes at a time when public debt in many countries has reached all-time highs, leading to concerns by policymakers that high-debt countries, such as Italy, could soon be plunged in a debt spiral (Frankel, 2023; IMF, 2023).

Motivated by this concern, in this paper we study the implications of a rise in safe interest rates in a sovereign default framework where the government chooses both debt issuance and default optimally (Eaton and Gersovitz, 1981). An intellectual tension in the above narrative is that higher interest rates are typically thought to make debt issuance less attractive, which reduces debt levels over time. Indeed, the fact that higher interest rates reduce borrowing is commonly thought to be a main channel through which a monetary tightening reduces economic activity. Would higher interest rates then not incentivize governments to pay back their debts, making a debt crisis (after some adjustment pain) less likely going forward?

The recurrent theme of our analysis is that this intuition is flawed, at least when debt levels are high to begin with. The reason is that higher interest rates work on debt issuance not only through the substitution effect just described, but also through an income effect wish pushes in the opposite direction. Indeed as we show in the paper, when debt levels are high enough, the income effect *dominates* the substitution effect, such that a rise in interest rates incentivizes the government to *issue even more* debt going forward. For high enough debt and interest rates, the government may even abandon entirely its attempts to pay back its debts and enter a slow moving debt spiral (Lorenzoni and Werning, 2019). In our quantitative application of the model to Italy, we find the seeds for a debt spiral are laid by a long phase of low interest rates: they imply that debt levels climb over time, making a future interest rate normalization more difficult. We also inspect possible strategies out of the low interest rate - high public debt dilemma. We find interest rate normalization must be credible, and occur at a speed that is intermediary: a too fast normalization leads to debt spirals, but a too slow one undermines incentives by the government to repay.

To make these points, we build a sovereign default model along the lines of Bocola and Dovis (2019). The government has access to exogenous and stochastic tax revenue and issues defaultable long-term debt to maximize utility over a stream of government spending. Bonds are priced by risk-neutral, competitive lenders. The model is real and the safe interest rate an exogenous variable.

[1] For instance, expected real interest rates on German government bonds with 10 years maturity have increased since the start of 2022, from -2% then to 0% in January 2024 (click here).

[2] For instance, the trend for de-globalization and protectionism, as well as the energy transition, could lead to higher interest rates (IMF, 2023, Chapter 2). The changing demographics may also contribute to higher interest rates in the future (Goodhart and Pradhan, 2017).

Following Chatterjee and Eyigungor (2015) we also allow for renegotiation and therefore a positive recovery value after default. As shown in Lorenzoni and Werning (2019), the combination of long-term debt and a positive recovery value is a necessary ingredient for slow moving debt spirals to exist, which are a key feature of our model.[3]

We start by studying a two-period version of the model, which allows us to shed light on the main economic forces at play in a sequence of four propositions. Proposition 1 establishes that, if a debt spiral exists, it occurs when debt levels exceed a *tipping point*, consistent with the analysis in Lorenzoni and Werning (2019). It becomes optimal for the government to let debt levels grow until an eventual default, as attempting to reduce debt starting from a very high level would be prohibitively costly in terms of current-period utility.[4] In the language of Ghosh et al. (2013), the government displays "fiscal fatigue" when debt levels are very high.

The remaining three propositions characterize how the government responds to a rise in safe interest rates. Proposition 2 establishes the existence of a *critical threshold* for the level of debt below which the rate hike reduces debt issuance going forward, and above which the rate hike increases debt issuance going forward. The threshold exists because of the presence of competing substitution and income effects of changes in interest rates. The substitution effect captures the government's reduced incentives to issue debt at a higher interest rate, prompting it to issue less debt going forward. The income effect captures that debt levels grow faster at a higher interest rate. Because the income effect becomes stronger, the larger is the level of preexisting debt, it starts to dominate the substitution effect (inherently a marginal effect) when debt levels are high enough. We derive the threshold analytically and explore its dependency on the model's parameters. For instance, we find the threshold is lower when the government has less fiscal space, which we define as tax income net of a non-discretionary component of government spending. This implies that a government with less fiscal space is more likely to respond to higher interest rates by higher debt issuance going forward.

In Proposition 3 we show that the presence of default risk leads to *amplification*. Assume that preexisting debt is high enough for the income effect to be dominant. This implies a safe-rate hike leads to more debt issuance. Because more debt raises future default risk, the effective interest rate on government debt (composed of the safe rate plus the compensation due to default risk) rises more than one-by-one with the rise in the safe interest rate. Because the income effect of a rise in interest rates is dominant, the result is even more debt issuance. Through this vicious circle, a safe-rate hike can have powerful effects on debt issuance and default risk.

Finally, Proposition 4 studies how the tipping point above which the government chooses the debt spiral adjusts when the safe rate is increased. We find that, in general, whether the tipping point rises or falls is ambiguous. However, for plausible parameters, we find that the tipping point

[3] Lorenzoni and Werning (2019) also highlight the existence of multiple equilibria, that is, expectations of a slow-moving crisis can be self-fulfilling. We sidestep the possibility of multiplicity in our study. Slow moving crises in our setting are therefore always fundamental, and can be triggered when debt levels are very high.

[4] In the two-period model, entering the spiral implies a default occurs in the second period. In the infinite-horizon model, the debt spiral plays out slowly and the default happens multiple periods in the future, as in Lorenzoni and Werning (2019).

is reduced, implying a safe-rate hike makes it more likely that the government "gives up" and chooses optimally the debt spiral. Taken together Propositions 2-4, we thus see that rising interest rates tend to reduce debt issuance when debt levels are low, but increase it when they are high. For even higher debt levels, in turn, rising interest rates may even plunge the economy in a slow moving debt spiral.[5]

Equipped with those theoretical insights, we turn to the infinite horizon version of our model, which we calibrate to Italy. We first show that the model describes reasonably well Italian debt dynamics: feeding the observed business cycle and path for safe interest rates into the model, it predicts well the evolution of Italian debt to GDP over the period 1999-2019.

We then ask the model what are the reasons for a debt spiral to occur by computing the average path leading up to a spiral from a long simulation. This analysis reveals that, on average, a debt spiral is triggered by a long phase of low interest rates, which then suddenly revert. Specifically, real interest rates are between 1 and 2 percentage points below their long-run mean for around 15 years. During this time, debt levels are rising. This reflects a dominant substitution effect when debt levels are initially low (Proposition 2). After 15 years, interest rates start to climb, by around 3-4 percentage points within a time frame of 5 years. Because the interest rate reversal coincides with a high level of debt, debt levels are not falling back. Rather, in line with Propositions 2-4, the rate hike triggers *even more* debt issuance and, in fact, a debt spiral.

This prediction of the model is in line with a recent literature stressing that, following a long phase of low interest rates, interest rate normalization becomes more difficult. Boissay et al. (2021) inspect a New Keynesian model with financial crises and stress that: "financial crises may occur after a long period of unexpectedly loose monetary policy as the central bank abruptly reverses course" (see also Akinci et al. (2021) and Coimbra and Rey (2023)). Grimm et al. (2023) provide some empirical support for this model prediction. In Mian et al. (2021), low interest rates reduce the natural rate of interest due to an accumulation of debt in the economy, implying the central bank cannot normalize interest rates without negative effects on aggregate demand. Focusing on public debt, we also stress that additional borrowing which is triggered by low interest rates makes interest rate normalization more difficult. In our model, this happens because a high level of debt implies the government has incentives to respond to higher interest rates by issuing an *even higher* level of debt.

Finally, we use the last part of the paper to inspect possible strategies for interest rate normalization in the face of high public debt. We make two points. The first point is that interest rate hikes which are expected to persist into the future strengthen the substitution effect, and therefore imply that debt levels are more likely to decline once interest rates rise. Assume for instance that debt levels are initially 140% of GDP and interest rates are -2%. Starting from this point, a rise in interest rates that is expected to be reversed in the next period leads to even more debt issuance in our calibrated model, as the income effect is dominant. In contrast, if the same rate hike signals

[5] In principle, there is also the possibility that higher interest rates trigger an immediate default. In the infinite horizon model, this never happens in equilibrium, because default only occurs at debt levels at which the economy is already in a slow moving debt spiral.

a path of rising interest rates in the future, the substitution effect is now dominant and debt levels decline. In this sense, interest rate normalization must therefore be *credible*.

Our second point relates to the *speed* of interest rate normalization, conditional on the normalization being credible. Our model suggests that interest rates should be reversed at a speed that is intermediary. A too fast normalization implies the government finds it optimal to enter the debt spiral, which echoes Proposition 4. Conversely, a too slow normalization fails to make use of the substitution effect, which as noted before is dominant. Stated differently, expectations of low interest rates for the future undermine the government's incentives to repay, locking the economy in a state of high debt and low interest rates.

To summarize, our analysis thus suggests that a successful interest rate normalization is a balancing act between preventing a debt spiral and providing sufficient incentives for the government to repay. This is resolved if interest rates revert at a speed that is intermediary, which additionally must be credible, i.e. perfectly anticipated and believed by the government.

Literature review. We connect to the sovereign default literature studying the implications of movements in the safe interest rate.[6] Guimaraes (2011) highlights that, in sovereign default models, the safe interest rate is the more important driver of the incentive-compatible level of debt (the maximum debt level before a default occurs), compared to movements in output. Johri et al. (2022) and Centorrino et al. (2022) build sovereign default models to study the conditions under which emerging market spreads increase following a rise in safe interest rates. Both papers are in the tradition of earlier research emphasizing the importance of changes in global safe rates for the business cycle in emerging markets (Neumeyer and Perri, 2005; Uribe and Yue, 2006). In contrast to those studies our focus is less on adjustments of spreads, but more on how changes in safe rates impact the level of public debt going forward. Our main innovation here is to uncover a strong state dependency of the effects of changes in safe interest rates: when debt levels are high, higher safe rates may lead to even more debt due to a dominant income effect, and possibly trigger a debt spiral.[7]

A recent wave of sovereign default models emphasize that governments may fail to deleverage when economic conditions deteriorate. In Conesa and Kehoe (2017), governments may run fiscal deficits and increase their debts in a recession in order to smooth consumption, a "gamble for redemption". In Müller et al. (2019), the combination of moral hazard and the need for structural reforms implies the government may increase its borrowing in a recession. In Bianchi et al. (2019),

[6] Our model is in the tradition of classic sovereign default models with incomplete asset markets, where the government optimizes both over the level of debt and over default. Seminal works are Eaton and Gersovitz (1981), Aguiar and Gopinath (2006) and Arellano (2008). As in Chatterjee and Eyigungor (2012) and Hatchondo and Martinez (2009), our government issues long-maturity bonds. Our model is most closely related to Bocola and Dovis (2019), Chatterjee and Eyigungor (2015) and Lorenzoni and Werning (2019). To the best of our knowledge, ours is the first paper to study slow moving debt crises - which have been introduced by Lorenzoni and Werning (2019) - in a quantitative sovereign default model.

[7] Some recent papers have started to integrate Keynesian elements into sovereign default models, in order to study the transmission channel of monetary policy in such a setting. One channel through which monetary policy operates is a change in safe (real) interest rates, which links our paper with those contributions. However, of course monetary transmission also works through other channels, notably the response of inflation and the fact that incomes become demand-determined. See Arellano et al. (2020) for a small open economy "NK-default" model, de Ferra and Romei (2020) for a model of a monetary union and Na et al. (2018) for a study of optimal default and devaluation.

the government may optimally increase debt in a recession to reap a Keynesian stabilization gain. In our paper, we show it is due to a dominant income effect that governments may wish to borrow more when their debts are already large and interest rates rise. As we explain in the text, a key ingredient to generate large income effects is the existence of a non-discretionary (subsistence) component of government spending - a proxy for adjustment frictions in government finances more generally. This links our analysis with Bocola et al. (2019) and Paluszynski and Stefanidis (2023), who also emphasize that subsistence consumption generates "borrowing into debt crises".[8] Ghosh et al. (2013) provide empirical evidence for this mechanism. They show that, when debt levels are high, countries' responsiveness of the primary balance to the debt stock shrinks and may even turn negative, which they dub "fiscal fatigue". Our model builds on this insight, and puts it into a framework where the government optimizes both over the level of debt and over default.[9]

Last, our paper connects to the literature stressing that a long phase of low interest rates may lead to financial stability risk (referenced above). We show this issue may also arise in the sovereign debt market. Furthermore, we also inspect possible strategies for interest rate normalization after a long phase of low interest rates. We find interest rate normalization must be credible, and it must occur at a speed that is intermediary.

The rest of the paper is structured as follows. In Section 2 we present a tractable two-period sovereign default model in which debt spirals are possible. In Section 3, we use this model to study analytically the implications of rises in the safe interest rate. The infinite-horizon, quantitative version of the model is presented in Section 4. In Section 5 we study the predictions of the quantitative model, and discuss strategies for interest rate normalization. Finally, Section 6 concludes. An appendix collects the proofs of all propositions as well as additional derivations.

2 A tractable sovereign default model with debt spirals

This section provides a tractable two-period sovereign default model. We use the model to study analytically the effects of changes in the safe real interest rate on borrowing choices by the sovereign and the risk of a sovereign default. A key feature of the model is that it allows for debt spirals akin to Lorenzoni and Werning (2019). In the two-period framework, the debt spiral materializes as a tipping point above which the government chooses infinite new debt, implying it defaults with certainty in the second period. The key insights derived in the two-period model will carry over to the quantitative, infinite horizon version of the model which we present in Section 4.

