Appendixes—For Online Publication.

A Appendix: Details on the model

A.1 Deriving the present value budget constraint

Here we derive the present value budget constraint, equation

$$(1-\Theta_t)\frac{B_t}{P_t} = \sum_{j=0}^{\infty} \beta^j E_t \left(\frac{C_{t+j}}{C_t}\right)^{-1} \tau_{t+j}. \tag{A.4}$$

from Section 2. The exposition follows closely Uribe (2006). First we define $\tilde{B}_t := I_t^{-1} B_{t+1}$, yielding the flow budget constraint

$$\tilde{B}_t = I_{t-1}\tilde{B}_{t-1}(1-\Theta_t) - P_t\tau_t.$$

Multiplying the left and right-hand side with $I_t(1-\Theta_{t+1})$, and iterating forward j periods, the budget constraint becomes

$$\begin{aligned} & I_{t+j}\tilde{B}_{t+j}(1-\Theta_{t+j+1}) \\ & \quad = \left(\prod_{h=0}^{j} I_{t+h}(1-\Theta_{t+h+1})\right) I_{t-1}\tilde{B}_{t-1}(1-\Theta_t) - \sum_{h=0}^{j}\left(\prod_{k=h}^{j} I_{t+k}(1-\Theta_{t+k+1})\right) P_{t+h}\tau_{t+h}. \end{aligned}$$

Now divide both sides by P_{t+j+1} and multiply by $(C_{t+j+1}/C_t)^{-1}$

$$\begin{aligned} & I_{t+j}\frac{\tilde{B}_{t+j}}{P_{t+j+1}}\left(\frac{C_{t+j+1}}{C_t}\right)^{-1}(1-\Theta_{t+j+1}) \\ & \qquad = \left(\prod_{h=0}^{j} I_{t+h}\frac{P_{t+h}}{P_{t+h+1}}\left(\frac{C_{t+h+1}}{C_{t+h}}\right)^{-1}(1-\Theta_{t+h+1})\right) I_{t-1}\frac{\tilde{B}_{t-1}}{P_t}(1-\Theta_t) \\ & \qquad\quad - \sum_{h=0}^{j}\left(\prod_{k=h}^{j} I_{t+k}\frac{P_{t+k}}{P_{t+k+1}}\left(\frac{C_{t+k+1}}{C_{t+k}}\right)^{-1}(1-\Theta_{t+k+1})\right)\left(\frac{C_{t+h}}{C_t}\right)^{-1}\tau_{t+h}. \end{aligned}$$

Now take conditional time$-t$ expectations E_t on both sides, use the law of iterated expectations $E_t(\cdot) = E_t(E_{t+h}(\cdot)),\ h \geq 0$, and exploit that

$$\beta E_t I_t \frac{P_t}{P_{t+1}}\left(\frac{C_{t+1}}{C_t}\right)^{-1}(1-\Theta_{t+1}) = 1$$

to arrive at

$$\begin{aligned} & E_t I_{t+j}\frac{\tilde{B}_{t+j}}{P_{t+j+1}}\left(\frac{C_{t+j+1}}{C_t}\right)^{-1}(1-\Theta_{t+j+1}) \\ & \qquad = \beta^{-j-1} I_{t-1}\frac{\tilde{B}_{t-1}}{P_t}(1-\Theta_t) - \sum_{h=0}^{j}\beta^{-h-j-1}E_t\left(\frac{C_{t+h}}{C_t}\right)^{-1}\tau_{t+h}. \end{aligned}$$

Finally, multiply both sides by β^{j+1}, take the limit $j \to \infty$ and use the transversality condition

$$\lim_{j\to\infty} \beta^{j+1} E_t I_{t+j} \left(\frac{C_{t+j+1}}{C_t}\right)^{-1} (1-\Theta_{t+j+1}) \frac{\tilde{B}_{t+j}}{P_{t+j+1}} = 0$$

to arrive at

$$(1-\Theta_t) I_{t-1} \frac{\tilde{B}_{t-1}}{P_t} = \sum_{h=0}^{\infty} \beta^h E_t \left(\frac{C_{t+h}}{C_t}\right)^{-1} \tau_{t+h}.$$

Substituting back $B_t = \tilde{B}_{t-1} I_{t-1}$ yields expression (A.4).