[8] Other recent sovereign default frameworks featuring subsistence consumption are Adam and Grill (2017) and Bianchi et al. (2018).

[9] Similar to us, Ghosh et al. (2013) develop a sovereign default framework and show the government enters a debt spiral when debt levels are sufficiently high. A key difference is that the government in our framework is fully optimizing (it chooses to enter the spiral optimally), whereas in Ghosh et al. (2013) the government follows a fiscal rule. We study an optimizing government as otherwise the response of borrowing to a change in the safe interest rate would be purely mechanical, and depending on the specific assumptions imposed on the fiscal rule.

2.1 Government's problem

Consider the problem of a government which lives for only two periods. To model the government's problem we follow closely Bocola and Dovis (2019). The government receives tax revenues every period and decides the path of spending, G in the first and G' in the second period. Tax revenues are a constant share $0 < \tau < 1$ of the output produced in the economy, $Y > 0$, which is exogenous and assumed to be constant across the two periods. The government starts with outstanding debt B. It chooses new debt B' to maximize utility, by taking as given the bond price schedule $q(B')$ and the safe interest rate R. In the second period, the government chooses to repay or to default on its debt.[10]

The budget constraint in the first period is given by

$$G = \tau Y + \frac{q(B')}{R}(B' - (1-\mu)B) - \mu B, \tag{1}$$

where $0 < \mu < 1$. A key assumption is that not all outstanding debt B matures in the first period. Rather, only a fraction μ of the debt matures and needs to be serviced in the first period. The remainder is only due in the second period, and thus becomes equivalent to one-period debt B' that is issued in the first period. This explains that $-(1-\mu)B$ is also multiplied with the price of debt $q(B')/R$. This formulation is a convenient way to introduce long-term debt into a two-period sovereign default model. Specifically, it retains the two key features of long-term debt; i) the fact that only a fraction of debt needs to be serviced in a given period and ii) the debt dilution effect, that is the fact that past creditors' claims are diluted by the issuance of new debt triggering a decline in the bond price $q(B')$ (Chatterjee and Eyigungor, 2012).

We view the first period as the "short run", the second period as the "long run". Consequently, we assume all uncertainty is resolved after the default decision has been made in the second period. Moreover, conditional on repayment, the government rolls over its debt B' forever, at the interest rate $1/\beta$, where $0 < \beta < 1$ captures the government's impatience. Conditional on repayment, the government's budget constraint in the second period is thus given by

$$G' = \tau Y - (1-\beta)B'. \tag{2}$$

In case the government defaults, it enters a renegotiation process with creditors, which we assume results in a recovery value $\kappa \geq 0$ of output. The fact that recovery is independent of outstanding past debt is a typical feature of renegotiation models.[11] We introduce a recovery value for two

[10] As in Bocola and Dovis (2019), we assume the government optimizes over government spending, making total debt the relevant state variable. This is in contrast to models of *external* debt, which assume the government borrows internationally on behalf of the private sector (Arellano, 2008). As discussed in Bocola and Dovis (2019), the former appears the more appropriate framework for modeling a country inside the euro area.

[11] See for instance Yue (2010). In the quantitative model in Section 4, we model properly the renegotiation process along the lines of Chatterjee and Eyigungor (2015). Note the assumption that the recovery value is always equal to κ violates the government's participation constraint in case it defaults at debt levels $B' < \kappa$, because renegotiation would entail a negative haircut. However, this is irrelevant for the results, because the government never chooses to default when $B' < \kappa$ along the equilibrium path. Therefore, we could equivalently have assumed the recovery value to be equal to $\min\{B', \kappa\}$.

reasons. First, a haircut of less than 100% is a typical feature of actual default events (Cruces and Trebesch, 2013). Second and more importantly - as explained in Lorenzoni and Werning (2019) and as will become clear below - the presence of a recovery value is a necessary condition for debt spirals to exist. In addition to the recovery value, we assume the presence of a stochastic default cost $X \geq 0$, which is the only source of uncertainty in the two-period model. Conditional on default, the government's budget constraint in the second period is thus given by

$$G'_\delta = \tau Y - (1-\beta)(\kappa + X). \tag{3}$$

Denoting u the utility function of the government, the government's problem in the first period can now be compactly expressed as

$$\max_{B'} \left\{ u(G) + \frac{\beta}{1-\beta} \mathbb{E} \max \left\{ u(G'), u(G'_\delta) \right\} \right\} \tag{4}$$

subject to the budget constraints (1)-(3). In this problem, $\mathbb{E}$ denotes expectation with respect to the random variable X.

2.2 Lenders

The bond B' issued by the government is priced by competitive lenders. Comparing the budget constraints (2) and (3), we see that the government defaults whenever $X < B' - \kappa$. In words, the government defaults when the default cost turns out to be low enough. We use this to define the default indicator

$$\delta'(B') = \mathbb{1}\{X < B' - \kappa\}. \tag{5}$$

The default probability is $\mathbb{E}\delta'(B')$, which is (weakly) increasing in B'.

In the case of repayment, in present value terms each bond earns a repayment of 1. In the case of default, in present value terms each bond earns a repayment of κ/B', reflecting that the recovery value κ is split equally among lenders. Hence $q(B')$ is given by

$$q(B') = \mathbb{E}(1 - \delta'(B')) + \mathbb{E}\delta'(B') \frac{\kappa}{B'}. \tag{6}$$

2.3 Equilibrium

Equilibrium involves optimality and rational expectations. Given a price function $q(B')$, equilibrium is a level of borrowing B' and a default rule $\delta'(B')$ solving equations (4)-(5). In turn, given the optimal borrowing and default rule, the price function must be compatible with (6).

2.4 Debt spirals

In their landmark contribution, Lorenzoni and Werning (2019) show that governments issuing long-term debt may be subject to slow moving debt crises. The term "slow moving" refers to the fact that these crises play out gradually, which differentiates them from rollover crises which an earlier literature has studied (Calvo, 1988; Cole and Kehoe, 2000). The term "debt crisis", in turn, refers to the fact that the government chooses a level of debt for which a future default is highly likely or unavoidable.

Our two-period model generates equilibria similar to Lorenzoni and Werning (2019), in the sense that a tipping point for the debt level exists above which the bond price drops discretely and the government finds it optimal to choose certain future default. As our model only has two periods, there is no time for these crises to move slowly. Instead, the government immediately borrows as much as possible and then defaults in the second period. Still, the underlying mechanism is the same, so working this out in a two-period model is useful. In the infinite horizon version of the model in Section 4, the spiral will indeed be slow moving.

Notice also that our two-period model has a unique bond price schedule, given by (6). This is in contrast to Lorenzoni and Werning (2019), who show that slow moving debt spirals can also be triggered by animal spirits. More precisely, they show the presence of equilibrium multiplicity, because multiple bond price schedules are compatible with equilibrium at intermediate debt levels. In the case of bad expectations, the schedule involves low bond prices, and the government finds it optimal to enter the debt spiral. As our two-period model reveals, equilibrium multiplicity is distinct from the presence of tipping points, and is therefore not a necessary feature of a model with debt spirals.[12]

We next work out the debt spiral. Note that if the government borrows $B' > \tau Y/(1-\beta)$, this implies a certain default in the second period (as this involves $G' < 0$ conditional on repayment, see budget constraint (2)). The bond price is therefore given by $q(B') = \kappa/B'$, and the government's utility becomes

$$V_{\mathcal{S}} \equiv \max_{B'} \left\{ u\left(\tau Y + \frac{\kappa}{R}\left(1-(1-\mu)\frac{B}{B'}\right) - \mu B\right) + \frac{\beta}{1-\beta}\mathbb{E}u\left(\tau Y - (1-\beta)(\kappa + X)\right)\right\}. \tag{7}$$

With long-term debt ($\mu < 1$) and a positive recovery value ($\kappa > 0$), the term in curly brackets is strictly increasing in debt issuance B'. This happens because the continuation value is independent of B' (since the government defaults with certainty in the second period), while resources raised through debt issuance are strictly increasing in B'. How can further issuance bring new resources for consumption, despite the fact that default happens with certainty next period? Key for this is the presence of a positive recovery value, and the debt dilution effect. New creditors are willing to lend to the government, because they expect to tap into the recovery value, implying the bond price remains strictly positive.[13] The decline in the bond price brought about by a higher B', in turn, dilutes old creditors, which relaxes the budget constraint of the government. Because current

[12] In our infinite horizon model in Section 4, multiplicity is possible in principle. However, we never encountered it in our numerical solution. We expand on this point in Section 4 and in Appendix C.

[13] Chatterjee and Eyigungor (2015) argue that it is implausible that new bonds can be issued when next-period default risk is very high, due to the presence of underwriting standards. We agree with them. In the infinite horizon version of the model in Section 4, entering the debt spiral will not imply next-period default probabilities to be very high, because of the slow moving nature of the crisis. Hence this model will be compatible with the presence of underwriting standards. In this sense, the two period model should be taken as an illustration of the mechanism, rather than as a realistic description of an actual debt spiral.

consumption is increasing in B', only the limit point $B' = +\infty$ can be an optimal choice for the government.

To summarize, when solving its maximization problem (4), the government ends up comparing welfare conditional on an interior optimum (a level of B' implying default risk is less than 1), with the boundary case of the debt spiral $B' = +\infty$, which implies a certain default in the second period and delivers utility $V_{\mathcal{S}}$. The following proposition establishes conditions for the existence of a debt spiral.

Proposition 1 *If a debt spiral exists, it is associated with a tipping point $\bar{B}$. That is, the spiral occurs if and only if $B > \bar{B}$. Moreover, a debt spiral does not exist if $\kappa = 0$ or if $\mu = 1$.*

Proposition 1 reveals that, if a debt spiral exists, it is associated with a tipping point, that is, debt levels must be high enough to begin with. Moreover, the existence of a tipping point rests on the presence of long-term debt and a positive recovery value.[14]

Numerical illustration. We conclude the description of the model by providing a simple numerical example. We choose the frequency to be annual and set $\beta = 0.96$ and $R = 1/\beta$, a (safe) real interest rate of 4%. We set some of the remaining parameters following the analysis in Bocola and Dovis (2019), who calibrate their model to Italy. First, we normalize output to unity ($Y = 1$) and choose $\tau = 0.42$, that is, the government's tax base is 42% of GDP. Second, we use the utility function

$$u(G) = \frac{(G - \underline{G})^{1-\sigma} - 1}{1 - \sigma}, \tag{8}$$

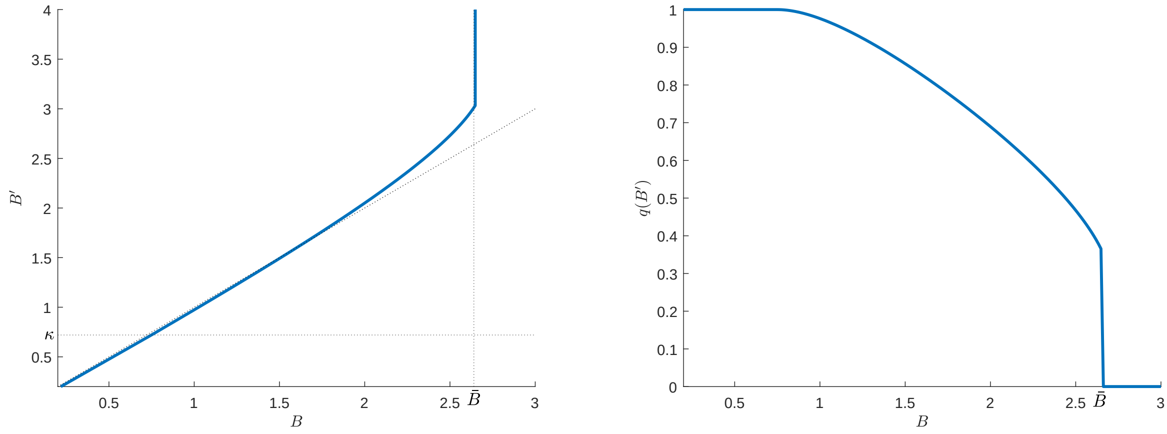
where $\sigma > 0$ and $\underline{G} \geq 0$ are parameters. The presence of $\underline{G}$ captures a non-discretionary part of government spending. As explained in Bocola and Dovis (2019), this can capture components of public spending that are hardly modifiable by the government in the short run, such as wages of public employees and pensions. As in their paper, we set $\underline{G}$ to equal 68% of the government's tax base, or $\underline{G} = 0.68 * 0.42 = 0.28$. For reasons that become apparent shortly, we also focus on log utility, that is, we set $\sigma = 1$. Third, we set the fraction of maturing debt to $\mu = 0.14$, consistent with an average maturity of Italian debt of 7 years.

The remaining parameters are the distribution of the default cost X and the recovery value κ. For X we simply assume a uniform distribution,

$$X \sim \mathcal{U}\left(\left[0, \frac{\tau Y - \underline{G}}{1 - \beta} - \kappa\right]\right). \tag{9}$$

Here we assume the default cost can never be so large that government spending is below the subsistence value after a default occurred. In turn, for the recovery value we use the illustrative value $\kappa = 0.75$, or 75% of GDP. Notice that lenders are paid this amount in present value terms, receiving $(1 - \beta) * \kappa$ (or 3% of GDP) in every period (see (3)).

[14] Hatchondo et al. (2014) study the implications of voluntary sovereign debt exchanges. They find debt exchanges lead to a positive expected recovery value upon default, and show this gives governments incentives to issue infinite amounts of debt when debt levels exceed a tipping point. They go on to show that this possibility leads to ex-ante lower bond prices, and that for this reason, ruling out the possibility of voluntary debt exchanges can be welfare improving.



(a) Borrowing. **(b) Bond price.**

Figure 1: Equilibrium in two-period model.

Figure 1 shows the equilibrium of the two-period model with these parameters, consisting of a level of borrowing B' (Figure 1a) and an associated price function $q(B')$ (Figure 1b). We plot the equilibrium against the initial level of debt B. Because Y has been normalized to 1 in both periods, the level of debt in both periods can be interpreted as debt to GDP.

A first insight from the figure is that, when B is low enough, equilibrium default risk is zero. To understand this point, assume for a second that default risk is zero. Then the Euler equation $u'(G) = u'(G')$ holds (where we used that $R = 1/\beta$), which implies that $B' = (\mu + \beta(1-\mu))B$. Because it is not optimal to default for $B' < \kappa$, we conclude that for $B < \kappa/(\mu + \beta(1-\mu))$, equilibrium default risk is indeed equal to zero.

Once B exceeds this threshold, default risk turns positive (the bond price falls below 1), and is strictly increasing in the level of B. Interestingly, B' remains close to the 45 degree line up to 200% initial debt to GDP, despite the fact that default risk at this point is already 30%. The government thus responds to the lower bond price by sacrificing current spending. Beyond this point, further reductions in current spending are no longer optimal, and the government responds to the lower bond price by simply issuing additional debt, implying B' starts to move above the 45 degree line. Finally, the tipping point for the debt spiral $\bar{B}$ is reached at around 270% debt to GDP. At this point, the government gives up its attempts to contain default risk in the second period. Rather, it decides to max out revenue from debt issuance for the first period, while accepting certain default (the lowest-possible continuation value) for the second period.

How do the predictions of our model compare with the data? Interestingly, the model makes a prediction consistent with the evidence in Ghosh et al. (2013). In a sample of advanced economies, they document the existence of a non-monotonic relationship between the primary balance and the level of outstanding public debt. Specifically, they report that the primary balance first increases with the level of debt, but the responsiveness eventually weakens and then actually *decreases* at very high debt (which they dub "fiscal fatigue"). Our model makes exactly these predictions: the primary balance (given by $\tau Y - G$) rises in B, first at an increasing, then at a decreasing rate.

Once B hits the tipping point, the primary balance falls. This happens because the government stops devoting resources to debt repayment when entering the debt spiral.

3 Safe interest rates, debt issuance and default risk

What is the effect of higher safe interest rates on debt issuance and the risk of a sovereign default? Can a higher safe rate plunge the economy in a debt spiral? We now use our two period model to look into this issue.

We highlight three results. First, we establish that governments may respond to higher interest rates by reducing their borrowing, but only when debt levels are low to begin with. Instead, when debt levels exceed a threshold level, governments respond to higher interest rates by issuing even more debt. Second, the presence of default risk can lead to amplification. That is, debt levels may rise by more following a rise in interest rates, once the endogenous increase in default risk is taken into account. Third, for even higher levels of debt, a rise in interest rates may even plunge the economy in a debt spiral.

3.1 Rise in safe rate may raise or reduce borrowing

We start by showing that a higher interest rate has ambiguous effects on the borrowing choice by the government, depending on the initial level of debt. Assume that $B < \bar{B}$, such that choosing the debt spiral is not optimal. The equilibrium is then characterized by an interior choice of B', solving the maximization problem (4). Under standard regularity conditions, the optimal choice is described by an Euler equation[15]

$$u'\left(\tau Y + \frac{q(B')}{R}(B' - (1-\mu)B) - \mu B\right) q(B')(1 - \varepsilon(B')) = \beta R \mathbb{E} u(G')(1 - \delta'(B')), \tag{10}$$

where

$$\varepsilon(B') \equiv -\frac{\partial q(B')/\partial B'}{q(B')}(B' - (1-\mu)B)$$

denotes the (negative) elasticity of the bond price with respect to new debt issuance. As explained in Aguiar et al. (2016), the elasticity term appears because the government internalizes the effects of its borrowing decisions on the price of debt.

Equation (10) can be seen as defining the optimal choice of B' implicitly, as a function of the safe rate R. The safe rate appears in two places, highlighting that two economic forces shape its impact on the optimal B'. First, it multiplies the time preference rate β - a substitution effect. Second, it enters the current marginal utility of consumption - an income effect. Intuitively, the substitution effect captures the fact that borrowing becomes more expensive when the interest rate is higher, which leads the government to reduce B'. The income effect, in turn, captures that the same amount of borrowing raises less resources when the interest rate is higher, implying a drop

[15] We derive the Euler equation in Appendix B.

in current consumption. Through this force, the government raises B' in response to rises in R, in order to cushion the drop in current consumption.[16]

A key insight of our analysis is that a critical threshold for initial debt B exists above which the income effect is dominant. To work out the threshold, assume that the utility function is given by (8), and also assume $\sigma = 1$, such that $u(G) = \log(G - \underline{G})$. In this case the threshold takes a particularly nice form, as we show in the following proposition.[17]

Proposition 2 *Assume the utility function is $u(G) = \log(G - \underline{G})$, and that the government takes borrowing decisions according to the Euler equation (10). Then a threshold for B/Y exists above which the impact of a change in R on B' flips. The threshold is*

$$\mathcal{T} = \frac{1}{\mu}\left(\tau - \frac{\underline{G}}{Y}\right). \tag{11}$$

When $B/Y < \mathcal{T}$, a rise in R reduces B' (the substitution effect dominates). When $B/Y > \mathcal{T}$, a rise in R increases B' (the income effect dominates).

Proposition 2 splits the state space into two regions: a region of low debt, where a rate hike leads to even lower debt going forward, and a region of high debt, where a rate hike leads to even higher debt going forward. What is the intuition for this result? The substitution effect is inherently a marginal effect that operates even when the debt stock is zero. In contrast, the income effect scales with the level of debt: the higher the debt stock, the more the same rise in interest rates eats into first-period consumption. This makes it clear that the income effect may start to dominate the substitution effect when the stock of debt is high enough.

We next explain which parameters shape the threshold in (11).

Debt maturity. The first determinant is the parameter μ, which governs the fraction of debt which matures in every period. When only little debt matures, a small part of the total stock of debt must be refinanced. Hence the rise in the interest rate only affects this small part of the debt, making the income effect weaker. It follows that a low μ increases the threshold $\mathcal{T}$ above which the income effect dominates. As pointed out by Chatterjee and Eyigungor (2012), the fact that government finances become insulated from changes in the price schedule $q(B')$ is what enables models with long-term debt to explain large equilibrium holdings of debt. For the threshold that we derived, what matters is that long-term debt also insulates government finances from changes in the safe interest rate R.

[16] The fact that interest rate changes have contrasting effects on borrowing should not come as a surprise. Indeed this result squares with undergraduate teaching of the effects of interest rate changes in the two-period consumption model. In this model, the substitution and income effects are typically explained in terms of consumption. The substitution effect of higher interest rates is that first-period consumption becomes more expensive relative to second-period consumption. Hence first-period consumption falls, whereas second-period consumption rises. In turn, for a *borrowing* agent, the income effect is that the agent becomes poorer, which reduces consumption in both periods. The overall effect on second-period consumption is therefore ambiguous. Because borrowing and second-period consumption are directly related (second-period consumption equals second-period income net of debt repayment, as in budget constraint (2)), the ambiguous response of second-period consumption translates into an ambiguous response of debt repayment (the face value of debt).

[17] We discuss the case $\sigma \neq 1$ below. See also Appendix B.

Fiscal space. The second determinant of $\mathcal{T}$ is $\tau - \underline{G}/Y$, the difference between the tax base and the non-discretionary part of government spending. This term can be thought of as the "fiscal space" of the government - the extent to which the government can adjust its primary balance in a discretionary manner. As we can see, a lower fiscal space reduces the threshold, and therefore acts like a strengthening of the income effect. The intuition is simple. Assume the government's primary balance is characterized by strong adjustment frictions, implying it can not easily be raised in response to a rise in the interest rate. Then the higher interest rate must be absorbed by more debt issuance going forward. A lower fiscal space therefore reduces the threshold $\mathcal{T}$ above which a rise in interest rates leads to more debt issuance going forward.

How large is the threshold $\mathcal{T}$ in a reasonable calibration? Using the earlier parameters $\mu = 0.14$, $\tau = 0.42$ and $\underline{G}/Y = 0.28$, we obtain $\mathcal{T} = 1/0.14 * (0.42 - 0.28) = 1$ - or exactly 100% debt to GDP. This computation suggests that the threshold may be in a range for debt that is empirically relevant.[18]

We conclude this section with a brief discussion of the case $\sigma \neq 1$. A detailed analysis can be found in Appendix B. As we show there, our main insight that a threshold $\mathcal{T}$ separating income and substitution effect exists is still valid. Moreover, the threshold tends to be lower, the higher is σ (intuitively, reflecting a weaker substitution effect as the intertemporal elasticity of substitution, which depends on $1/\sigma$, is reduced). However, we also show that $\mathcal{T}$ now depends on the equilibrium B', implying it becomes an endogenous variable. Even in the two period model, this makes the analysis analytically intractable, and computing the level of debt above which the income effect dominates can only be done numerically.

3.2 Default risk and amplification

The threshold we derived in equation (11) is independent of default risk. In fact, this threshold would be exactly identical, in case we assumed away the possibility of default in the second period. Does this imply default risk is irrelevant? In this section, we will argue that default risk amplifies the effect of the safe-rate hike on debt issuance in the region where the income effect is dominant. That is, a rise in the safe rate will increase borrowing (and therefore default risk) by more, once the endogenous feedback of default risk into borrowing decisions is taken into account.

To see this point, we rearrange the Euler equation (10) as follows

$$u'\left(\tau Y + \frac{1}{I(B')}(B' - (1-\mu)B) - \mu B\right) = \eta(B')I(B')u'(G'), \tag{12}$$

where we denote $I(B') \equiv R/q(B')$ the total interest rate on government debt (composed of the safe interest rate plus the compensation due to default risk), and $\eta(B')$ the "effective patience" by

[18] In related work, Johri et al. (2022) study the response of borrowing in a sovereign default model to a rise in safe interest rates and find borrowing always declines. Equation (11) makes it clear why this is the case. In their model, they abstract from adjustment frictions in the primary balance by assuming $\underline{G} = 0$, and they also assume $\tau = 1$ (the government's tax base is the entire GDP). The threshold for transitory interest changes is then $1/0.14 = 7$, or about 700% debt to GDP. As we discuss in section 5.2, persistence in the interest process further strengthens the substitution effect. This implies that the threshold is above all reasonable debt levels in their setting.

the government, given by

$$\eta(B') \equiv \beta \frac{1 - \mathbb{E}\delta'(B')}{1 - \varepsilon(B')}. \tag{13}$$

The first insight is that $I(B')$ enters the Euler equation symmetrically as the safe interest rate R. Through the interest rate channel, the model thus makes a clear prediction: default risk *amplifies* the effect of R on B' in the region where the income effect dominates, and *dampens* it in the region where the substitution effect dominates. Intuitively, when the income effect dominates, a rise in R triggers a rise in B'. Because this raises default risk ($q(B')$ declines), the total interest rate $I(B')$ endogenously increases further. Since the income effect dominates, the rise in $I(B')$ will trigger a further increase in B', leading to a further rise in $I(B')$, and so forth. In brief, the interest rate on government debt rises more than one-by-one with the safe interest rate, which triggers an additional issuance of debt.[19]

But there is another effect of changing default risk on the borrowing choice by the government, running through what we called effective patience. As it is intuitive, a decline in patience implies the government will borrow more. Now assume again the income effect dominates, implying a rise in R raises B'. Does the rise in B' make the government more patient, or more impatient? From equation (13), there are two effects to consider. First, the default probability $\mathbb{E}\delta'(B')$ rises, making the government more impatient. Intuitively, this happens because when the default probability is higher, the government cares less about the continuation value, and more about present consumption. Second, the rise in B' may also steepen the price curve (a higher elasticity $\varepsilon(B')$), making the government more patient. Intuitively, what matters for the substitution effect of the total interest rate in a sovereign default model is not just the level of the interest rate, but also the slope of the price curve.

Which of the two effects is stronger? A general characterization is difficult, as the properties of $\mathbb{E}\delta'(B')$ and $\varepsilon(B')$ depend on the distribution that is assumed for the cost of default X. However, we can obtain a first-order approximate result that holds for any distribution of X, as we show in the following proposition.

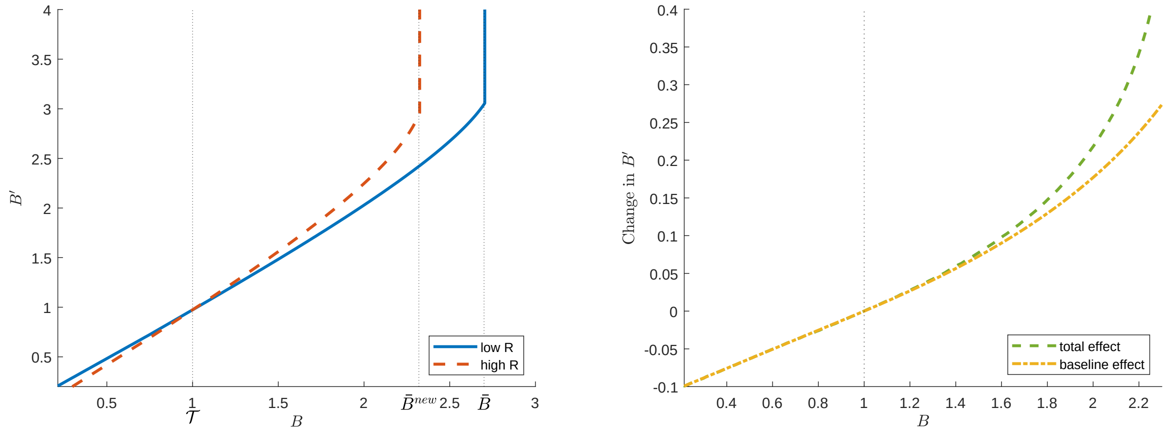
Proposition 3 *Let $\beta R = 1$ and consider the approximation point $(B, B') = (\kappa/(\mu + \beta(1-\mu)), \kappa)$. At this point, up to a first-order approximation, a rise in B' impacts $\eta(B')$ as follows*

$$\eta(B') \approx \beta + \beta\lambda \left(\frac{\mu + \beta(1-\mu) - 2(1-\mu)}{\mu + \beta(1-\mu)} \right) (B' - \kappa), \tag{14}$$

where $\lambda \geq 0$ is the hazard rate of the distribution function of X at zero.

The key insight from equation (14) is that $\eta(B')$ falls in B' when debt maturity is long enough. Specifically, the term multiplying $B' - \kappa$ is negative whenever $\mu + \beta(1-\mu) < 2(1-\mu)$, or approximately $\mu < 1/2$. Intuitively, with a long debt maturity, the elasticity is not very responsive to

[19] In turn, when the substitution effect dominates, the rise in $I(B')$ is endogenously dampened following a rise in R, because B' and therefore default risk declines. However, because the substitution effect dominates when debt levels (and therefore default risk) are small to begin with (see Proposition 2), these equilibrium effects are less relevant for the region where the substitution effect dominates, compared to the region where the income effect dominates.



(a) Borrowing. **(b) Amplification.**

Figure 2: Effects of higher safe interest rate in two-period model.

changes in default risk. Hence when B' rises, the default probability $\mathbb{E}\delta'(B')$ rises faster than the price schedule steepens (than the elasticity $\varepsilon(B')$ rises). The total effect is that effective patience of the government *declines* when B' is higher.[20]

Let us summarize the insights obtained in this section. When debt levels are high enough for the income effect to be dominant, a rise in the safe rate R has the effect of raising debt issuance B' (Proposition 2). This raises default risk, which leads to even more debt issuance. The amplification occurs through two channels; first, the endogenous rise in the total interest rate $I(B')$; second, the fact that the government becomes more effectively more impatient (Proposition 3).

We conclude this section by going back to our numerical example. We use the same parameters as in the previous section, and study the implications of a sharp rise in the safe interest rate. We target a rise in the safe rate such that, at a debt to GDP ratio of 150%, a simple rollover of the additional interest burden would increase debt to GDP by 10 percentage points.[21] The result is in Figure 2a. In line with Proposition 2, we find that new borrowing B' *tilts* around the threshold point $\mathcal{T}$, which equals 1 under the chosen parameters. That is, to the left of this point, the rise in R reduces new borrowing, while to the right of this point, the rise in R leads to additional debt issuance. In Figure 2b, in turn, we decompose the total effect into a baseline effect plus the effect that is due to the presence of default risk. The total effect (green dashed line) is the difference between the red dashed and blue solid lines in Figure 2a. To compute the baseline effect, we compute the level of B' that satisfies the Euler equation (12), while keeping the total interest rate $I(B')$ and effective patience $\eta(B')$ at their levels before the change in B'. This procedure isolates

[20] As we explain in the proof of the proposition, the appealing property of the approximation point $(B, B') = (\kappa/(\mu + \beta(1-\mu)), \kappa)$ is that default risk is zero and the recovery rate is one at this point. At the same time, this point contains information about the impact of a *rise* in B' on default risk and on the recovery rate. Because $\eta(B')$ is not differentiable at $B' = \kappa$, equation (13) is to be understood as using the right-derivative of $\eta(B')$ at the approximation point.

[21] The implied interest rate is given by $R = 2.04$. It is very large, because we squeeze all the effect of the interest rate hike in a single period. In the infinite horizon model in Section 4, much smaller interest rate hikes will be needed to produce strong effects on debt issuance, because the interest rate hikes are going to be persistent.

the effect on B' that is purely due to the change in the safe interest rate. In line with our insights from this section, the total effect is always larger than the baseline effect; hence, the presence of default risk leads indeed to amplification.[22]

3.3 Back to the debt spiral

A large concern is that rising safe interest rates could plunge high-debt countries such as Italy in a spiral of ever rising debt. In the previous two sections we have already shown that, when debt levels are high enough, a rise in interest rates leads to even more debt and a higher risk of default going forward. We will now complete the picture by arguing that the tipping point for the debt spiral may also be reached earlier when interest rates are higher. Hence a rise in safe rates may indeed plunge the economy in a slow moving debt spiral.

We start by going back to Figure 2. As Figure 2a illustrates, in our numerical example, the tipping point for the spiral $\bar{B}$ has indeed been reduced by the rise in the interest rate, from 270% debt to GDP to about 230%. Let's see why.

The defining feature of the tipping point $\bar{B}$ is that, at $B = \bar{B}$, the value from choosing the interior optimum is the same as the value from choosing the debt spiral. Denote $V_{\mathcal{I}}(B)$ the value from choosing the interior optimum - the value that is implied when B' satisfies the Euler equation (10). Then $\bar{B}$ is defined by the condition

$$V_{\mathcal{I}}(\bar{B}) = V_{\mathcal{S}}(\bar{B}), \tag{15}$$

where $V_{\mathcal{S}}$ has been defined in Section 2.4. Because both values also depend (negatively) on the safe rate R, by using implicit differentiation, we can back out the impact of changes in R on $\bar{B}$. We summarize our results in another proposition.

Proposition 4 *Assume a debt spiral exists, described by the tipping point* (15). *Then $\partial\bar{B}/\partial R < 0$ (a rise in the safe rate reduces the tipping point $\bar{B}$), if and only if*

$$u'(G)(G + \mu\bar{B} - \tau Y)\Big|_{\mathcal{S}} < u'(G)(G + \mu\bar{B} - \tau Y)\Big|_{\mathcal{I}}. \tag{16}$$

The left-hand side is the marginal-utility-weighted budget deficit at the point $\bar{B}$ when choosing the debt spiral, the right-hand side, when choosing the interior optimum.

Proposition 4 reveals that the impact of a rise in R on the tipping point is in fact ambiguous. To understand this, note that $G\big|_{\mathcal{S}} > G\big|_{\mathcal{I}}$ - the government consumes more conditional on entering the spiral because it does not devote resources to debt repayment. This reduces marginal utility, but increases the budget deficit, on the left-hand side of equation of (16). Whether the inequality holds or not is therefore ambiguous.

[22] For low debt levels, the baseline and the total effect become identical. Of course, this reflects that default risk is low for low debt levels, hence any dampening or amplifying force due to default risk becomes irrelevant.

What is the intuition? Entering the debt spiral means choosing a budget deficit that is particularly large. This makes a rise in interest rates particularly damaging to first-period consumption. Through this force, choosing the spiral when interest rates are high is particularly unattractive. At the same time, recall that the level of consumption is lower when choosing the interior optimum. This implies the rise in interest rates is particularly damaging in *utility terms* when choosing the interior optimum (repaying becomes even more painful with higher interest rates). Through this force, choosing the debt spiral becomes more attractive when interest rates are higher.

In our numerical illustration, the utility effect is larger than the primary deficit effect, and therefore the tipping point is reduced. However, we find it interesting that the effect is in general ambiguous. In fact, should the primary deficit effect dominate, a rise in interest rates would make the emergence of a debt spiral less likely.

4 Infinite horizon model

We now move forward to the infinite horizon model. Its main building blocks are unchanged relative to the two period model: a government issues defaultable long-term debt to maximize a stream of utility values $u(G)$ and haircuts are less than 100% due to the possibility of recovery. Both features combined implies the possibility of slow moving debt spirals.[23] As before, we model the government's problem following closely the analysis in Bocola and Dovis (2019). Moreover, to model recovery we follow the renegotiation framework in Chatterjee and Eyigungor (2015).

4.1 Environment and recursive equilibrium

Conditional on repayment, the resource constraint of the government is

$$G = \tau Y + q(s, B')(B' - (1 - \mu)B) - \bar{\mu}B, \tag{17}$$

which looks almost like in the two period model. To repeat, B is the stock of debt in the current period, B' is debt chosen for next period, G is government spending, τY is tax income and $q(s, B')$ is the price of debt.[24] For the parameters we assume $0 < \tau < 1$ and $0 < \mu < 1$. For the coupon we assume $\bar{\mu} \equiv \mu + \bar{R} - 1$, where $\bar{R} > 1$ is the safe interest rate in steady state. This coupon structure implies that the bond price is 1 in the steady state, independent of the bond's maturity structure. In contrast to the baseline model, output Y and the safe rate R are now stochastic. The exogenous state vector is denoted by $s \equiv (Y, R)$. Note that the bond price $q(s, B')$ depends on new borrowing

[23] As in our two-period model, we do not focus on equilibrium multiplicity of the types studied in Lorenzoni and Werning (2019) and Aguiar and Amador (2020). In fact, we have never encountered multiple equilibrium bond price schedules in our numerical implementation, even though they may in principle exist in our framework (see Appendix C for details). How a change in safe interest rates impacts multiplicity of these types is therefore an interesting open question, that we reserve for future research. Some progress in this direction is made in Bacchetta et al. (2018), albeit in a setting where the government does not fully optimize.

[24] Note we now define the bond price schedule $q(s, B')$ as *inclusive* of the safe interest rate R, see (25) below. This is the standard formulation in the literature. In the two period model in Section 2, we departed from this practice to make our analytic results on the effects of changes in R on B' more transparent.

B', but also on the exogenous state vector s.

As before, we denote $u(G)$ the flow utility from government spending. The value from repaying the debt, denoted $V_r(s,B)$, is then

$$V_r(s,B) = \max_{B' \in \mathbb{B}} \left\{ u(G) + \beta \mathbb{E} V(s', B') \right\}, \tag{18}$$

where G is given by (17) and where $V(s',B')$ is the value from entering next period in good financial standing (i.e. not in a state of default). For future reference, we denote $\mathcal{B}(s,B)$ the optimal debt choice conditional on repayment (the policy function for B' that solves equation (18)).

Conditional on default, the resource constraint of the government is

$$G_\delta = \tau Y - \max\{0, d_0 \tau Y + d_1 (\tau Y)^2\}. \tag{19}$$

where $d_0 > 0$ and $d_1 > 0$ are parameters. When in default, the government's only source of revenue is tax income, of which an increasing and convex share is lost. This standard assumption captures in a simple way that sovereign default entails severe disruptions in the economy. The value from default, denoted $V_\delta(s,B)$, solves the recursive expression

$$V_\delta(s) = u(G_\delta) + \beta \mathbb{E} \left\{ (1-\psi) V_\delta(s') + \psi V_r(s', Z(s')) \right\}, \tag{20}$$

where G_δ is given by (19). When in default, the government suffers exclusion from financial markets for a random number of periods. The parameter $0 < \psi < 1$ is the probability of regaining access to financial markets in the next period. When this happens, the government enters a renegotiation process with creditors, which we specify below. In case the renegotiation is successful (which in equilibrium is always the case), the government regains good financial standing, and owes creditors an amount of debt equal to $B' = Z(s')$.

The renegotiation works as follows. In case renegotiation starts (with probability ψ), creditors as a group make a take-it-or-leave-it offer to the sovereign.[25] If the renegotiation fails, the sovereign is condemned to permanent autarky. The value from permanent autarky is

$$V_a(s) = u(\tau Y - \alpha) + \beta \mathbb{E} V_a(s'), \tag{21}$$

where $\alpha > 0$ is a parameter that captures additional costs of complete autarky, and can be used to calibrate the debt recovery rate. The group of creditors attempts to max out the new level of debt owed by the government, conditional on the government's participation constraint that the value from emerging from default $V_r(s,B)$ is at least as large as the value from choosing autarky. Because $V_r(s,B)$ is strictly declining in B, this implies the amount of negotiated new debt $Z(s)$ is

[25] Following Chatterjee and Eyigungor (2015), we thus assign the entire bargaining power in the renegotiation phase to the lenders. More general frameworks where the bargaining power is a parameter can be found in Yue (2010) and Prein (2022).

defined by the following condition:[26]

$$V_r(s, Z(s)) = V_a(s). \tag{22}$$

Conditional on entering the period in good financial standing (the value of which is $V(s,B)$), the government chooses to default or to repay its debt

$$V(s, B) = \hat{\mathbb{E}} \max \left\{ V_r(s, B) + \varsigma\varepsilon_r, V_\delta(s) + \varsigma\varepsilon_\delta \right\}, \tag{23}$$

where ε_r and ε_δ are default taste shocks that are i.i.d. Gumbel (Extreme Value Type I) distributed. The parameter $\varsigma > 0$ controls their importance and $\hat{\mathbb{E}}$ is the expectation operator with respect to these shocks. We introduce default taste shocks as a third source of exogenous variation (in addition to fluctuations in Y and R), because non-fundamental default risk (for instance due to multiplicity or the possibility of "runs") appears to be an important driver of spreads in sovereign debt markets, particularly in countries inside the euro area (De Grauwe and Ji, 2013; Bocola and Dovis, 2019; Bianchi and Mondragon, 2021). Below we will therefore calibrate ς to aid our model to come closer to the observed behavior of spreads.[27]

Solving the expectation in (23) explicitly, we see the government chooses to default with probability

$$P(s, B) = \frac{\exp(V_\delta(s)/\varsigma)}{\exp(V_\delta(s)/\varsigma) + \exp(V_r(s, B)/\varsigma)}. \tag{24}$$

This allows us to define the bond price schedule by risk neutral lenders

$$q(s, B') = \mathbb{E}\left\{ (1 - P(s', B'))\frac{\bar{\mu} + (1-\mu)q(s', \mathcal{B}(s', B'))}{R} + P(s', B')\frac{\frac{\kappa(s')}{B'}}{R} \right\}. \tag{25}$$

In this expression, $\kappa(s')$ is the aggregate value of repayment, which is split equally among lenders and hence is divided by B'. It is given by the recursive expression

$$\kappa(s) = \mathbb{E}\left\{ (1-\psi)\frac{\kappa(s')}{R} + \psi\frac{q(s', Z(s'))Z(s')}{R} \right\}. \tag{26}$$

Intuitively, in case the government defaults next period, it is only in the *subsequent* period that the creditors can elicit repayment, with probability ψ. Hence expected repayment is $\psi q(s', Z(s'))Z(s')$, and it needs to be discounted using the safe interest rate R.

We are now ready to define equilibrium in the standard way. For a given law of motion governing the exogenous state s, equilibrium is a collection of value functions (V, V_δ, V_r, V_a), a collection of

[26]The assumption here is that creditors want to elicit as much new debt as possible, which is indeed optimal for them as long as the economy is on the "correct" side of the Laffer curve such that the value of debt is strictly rising in B'.

[27]Note default taste shocks of the type discussed are perfectly standard in the literature, and are typically used to aid the numerical convergence process (e.g., Arellano et al., 2020). Here we use these shocks to aid convergence, but also to bring our calibrated model closer to the data. We also use taste shocks on the borrowing choice B' - again in line with the literature - however those shocks we calibrate as small as possible to guarantee convergence of our algorithm, without them affecting the properties of the solution of the model.

policy functions $(P, \mathcal{B}, Z, \kappa)$, and a price function q, such that

1. Given the price function, the value and policy functions solve (18), (20)-(24) and (26).
2. The price functions satisfies (25).

4.2 Calibration

Our calibration is quarterly and aimed at Italy during its time in the euro area, before the outbreak of Covid in 2020. We first fix some parameters externally before calibrating the remaining parameters through simulations of the model.

We set some parameters the same way we did in the two period model (where we followed Bocola and Dovis 2019). First, we again assume $\tau = 0.42$, implying the tax base is 42% of GDP. The flow utility of government spending is again specified as in (8), and we again set $\underline{G} = 0.28$ for the non-discretionary part of government spending. Third, we again target an average maturity of Italian debt of 7 years, which in the quarterly calibration implies $\mu = 0.035$. Fourth, also following Bocola and Dovis (2019), we set $\psi = 0.05$ to match an average exclusion from financial markets of 5 years.

Another parameter we set is σ, the curvature of the utility function. In typical default models, this parameter is set to $\sigma = 2$ to target an intertemporal elasticity of substitution (EIS) of 0.5. However, with the utility function (8) the EIS is $\sigma^{-1}(G - \underline{G})/G$, which is different from σ^{-1} due to the presence of subsistence consumption. Intuitively, the presence of subsistence consumption effectively reduces the EIS, as it makes the government less tolerant to accept fluctuation in consumption. In fact, given the parameters we chose, the EIS is around 1/3 in our model once we assume that $\sigma = 1$.[28] For this reason, we will keep the log utility specification from Sections 2-3 also for the analysis of the infinite horizon model.

We next specify the stochastic structure of the model. We assume that output Y_t and the safe rate R_t follow an autoregressive process of order 1 in logarithms

$$\log(Y_t) = \rho_Y \log(Y_{t-1}) + \sigma_Y \varepsilon_t^Y \tag{27}$$

$$\log(R_t) = (1 - \rho_R) \log(\bar{R}) + \rho_R \log(R_{t-1}) + \sigma_R \varepsilon_t^R, \tag{28}$$

where $(\varepsilon_t^Y, \varepsilon_t^\zeta)$ are independent standard normal random variables. We estimate the process (27) on linearly-detrended Italian real GDP during 1999Q1-2019Q4. This yields $\rho_Y = 0.92$ and $\sigma_Y = 0.009$. For the interest rate process (28) we proceed as follows. For the dynamics of long-term debt, what matters more than fluctuations in the short rate is how those transmit through the yield curve, i.e. how they translate into fluctuations in the long rate. We therefore pick (ρ_R, σ_R) in such a way that our model makes good predictions for the long rate.[29] We first extract data on 5-year ex-ante real

[28] Given our parameters, $(G - \underline{G})/G$ equals 1/3 in states when output equals its mean ($Y = 1$) and the primary balance is equal to zero ($G = \tau Y$). It even falls below 1/3 when the economy runs a primary surplus.

[29] In the model, the price of a safe long-term bond is

$$\tilde{q}(R) = \mathbb{E}\frac{\bar{\mu} + (1 - \mu)\tilde{q}(R')}{R}.$$

interest rates on German government debt during 1999Q1-2019Q4. We then set (ρ_R, σ_R) to match the unconditional autocorrelation and standard deviation of this time series, which are 0.987 and 0.019, respectively. This yields $\rho_R = 0.986$ and $\sigma_R = 0.001$. According to our estimates, changes in the short rate are thus expected to be very persistent, implying they transmit strongly into the yield curve. Finally, we set the steady state interest rate to 2% annually by setting $\bar{R} = 1.005$. Discretizing our process for R_t using the Tauchen (1986) methodology, we obtain that the long rate fluctuates slowly in the interval [-2.5%,7.3%], close to the values seen in the data over our period of study.[30]

We calibrate the remaining parameters $(\alpha, \beta, d_0, d_1, \varsigma)$ jointly by simulation.[31] First, we calibrate β by targeting an average debt to (annual) GDP ratio of 110%.[32] To calibrate the autarky parameter α, we target an average haircut after default of 59%. While Italy has not defaulted in our sample, this corresponds to the haircut suffered by Greek creditors in the 2012 default of Greece (Zettelmeyer et al., 2014). Last, we calibrate (d_0, d_1, ς) to match the behavior of spreads of Italian bonds over the safe interest rate during the time period 2000Q1-2012Q2: a mean spread of 0.61 percentage points (annualized), a standard deviation of 1.01 percentage points (again annualized), and a correlation with output of -0.21.[33]

A final point of the calibration is to specify the set $\mathbb{B}$ from which the government picks future B' conditional on repayment (equation (18)). If unrestricted, when default is imminent, the government borrows as much as it can, postponing the default decision to the next period (Chatterjee and Eyigungor (2015) call this behavior maximum dilution before default). Because such behavior is not observed in reality, we impose the following restriction on debt issuance:

$$\frac{B' - B}{4Y} \leq 0.05. \tag{29}$$

We therefore impose that debt cannot grow by more than 5% of GDP per quarter (or 20% of GDP per year). Intuitively, this restriction may capture the possibility of panic among investors, or the presence of underwriting standards. To be clear, this assumption is not what makes debt spirals slow moving in our quantitative model. They are slow moving (play out over multiple periods) also in the absence of this restriction. What condition (29) rules out is that debt levels diverge to plus infinity before a default occurs. Instead, with this restriction in place, defaults occur already at finite levels of debt.[34]

Solving this equation, we infer the long rate as $\bar{\mu}/\tilde{q}(R) - \mu$ (as in Chatterjee and Eyigungor, 2012).

[30]For the short rate this implies the interval [-4.2%,9.2%] for the discretized process. We report the long rate in the text because, as noted above, for the dynamics of long term debt the long rate is the more relevant variable.

[31]As is standard in the literature, we exclude the periods following a default when computing model moments. Specifically, we drop all periods during which the sovereign is excluded from financial markets as well as the 30 years after reentry.

[32]This corresponds to the average debt to GDP ratio of Italy during 1999-2012, a time when safe interest rates where close to 2%, the steady state level of interest rates in our calibrated model.

[33]For calibrating spreads, we only use data up to 2012 because the ECB took massive measures intervening in sovereign debt markets starting this year, notably the announcement "Whatever it takes". The same approach is taken by Bocola and Dovis (2019).

[34]Chatterjee and Eyigungor (2015) impose a similar restriction on debt issuance in their model, to capture underwriting standards in sovereign debt markets. Specifically, they impose that bonds cannot be issued when the default

Parameter	Value	Target	Data	Model
τ	0.42	tax base	-	-
μ	0.035	debt maturity	-	-
$\underline{G}$	0.28	non-discretionary spending	-	-
ψ	0.05	exclusion	-	-
σ	1	EIS < 0.5	-	-
ρ_Y	0.92	see text	-	-
σ_Y	0.009	see text	-	-
$\bar{R}$	1.005	see text	-	-
ρ_R	0.986	see text	-	-
σ_R	0.001	see text	-	-
β	0.98	debt to GDP (annual)	1.10	1.12
α	0.002	haircut	0.59	0.56
ς	0.70	spread (mean)	0.0061	0.0054
d_0	0.235	spread (std)	0.010	0.014
d_1	0.70	spread (corr with Y)	-0.21	-0.17

Table 1: Parameters and model fit.

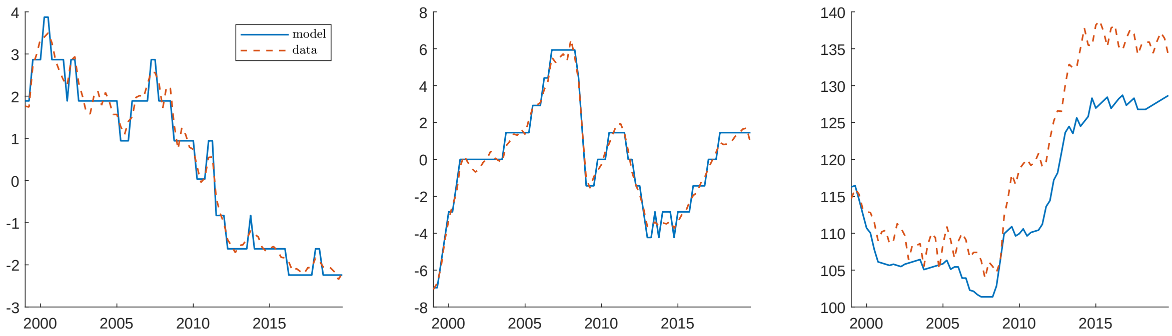
Table 1 summarizes the parameters, and also shows the model hits quite successfully the calibration targets. To obtain an additional, external, measure of model performance, we also consider the following experiment. Starting at the observed debt-to-GDP ratio of Italy in 1999, and feeding in the observed behavior of real GDP and long safe interest rates, how well does the model predict the evolution of debt to GDP during our sample period? Figure 3 shows the result.[35] We find our model explains quite well the time series of Italian debt, for instance the decline in the debt ratio before 2008, but also the subsequent build-up of debt. The only part our model misses is the massive debt uptick after the 2008 financial crises, which it underpredicts. Still, we take some comfort from this picture that our model produces reasonable debt dynamics, particularly in response to changes in (long) safe interest rates.

5 Quantitative results

In this section we study how debt accumulation and default incentives are shaped by movements in safe interest rates in our quantitative model. We start by studying the triggers of slow moving debt spirals in our model, and identify as the main culprit long periods of low interest rates which then suddenly revert. Thereafter, we study how interest rate normalization in the face of high

probability is higher than 75% in the following period. As it turns out, when using their restriction on bond issuance in our framework, slow moving debt crises do not exist. Because we think those crises capture an interesting possible scenario in the real world, we use the formulation (29) to restrict debt issuance, which preserves the presence of slow moving crises.

[35] Of course, the fact that output and the long-rate follow non-smooth paths in our model is due to the fact that both variables are discretized in our numerical implementation.



(a) Long safe rate **(b) Output (deviation from trend)** **(c) Debt to GDP**

Figure 3: External validity. Simulation of model using observed behavior of long safe real interest rates and Italian real GDP. Behavior of debt to GDP is non-targeted. All variables are in percent.

public debt is possible without triggering a debt crisis, by leveraging and extending our theoretical insights from Section 3.

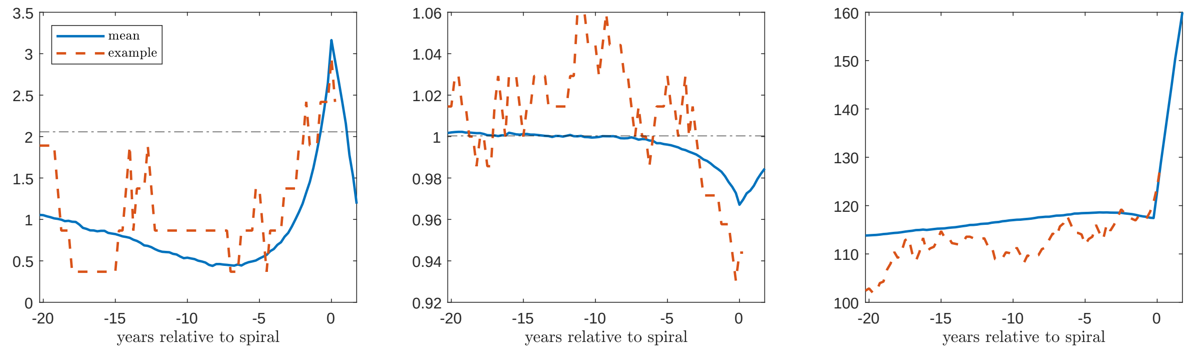
5.1 Dynamics of a debt spiral

What are the triggers of a debt spiral according to our model? To answer this question, we consider the following event study approach. We first simulate the model for a large number of periods. We then identify time periods when the economy enters a debt spiral, and denote this as year zero. Finally, we average across the 20-year windows preceding a debt spiral.[36] The result is shown in Figure 4. The left panel shows the safe long rate, the middle one shows output, the right panel shows debt to GDP. For interest rates and output, we also plot a thin line to indicate the average (steady state) values of the two variables. Because the mean hides a lot of variation, we also show one example transition leading up to a debt spiral (red dashed).

The figure reveals that the seed of a debt spiral is laid by a long phase of low interest rates: as much as 20 years before the spiral, the average level of safe long-term rates is already 1 percentage point below the steady state. Interest rates decline further, up to 1.5 percentage points below the steady state, up until 5 years before the spiral. Around 5 years before the spiral, interest rates start to suddenly revert. The average reversion leading to crisis is between 3-4 percentage points within a time frame of 5 years. We also observe that spirals are usually triggered by a simultaneous decline of output with the rise in interest rates.

We can understand the intuition for these results with the help of our theoretical insights from Section 3. Initially debt levels are comparatively low, implying the substitution effect of interest rate changes is dominant. This implies that debt levels climb once interest rates fall. Because the interest rate reversal coincides with a high level of debt, debt levels are not falling back. Rather, the rate hike triggers even more debt issuance and, in fact, a debt spiral.

[36]Similarly to the calibrated moments, we exclude paths on which default and reentry occur within 30 years before the debt spiral. The upward trend of debt observed in the figure is therefore not driven by increases in debt after reentry. After time zero, default happens on some simulated paths. We exclude these paths when computing the mean going forward.



(a) Long safe rate **(b) Output** **(c) Debt to GDP**

Figure 4: Dynamics preceding a debt spiral. Shown are simulated paths leading up to a debt spiral in the calibrated model. Blue: mean path, red dashed: example path. Years on horizontal axis, the start of the spiral is in year 0. Interest rate and debt to GDP in percent, output in levels.

These results are in line with a recent literature pointing out that a long phase of low interest rates can lead to heightened financial stability risk (Boissay et al., 2016, 2021; Grimm et al., 2023; Akinci et al., 2021; Coimbra and Rey, 2023). Our paper shows that low interest rates can also sow the seeds for financial stability risk in the sovereign debt market. Moreover, we highlight a new mechanism leading to the fragility of this market: the fact that when debt levels are high, higher interest rates may lead to even more debt, because the income effect of interest rate changes may become dominant.

5.2 Implications for interest rate normalization

We have shown that rising interest rates in the face of high public debt may imply that debt levels climb even more, possibly giving rise to a debt spiral. In Section 5.1 we have further shown that, according to our model, the seeds for this problem are laid by a long phase of low interest rates. In this last section of the paper we seek to uncover strategies out of this dilemma. Specifically, we ask if interest rate paths exist that lead the economy successfully out of a long phase of low interest rates.[37]

To do so we inspect policy functions for the change in debt (relative to GDP) $\Delta B'/4Y$, which we plot against i) the current level of debt $B/4Y$ and ii) the current level of the safe long-term rate, while keeping output at its steady state level ($Y = 1$). Because the resulting policy function is three-dimensional, we visualize it with the help of a heat map diagram: a rise in debt levels ($\Delta B'/4Y > 0$) is indicated by red, and a fall in debt levels is indicated by green ($\Delta B'/4Y < 0$). Furthermore, we indicate the states which trigger a slow moving debt spiral by light gray, and those states which trigger an immediate default by dark gray.[38]

To illustrate the mechanics of the model, we start by making the following assumption for the

[37] To be clear, we are not conducting a normative analysis in this section, such as how interest rates can be raised optimally to speed up deleveraging while avoiding a debt spiral. This is beyond the scope of this paper.

[38] More specifically, because default is probabilistic in our model due to the presence of default taste shocks (Section 4), the dark gray zone indicates the region where the probability of immediate default is higher than 50%.

interest rate process. We assume that, independent of the current level of interest rates, interest rates in the next period are expected to be equal to -2%. Moreover, in all periods after next period, interest rates are expected to follow the law of motion (28). This assumption implies that changes in *current* interest rates leave expectations about the *future* path of interest rates unchanged, which parallels the experiment conducted in our theoretical analysis in Section 3.

The result is in Figure 5a. First notice that the figure is mostly red on the left side and turns green when moving to the right. Intuitively, when debt levels are low the economy is ramping up additional debt (red), whereas when debt levels are high the debt stock converges toward its long-run mean from above (green). Moreover, when debt levels exceed a tipping point, it is no longer optimal to reduce debt levels going forward. Rather, in line with Proposition 1, it becomes optimal for the government to enter a debt spiral (gray).

What about the effect of a rise in interest rates? For concreteness, assume the interest rate is currently equal to -2%, perhaps reflecting the end of a long phase of low interest rates. What are the implications of a rise in interest rates above that value? Consider first the left side of Figure 5a. Starting from -2%, a rate rise moves the economy from red to orange. This indicates that debt issuance is *reduced* as interest rates rise (the substitution effect dominates). In contrast, at high levels of debt to GDP, say 140%, the same rate rise moves the economy from bright into mud green. That is, higher interest rates now imply that debt issuance *rises* (the income effect dominates). These results are all in line with Proposition 2: there is a threshold close to 90% debt to GDP, at which the effect of a rate hike on debt issuance flips from negative to positive.[39]

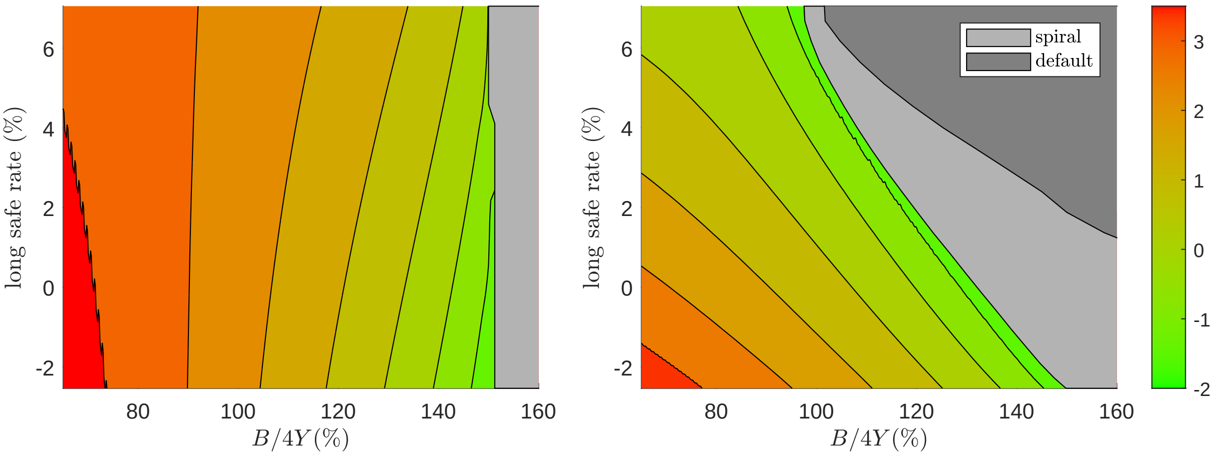
Next, we investigate the spiral region. The gray area is separated by an almost vertical line from the green zone at around 150% debt to GDP. Close inspection would reveal that the line is very slightly negatively sloped, so an interest rate increase makes a debt spiral more likely, but this effect is not economically significant. In the language of Proposition 4, the utility effect and the primary deficit effect offset each other almost entirely, so the tipping point is nearly independent of the interest rate level.

Figure 5a seems to paint a dire picture for the possibility of interest rate normalization. The debt levels of many countries are far above the threshold of 90%, implying that higher interest rates will entail more debt issuance (a slower debt reduction) going forward.[40] However, this picture is incomplete, because so far we have only considered interest rate changes that leave expectations about the future level of the interest rate unchanged.

We now complete the picture by assuming that current movements in interest rates also affect expectations about the path of interest rates in the future. Specifically, we again construct the policy function for the change in debt, but now assume that, starting from the current interest rate

[39]In fact, Proposition 2 applies to the infinite horizon model as well, as we showed in an earlier version of the paper. For our parameters the theoretical threshold for debt is $(1/\tilde{\mu})\,(\tau - \underline{G}/Y)\,/4 = 87.5\%$ debt to GDP. We find exactly this value for the threshold numerically, which gives us confidence in the accuracy of our numerical solution.

[40]This being said, the figure also seems to suggest that one can just wait, as debt levels will fall over time starting from a high level independent of the current level of interest rates (as the right side of the figure is green). However, these dynamics are triggered by expectations that interest rates will revert to the steady state value of 2% over the long term. If instead, expectations are anchored at -2% interest rates indefinitely, debt levels will not fall over time even starting from very high levels. We explore the effect of interest rate expectations in the last part of this section.



(a) Transitory change in interest rate. **(b) Persistent change in interest rate.**

Figure 5: Policy function for borrowing. Policy function for $\Delta B'/4Y$ (in percentage points), plotted against $B/4Y$ and the safe long interest rate (both in percent), by keeping fixed $Y = 1$. Interest rate changes are assumed to leave interest rate expectations unchanged (left panel), or to change interest rate expectations according to the calibrated law of motion (28) (right panel).

level, interest rates are expected to follow the calibrated law of motion (28) in all future periods. Recall that we estimated $\rho_R = 0.986$ (which is the first order autocorrelation of both the short and the long term interest rate), so an interest rate increase in the current period is expected to have an almost equally strong effect on interest rates in all the subsequent periods.

The result is shown in Figure 5b. The first insight from this figure is that, relative to Figure 5a, all contour lines of the heat map are strongly tilted to the left. That is, the negative effect of higher interest rates on debt issuance is far stronger. In fact, at the calibrated level of persistence of the interest rate process, we find that *all* contour lines are now negatively sloped, which implies that the substitution effect dominates everywhere until the point where an interest rate hike triggers a debt spiral. Intuitively, a persistent interest rate increase makes current debt issuance less attractive, not only because current interest rates are high, but also because interest rates will still be high when debt matures in the future. This works like a strengthening of the substitution effect, at the expense of the income effect.

Persistence of the interest rate hike also changes the properties of the tipping point for entering the spiral. In stark contrast to Figure 5a, it is negatively sloped here, which implies that keeping interest rates low persistently might prevent a debt spiral from occurring. Again, linking this to Proposition 4, the negative effect on utility of not entering the spiral is much stronger for more persistent interest rate hikes, because debts have to be repaid at higher interest rates in the future. The primary deficit effect, which makes the spiral less attractive, is not scaled up to the same extent. Thus, a higher interest rate makes entering the spiral relatively more attractive.

Taken together Figures 5a-5b, we draw the following two lessons for the possibility of a successful interest rate normalization in the face of high public debt.

First, the interest rate normalization must be *credible*. By this we mean that a current rise in the interest rate must be accompanied by expectations of a further rise in interest rates in the future. If a current rise in the interest rate is expected to be reversed in the future (Figure 5a), it is optimal for governments to respond by issuing even more debt during the time when interest rates are elevated. It is only when the current rate hike triggers expectations of higher interest rates in the future (Figure 5b) that the substitution effect is strong and governments respond to higher interest rates by reducing their debt.

Second, conditional on the interest rate normalization being credible, the actual *speed* of normalization must be *intermediary*. To see this point, look again at Figure 5b and take as a starting point an interest rate of -2% and a debt level relative to GDP of 140%. As we can see, an immediate rate hike that is larger than half a percentage point will plunge the economy in a debt spiral. This implies that, to avoid the occurrence of debt spirals, interest rate normalization cannot be too fast. On the other hand, by raising interest rates by too little, the substitution effect remains weak which slows down debt reduction by the government. This can be seen by the fact that the contour lines are downward sloping, so a higher interest rate implies a faster debt reduction. In brief, a successful interest rate normalization is a balancing act between preventing a debt spiral, and providing sufficient incentives for the government to repay.

We illustrate the benefit of an intermediate speed of interest rate normalization with the following additional experiment. We assume interest rates are -2% initially and distinguish between three different initial levels of debt to GDP, equal to 130%, 145% and 160%, respectively. We then ask how the government adjusts when interest rates are normalized at different speeds to their 2% steady state level. The two statistics we look at are i) is a debt spiral avoided along the way of the interest rate normalization, and if yes, ii) how many years does the government take to reduce its debt to GDP ratio halfway to the steady state of 110%.[41]

The result is in Table 2. In the columns we vary the speed of interest rate normalization, which we express in terms of half lives of the interest rate process. For instance, "2 years" indicates that the interest rate process takes two years to reach 0% (halfway from -2% to 2%). In the rows we vary the initial level of debt. A "×" indicates that the interest rate normalization triggers a debt spiral along the way. Otherwise, the table contains the half life of the government's debt reduction to the steady state, as explained in the previous paragraph.

Focus for instance on the row with 145% initial debt to GDP. As we can see, an interest rate normalization with half lives of 1 year or 2 years is too fast to bear for the government, which chooses to enter a debt spiral. In contrast, half lives of 5 and 10 years both imply that a debt spiral is avoided, and debt levels converge back to their steady state in the long term. For the 5 year half life of the interest rate process, in turn, it takes 11 years for debt levels to fall down to 127.5% of GDP (halfway from 145% to 110%), while for the 10 year half life of the interest rate process, it takes almost 30 years for debt levels to come down by half. We thus confirm our earlier statement

[41] Technically, we implement this experiment using MIT shocks. That is, the government has a perfect foresight path of the interest rate going forward. This methodology ensures that the government has rational expectations, as it correctly anticipates the speed of interest rate normalization. For output, while we assume output is expected to vary according to the estimated process (27), in equilibrium we keep it at its mean value $Y = 1$ during the phase of interest rate normalization.

Speed of normalization (half life)	1 year	2 years	5 years	10 years
Initial debt to GDP				
130%	×	5 years	16.25 years	36.25 years
145%	×	×	11 years	29.5 years
160%	×	×	×	23.5 years

Table 2: Interest rate normalization. Shown are the years needed to reduce debt to GDP halfway to the steady state of 110%, starting from different debt levels, and starting from an interest rate of -2%, for different speeds of interest rate normalization (in half lives in years). A × indicates that the government enters a debt spiral.

that a too fast interest rate normalization leads to debt spirals, but a too slow one undermines the incentives by the government to deleverage.

We conclude this section by discussing two caveats. When we say that the speed of interest rate normalization should be intermediary, we are making a normative statement by assuming that it is desirable both to avoid a debt spiral and to reduce debt ratios from initially very high levels. But of course, we do not look at welfare, so this statement must be taken with a grain of salt. Second, the exact numbers in Table 2 must be treated with caution. For instance, we have assumed the long run interest rate to be equal to 2%, and the government to be perfectly aware of this fact. In practice there is considerable uncertainty regarding the long run safe real interest rate. If it is 1% rather than 2%, for instance, our model would predict that interest rates can be normalized at faster speeds, without plunging the economy in a debt crisis. With these caveats in mind, however, we believe that our analysis sheds some much-needed light on possible strategies out of the low interest rate - high public debt dilemma.

6 Conclusion

As we write, many countries are suffering from record-high levels of public debt, and real interest rates are rising around the globe. In this paper, we study how high-debt countries may adjust to rising interest rates, by constructing a sovereign default model that we calibrate to a country inside the euro area. Our main finding is that countries may respond very differently to the same rise in interest rates, depending on their initial level of debt: countries with debt levels below a threshold may respond by borrowing less, which reduces default risk going forward. In contrast, high-debt countries may respond to the same rate rise by borrowing even more, possibly even entering a slow moving spiral of ever rising debt which culminates in an eventual default. We show this within a framework where the government fully optimizes with respect to its borrowing and with respect to its decision to default. We also work out the implications for interest rate normalization following a long period of low interest rates that has incentivized governments to pile up large amounts of public debt in the first place.

The current analysis opens up many follow up questions. For example, we have not touched on the issue of self-fulfilling debt crises. However, a big issue in the euro area is whether default can be the outcome of self-fulfilling beliefs or animal spirits (De Grauwe and Ji, 2013; Bocola and Dovis, 2019; Bianchi and Mondragon, 2021). Do higher interest rates make the emergence of self-fulfilling crises more or less likely?

Another direction for future work may be to conduct a normative analysis. For instance, it would be interesting to work out optimal interest rate normalization in our framework, by taking into account welfare of the sovereign (who suffers from higher interest rates), but also of the sovereign's lenders (who benefit from higher interest rates). To do so one might use the criterion of Pareto-improvements in sovereign debt markets recently introduced by Aguiar et al. (2019). Our findings may also be useful for studies of optimal monetary policy within the euro area which take account of the possibility of default in some member countries. de Ferra and Romei (2020) take a step in this direction, but much more is yet to be done.

Finally, we believe our insights about the differential response of borrowers to higher interest rates depending on their initial level of debt can be applied also to contexts beyond the sovereign debt market. For instance, these insights could prove relevant also for macroprudential regulation: macroprudential taxation aimed at reducing debt levels in the economy may backfire in case debt levels are high to start with, implying a large income effect of the tax increase. Similarly, our insights about when interest rate normalization can be successful should also be useful for designing regulation in other markets.

Appendix (for online publication)

A Appendix: Proofs of all propositions

A.1 Proof of Proposition 1

Proposition 1 *If a debt spiral exists, it is associated with a critical value $\bar{B}$. That is, the spiral occurs if and only if $B > \bar{B}$. Moreover, a debt spiral does not exist if $\kappa = 0$ or if $\mu = 1$.*

Proof.

Denote $V_{\mathcal{S}}(B)$ the value conditional on choosing the debt spiral (defined in (7) in the main text), and $V_{\mathcal{I}}(B)$ the value from choosing the interior optimum - the value which is implied when B' satisfies the Euler equation (10). Also define the associated policy functions $B'_{\mathcal{I}}(B)$, $G_{\mathcal{I}}(B)$ and $G_{\mathcal{S}}(B)$, where we omit the argument B if convenient.

Assume that there exist values $\underline{B}$ and $\tilde{B}$ such that $V_{\mathcal{S}}(\underline{B}) \leq V_{\mathcal{I}}(\underline{B})$ and $V_{\mathcal{S}}(\tilde{B}) > V_{\mathcal{I}}(\tilde{B})$. That is, at $\underline{B}$ choosing the interior optimum is weakly optimal, but at $\tilde{B}$, it is optimal to choose the spiral.

The spiral delivers the worst possible continuation value in period 2, but since $V_{\mathcal{S}}(\tilde{B}) > V_{\mathcal{I}}(\tilde{B})$, first period consumption must be higher under the spiral

$$G_{\mathcal{S}}(\tilde{B}) > G_{\mathcal{I}}(\tilde{B}).$$

Next, compare the derivatives of the two values with respect to B

$$\frac{\partial V_{\mathcal{S}}}{\partial B} = -\mu u'\left(G_{\mathcal{S}}\right)$$

and, using the envelope condition for the derivatives of $B'_{\mathcal{I}}$,

$$\frac{\partial V_{\mathcal{I}}}{\partial B} = -(\mu + (1-\mu)q(B'_{\mathcal{I}}))u'\left(G_{\mathcal{I}}\right).$$

Because $G_{\mathcal{S}}(\tilde{B}) > G_{\mathcal{I}}(\tilde{B})$, $u'\left(G_{\mathcal{I}}(\tilde{B})\right) > u'\left(G_{\mathcal{S}}(\tilde{B})\right)$. It follows that

$$\left.\frac{\partial V_{\mathcal{I}}}{\partial B}\right|_{B=\tilde{B}} < \left.\frac{\partial V_{\mathcal{S}}}{\partial B}\right|_{B=\tilde{B}}.$$

Thus at $\tilde{B}$, $V_{\mathcal{I}}$ is decreasing faster than $V_{\mathcal{S}}$. Since $V_{\mathcal{I}}(\tilde{B}) < V_{\mathcal{S}}(\tilde{B})$, the difference between the two functions is increasing. This implies that the inequality on the derivatives above holds for all $B > \tilde{B}$. Using that $V_{\mathcal{S}}(\underline{B}) \leq V_{\mathcal{I}}(\underline{B})$ and that both $V_{\mathcal{S}}(B)$ and $V_{\mathcal{I}}(B)$ are continuous functions, this proves the existence of a threshold $\bar{B}$.

Next we turn to the conditions for the possibility of a debt spiral. First, consider the values $V_{\mathcal{S}}$

and $V_{\mathcal{I}}$ for the case where all debt matures immediately ($\mu = 1$) evaluated at the optimal choices

$$V_{\mathcal{S}} = u\left(\tau Y + \frac{\kappa}{R} - B\right) + \frac{\beta}{1-\beta}\mathbb{E}u(G'_{\delta})$$

and

$$V_{\mathcal{I}} = u\left(\tau Y + \frac{q(B'_{\mathcal{I}})}{R}B'_{\mathcal{I}} - B\right) + \frac{\beta}{1-\beta}\mathbb{E}\max\{u(G'(B'_{\mathcal{I}})), u(G'_{\delta})\}.$$

Notice that the interior choice $B' = \kappa$ is available and leads to zero default risk, with $q(\kappa) = 1$. This is better than setting $B' = \infty$, as both choices deliver the same first period utility but second period utility is strictly larger at the interior choice.[42] Since $B'_{\mathcal{I}}$ is the optimal choice, it must deliver at least as much utility as $B' = \kappa$, so the debt spiral is never optimal with short-term debt.

Second, consider the case with no recovery value ($\kappa = 0$). Under the debt spiral, utility is:

$$V_{\mathcal{S}} = u(\tau Y - \mu B) + \mathbb{E}u(\tau Y - \kappa - X).$$

That is, no resources are raised by borrowing if default occurs with certainty as lenders receive no payoff. In this region, the value of B' has no effect on utility. As long as the government can issue any new debt in the first period ($B' > (1-\mu)B$), with $q(B'_{\mathcal{I}}) > 0$, it prefers this choice over certain default in period 2. If B is very high, it is optimal to choose certain default in period 2, similar to the debt spiral. However, this outcome is different from a debt spiral in two ways: (i) the debt choice is indeterminate and not necessarily infinite and (ii) it is not a voluntary choice but only occurs, if the government is unable to issue any new debt. ■

A.2 Proof of Proposition 2

Proposition 2 *Assume the utility function is $u(G) = \log(G - \underline{G})$, and that the government takes borrowing decisions according to the Euler equation (10). Then a threshold for debt to GDP exists above which the impact of a change in R on B' flips. The threshold is*

$$\mathcal{T} = \frac{1}{\mu}\left(\tau - \frac{\underline{G}}{Y}\right).$$

When $B/Y < \mathcal{T}$, a rise in R reduces B' (the substitution effect dominates). When $B/Y > \mathcal{T}$, a rise in R increases B' (the income effect dominates).

Proof.

The Euler equation is

$$u'\left(\tau Y + \frac{q(B')}{R}(B' - (1-\mu)B) - \mu B\right) q(B')(1 - \varepsilon(B')) - \beta R\mathbb{E}u(G')(1 - \delta'(B')) = 0,$$

which we can compactly write as $\Gamma(B', R) \equiv 0$. This implicitly defines a mapping $R \to B'$. Using

[42] If first period consumption is already negative for $B' = \kappa$, choosing the spiral is not possible and the problem is not interesting.

total differentiation with respect to R

$$\frac{\partial \Gamma(B',R)}{\partial B'}\frac{\partial B'}{\partial R}+\frac{\partial \Gamma(B',R)}{\partial R}=0.$$

The first summand contains our object of interest - the derivative $\partial B'/\partial R$ - as well as the indirect derivatives. The second summand captures the direct derivatives.

Because the equilibrium point is a local optimum, we know that the derivative of Γ with respect to B' at the equilibrium point is negative

$$\frac{\partial \Gamma(B',R)}{\partial B'}<0.$$

But this implies that

$$\frac{\partial B'}{\partial R}>0 \Leftrightarrow \frac{\partial \Gamma(B',R)}{\partial R}>0. \tag{A.1}$$

The derivative of Γ with respect to R is

$$\frac{\Gamma(B',R)}{\partial R}=\underbrace{u''(G)q(B')(1-\varepsilon(B'))\left(-\frac{q(B')}{R^2}\right)(B'-(1-\mu)B)}_{\text{Income effect}}\underbrace{-\beta\mathbb{E}u(G')(1-\delta'(B'))}_{\text{Substitution effect}}.$$

Note the income effect makes a positive contribution to $\partial\Gamma/\partial R$ (because $u''(G)<0$), whereas the substitution effect makes a negative contribution. By (A.1), the income effect thus implies that borrowing rises following a rise in R, whereas the substitution effect implies it falls. Which effect dominates? Using the Euler equation, we rewrite the previous expression as

$$\frac{\Gamma(B',R)}{\partial R}=u'(G)\frac{q(B')}{R}(1-\varepsilon(B'))\left(\frac{u''(G)}{u'(G)}\left(-\frac{q(B')}{R}\right)(B'-(1-\mu)B)-1\right).$$

Because the term before the brackets is positive, we thus obtain

$$\frac{\partial B'}{\partial R}>0 \Leftrightarrow -\frac{u''(G)}{u'(G)}(G-\tau Y+\mu B)>1, \tag{A.2}$$

where we also used (1).

Now we use the assumption $u(G)=\log(G-\underline{G})$, which implies $u'(G)=1/(G-\bar{G})$ and $u''(G)=-1/(G-\bar{G})^2$, to obtain

$$\frac{\partial B'}{\partial R}>0 \Leftrightarrow \frac{G-\tau Y+\mu B}{G-\bar{G}}>1.$$

Simple rearranging yields the final result. ■

A.3 Proof of Proposition 3

Proposition 3 *Let $\beta R = 1$ and consider the approximation point $(B, B') = (\kappa/(\mu+\beta(1-\mu)), \kappa)$. At this point, up to a first-order approximation, a rise in B' impacts $\eta(B')$ as follows*

$$\eta(B') \approx \beta + \beta\lambda \left(\frac{\mu + \beta(1-\mu) - 2(1-\mu)}{\mu + \beta(1-\mu)} \right) (B' - \kappa),$$

where $\lambda \geq 0$ is the hazard rate of the distribution function of X at zero.

Proof.

We first introduce some notation to simplify the algebra. Denote F the CDF and f the PDF of the default cost X. The hazard rate of X is defined as $\lambda \equiv f/(1-F)$. Now recall that a default occurs if and only if $X < B' - \kappa$, with associated default probability $F(B' - \kappa)$. In what follows, we always evaluate F, f and λ at the point $B' - \kappa$, but we suppress the argument to enhance readability. Moreover, we also define the haircut variable $\chi = 1 - \kappa/B'$.

With this notation, the bond price is simply

$$q(B') = 1 - \chi F.$$

Everything else equal, a larger haircut or a larger default probability imply the bond price is lower. Using the chain rule, we can compute the derivative of $q(B')$ with respect to B'

$$\frac{\partial q(B')}{\partial B'} = -\chi(1-F)\lambda - (1-\chi)\frac{F}{B'}.$$

In absence of a recovery value (the haircut is 100%, $\chi = 1$), the bond price is simply $1 - F$ and its derivative is $-(1-F)\lambda = -f$. When a recovery value is present, the two expressions are slightly more complicated.

Using these results, the elasticity of the bond price with respect to B' is

$$\varepsilon \equiv -\frac{\partial q(B')/\partial B'}{q(B')}(B' - (1-\mu)B) = \frac{\chi(1-F)\lambda + (1-\chi)\frac{F}{B'}}{1 - \chi F}(B' - (1-\mu)B) \equiv \tilde{\lambda}(B' - (1-\mu)B).$$

In absence of a recovery value ($\chi = 1$), we have $\tilde{\lambda} = \lambda$, and so the elasticity is $\varepsilon = \lambda(B' - (1-\mu)B)$. In this case, therefore, the elasticity is dictated by the hazard function λ. In case a recovery value is present, we can think of $\tilde{\lambda}$ as an "effective" hazard rate.

Now we turn to the main result. With the notation from this section, effective patience $\eta(B')$ is

$$\eta(B') = \beta\frac{1-F}{1-\varepsilon} = \beta\frac{1-F}{1-\tilde{\lambda}(B' - (1-\mu)B)}, \tag{A.3}$$

where recall that F and $\tilde{\lambda}$ are both functions of B'.

Is $\eta(B')$ decreasing or increasing in B'? To answer this question, we assume that $\beta R = 1$ and take a first order approximation of $\eta(B')$ at the point $(B, B') = (\kappa/(\mu + \beta(1-\mu)), \kappa)$. As we have shown in Section 2.4, at this point default risk is equal to zero, $F = 0$. In addition, the haircut is zero as well, $\chi = 0$, which also implies $\tilde{\lambda} = 0$. Hence $\eta(B') = \beta$ at the approximation point. At the same time, this point contains information about default risk once debt levels are marginally higher. This makes this a useful approximation point.

Taking a first-order Taylor expansion of (A.3) at the point $B' = \kappa$, and using again $f = (1-F)\lambda$ and $B = \kappa/(\mu + \beta(1-\mu))$, we obtain

$$\eta(B') \approx \beta \left(1 + \lambda \left(\frac{\mu + \beta(1-\mu) - 2(1-\mu)}{\mu + \beta(1-\mu)}\right)(B' - \kappa)\right),$$

where λ continues to be evaluated at $B' - \kappa$. Because $B' = \kappa$ at our approximation point, in the last expression λ is thus the hazard rate of X evaluated at zero. Note we take the derivative from the right: in fact $\eta(B')$ is flat at β to the left of the approximation point, and in this direction, its derivative with respect to B' is therefore equal to zero. ■

A.4 Proof of Proposition 4

Proposition 4 *Assume a debt spiral exists, described by the tipping point (15). Then $\partial\bar{B}/\partial R < 0$ (a rise in the safe rate reduces the tipping point $\bar{B}$), if and only if*

$$u'(G)(G + \mu\bar{B} - \tau Y)\Big|_{\mathcal{S}} < u'(G)(G + \mu\bar{B} - \tau Y)\Big|_{\mathcal{I}}.$$

The left-hand side is the marginal-utility-weighted budget deficit at the point $\bar{B}$ when choosing the debt spiral, the right-hand side, when choosing the interior optimum.

Proof. Let $\bar{B}$ be the threshold value for the spiral such that $V_{\mathcal{I}}(\bar{B}) = V_{\mathcal{S}}(\bar{B})$. As in the proof for Proposition 1 we have that $G_{\mathcal{S}}(\bar{B}) > G_{\mathcal{I}}(\bar{B})$, which implies that

$$\kappa > (B'_{\mathcal{I}}(\bar{B}) - (1-\mu)\bar{B})q(B'_{\mathcal{I}}(\bar{B})).$$

To determine in which direction the threshold for the debt spiral shifts, we can compute the derivatives of the values $V_{\mathcal{I}}$ and $V_{\mathcal{S}}$ with respect to the risk free rate R:

$$\frac{\partial V_{\mathcal{S}}}{\partial R} = -u'(G_{\mathcal{S}})\frac{\kappa}{R^2}$$

and

$$\frac{\partial V_{\mathcal{I}}}{\partial R} = -u'(G_I)(B'_{\mathcal{I}} - (1-\mu)B)\frac{q(B'_{\mathcal{I}})}{R^2},$$

using the envelope theorem for $B'_{\mathcal{I}}$. The threshold $\bar{B}$ will shift to the left (the spiral region becomes larger) if

$$\left.\frac{\partial V_{\mathcal{I}}}{\partial R}\right|_{B=\bar{B}} < \left.\frac{\partial V_{\mathcal{S}}}{\partial R}\right|_{B=\bar{B}}.$$

This is ambiguous, however. First we have that $u'\left(G_{\mathcal{I}}(\bar{B})\right) > u'\left(G_{\mathcal{S}}(\bar{B})\right)$, which reflects the fact that first period consumption under the interior choice is lower. The reduction in net income from the higher interest rate thus hurts more, if the government intends to repay. Second, as shown earlier, $\kappa > (B'_{\mathcal{I}}(\bar{B}) - (1-\mu)\bar{B})q(B'_{\mathcal{I}}(\bar{B}))$. That is, if the government enters the debt spiral, it issues more debt. The effect of the higher interest rate on utility is thus larger.

Noticing that $\frac{q(B')}{R}(B' - (1-\mu)\bar{B})q(B') = G + \mu\bar{B} - \tau Y$, both in the interior choice and in the spiral, completes the proof. ■

B Appendix: Additional derivations

B.1 Derivation of Euler equation (10)

The maximization problem is (4), which after inserting (1) and (2) can be written as

$$\max_{B'} \left\{ u\left(\tau Y + \frac{q(B')}{R}(B' - (1-\mu)B) - \mu B\right) + \frac{\beta}{1-\beta}\mathbb{E}\max\left\{u(\tau Y - (1-\beta)B'), u(G'_\delta)\right\}\right\}.$$

We did not replace G'_δ, because the maximization is with respect to B' and G'_δ is independent of B', see (3).

Using that default happens if and only if $X < B' - \kappa$ allows us to split the max operator and rewrite the expectation operator

$$\max_{B'} \left\{ u\left(\tau Y + \frac{q(B')}{R}(B' - (1-\mu)B) - \mu B\right) + \frac{\beta}{1-\beta}\left(\int_{X_{min}}^{B'-\kappa} u(G'_\delta) dFX + \int_{B'-\kappa}^{X_{max}} u(\tau Y - (1-\beta)B') dFX\right)\right\}.$$

where we denote F the CDF of the random variable X. Now we compute the derivative with respect to B' and set the resulting expression equal to zero

$$u'(G)\left(\frac{\partial q(B')/\partial B'}{R}(B' - (1-\mu)B) + \frac{q(B')}{R}\right) + \frac{\beta}{1-\beta}\int_{B'-\kappa}^{X_{max}} u'(G')(-(1-\beta)) dFX = 0.$$

In this expression, we ignored the derivatives in the two integral limits (see the Leibniz rule of differentiation) as these cancel each other out. Rewriting in terms of the elasticity $\varepsilon(B')$, and using that

$$\int_{B'-\kappa}^{X_{max}} u'(G')(-(1-\beta)) dFX = \int_{X_{min}}^{X_{max}} (1 - \mathbb{1}\{X < B' - \kappa\}) u'(G') dFX = \mathbb{E}u'(G')(1 - \delta'(B')),$$

where $\delta'(B')$ is defined in (5), yields the final result.

B.2 Threshold $\mathcal{T}$ when $\sigma \neq 1$

We start from equation (A.2) in the proof of Proposition 2 (see Appendix A.2), which we repeat for convenience.

$$\frac{\partial B'}{\partial R} > 0 \Leftrightarrow -\frac{u''(G)}{u'(G)}(G - \tau Y + \mu B) > 1,$$

When $u(G) = (G - \underline{G})^{1-\sigma}/(1-\sigma)$, we get $u'(G) = (G - \underline{G})^{-\sigma}$ and $u''(G) = -\sigma(G - \underline{G})^{-\sigma-1}$. Inserting yields

$$\frac{\partial B'}{\partial R} > 0 \Leftrightarrow \frac{\sigma}{G - \underline{G}}(G - \tau Y + \mu B) > 1,$$

and after rearranging

$$\frac{\partial B'}{\partial R} > 0 \Leftrightarrow \frac{B}{Y} > \frac{1}{\mu}\left(\tau - \left(\frac{1}{\sigma}\frac{\underline{G}}{Y} + \left(1 - \frac{1}{\sigma}\right)\frac{G}{Y}\right)\right) \equiv \mathcal{T},$$

which is the generalization of (11) to the case $\sigma \neq 1$. The key difference to before is that $\mathcal{T}$ depends on G and therefore on an endogenous variable. This reveals that, in general, the threshold depends not just on parameters but also on the state of the business cycle. Another insight is that the threshold becomes smaller, the bigger is σ (because $G > \underline{G}$, and because with a higher σ more weight is put on G/Y relative to $\underline{G}/Y$). The intuition for this is that a higher σ reduces the elasticity of intertemporal substitution (IES), making the substitution effect weaker. This implies the region of debt where the substitution effect dominates the income effect becomes smaller.

C Numerical algorithm and accuracy

Our numerical solution relies on discrete choice value function iteration augmented with extreme value taste shocks. As discussed for example in Gordon (2019) and Dvorkin et al. (2021), these shocks smooth out kinks in policy functions and bond price schedules which otherwise lead to convergence problems. Our solution algorithm closely follows Arellano et al. (2020) where a full description can be found.

For borrowing B we choose a discrete grid over the interval [0.5, 12] with 600 points. The grid is separated into 3 linearly spaced parts, from [0.5,3), [3, 6) and [6, 12], with 30, 440 and 130 gridpoints respectively. We place most gridpoints in the middle interval, where the economy spends most of the time during simulations. We denote the set of possible borrowing levels with $\mathbb{B}$.

We discretize the autoregressive processes for Y and R and using the Tauchen (1986) algorithm with a gridwith of 3 standard deviations for each state. We use 11 and 21 gridpoints respectively.

The algorithm involves extending the exogenous state with a taste shock specific to each possible borrowing choice, so $s = (Y, R, \xi, \{\psi_{B'}\}_{B' \in \mathcal{B}})$. The shocks $\psi_{B'}$ follow a Gumble (Extreme Value Type 1) distribution with standard deviation σ_ψ. The value function in repayment is then given by

$$V^r(B, i, s) = \max_{B' \in \mathbb{B}} \left\{U(C) + \beta \mathbb{E} V(B', i', s') + \psi_{B'}\right\}. \tag{C.1}$$

Before taste shocks realize, the policy functions for borrowing B' and default δ are probability distributions over the possible choices at each grid point (Y, R, B).[43] We chose a small value for $\sigma_\psi = 10^{-2}$ which still leads to reliable convergence of the algorithm. The effect of taste shocks on debt choice is then small. For example, at the state $B = 4.4$, $Y = 1$ and $R = 1.005$, the probability that a point in the interval [4.38, 4.42] is chosen is larger than 95%.

We iterate backward in time on the value function and bond price schedule until updating errors are smaller than 10^{-6}.

Equilibrium multiplicity Because we follow the timing assumptions of Eaton and Gersovitz (1981), multiple equilibria in the sense of static coordination failures (Cole and Kehoe (2000)) are ruled out in our setting. However, long-term debt can give rise to different types of equilibrium multiplicity studied in Lorenzoni and Werning (2019) and Aguiar and Amador (2020). To investigate whether multiplicity is present in our model, we run our algorithm twice: we start iterating from i) the risk free bond price schedule and ii) the risk free bond price schedule multiplied by 0.5. In both cases our algorithm converges to the same fixed point. While we cannot rule out that other equilibria exist in our model, we take confidence from the fact that the equilibrium appears to be unique for a wide range of plausible starting values.

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[43] See Arellano et al. (2020) for details on how to compute the choice probabilities.

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