A.2 Linearizing the model

Here we provide details on the linearization of our model introduced in Section 2. Lower-case letters denote log deviation of upper case letters from steady state, absolute deviation (scaled by the price level) in case of public debt and taxes. We linearize around purchasing power parity, zero inflation and zero default. Variables in the rest of the world are constant. Public debt can be non-zero in steady state, parameterized by $\lambda = \tau/(1-\beta) \geq 0$.

Log-linearizing the Euler equation $R_t = 1/\{E_t \rho_{t,t+1}\}$, the labor supply curve $W_t/P_t = C_t H_t^{\varphi}$ and the risk sharing condition $C_t/C^* = Q_t$ yields the conditions

$$c_t = E_t c_{t+1} - (r_t - E_t \pi_{t+1}) \tag{A.5}$$

$$w_t^r := w_t - p_t = c_t + \varphi h_t, \tag{A.6}$$

$$c_t = q_t, \tag{A.7}$$

where $\pi_t := p_t - p_{t-1}$ is CPI inflation. We approximate the real exchange rate $Q_t = (P^* \mathcal{E}_t)/P_t$ and the consumer price index $P_t = ((1-\omega) P_{H,t}^{1-\sigma} + \omega(\mathcal{E}_t P^*)^{1-\sigma})^{1/(1-\sigma)}$ as

$$q_t = e_t - p_t \tag{A.8}$$

$$p_t = (1-\omega) p_{H,t} + \omega e_t. \tag{A.9}$$

Aggregate demand $Y_t = (P_{H,t}/P_t)^{-\sigma}[(1-\omega)C_t + \omega Q_t^{\sigma} C^*]$ can be approximated by

$$y_t = -\sigma(p_{H,t} - p_t) + (1-\omega)c_t + \omega\sigma q_t + \mu_t,$$

where we introduce the demand shock μ_t that we use in Section 5. Combine this with (A.8) and (A.9) to obtain

$$y_t = (1-\omega)c_t + \omega\sigma(2-\omega)/(1-\omega) q_t + \mu_t \tag{A.10}$$

The aggregate supply block can be written as a New Keynesian Phillips curve

$$\pi_{H,t} = \beta E_t \pi_{H,t+1} + \kappa mc_t, \tag{A.11}$$

where marginal costs $MC_t = W_t/P_{H,t}$ are approximated as

$$mc_t = w_t - p_{H,t} = w_t^r - (p_{H,t} - p_t). \tag{A.12}$$

Furthermore, production technology $Y_t = H_t$ can be approximated as

$$y_t = h_t. \tag{A.13}$$

The policy rules $\mathcal{E}_t = 1$ and $(R_t^{-1}/\beta) = \Pi_{H,t}^{-\phi}$ can be readily log-linearized as

$$e_t = 0 \tag{A.14}$$

as well as

$$r_t = \phi\pi_{H,t}. \tag{A.15}$$

The government's flow budget constraint can be written as

$$\beta\frac{(I_t)^{-1}}{\beta}\frac{B_{t+1}}{P_{H,t}} = (1-\Theta_t)\frac{B_t}{P_{H,t-1}}\frac{P_{H,t-1}}{P_{H,t}} - \tau_t.$$

We log-linearize around $I = 1/\beta$ as well as $1 - \Theta = 1$, we linearize around $B/P_H = \lambda$ and $T/P_H = (1-\beta)\lambda$ to obtain

$$\beta b_{t+1} = (1-\psi)b_t + \lambda(\beta i_t - \pi_{H,t} - \theta_t) + \eta_t, \tag{A.16}$$

where we denote $-\theta_t := \log(1-\Theta_t)$, where we have used that the tax rule is already in linear form: $\tau_t - \tau = \psi b_t$, and where we introduce the deficit shock η_t that we use in Section 5. Using Euler equation (A.5), the bond price schedule $1 = I_t E_t \rho_{t,t+1}(1-\Theta_{t+1})$ can be log-linearized to

$$i_t = r_t + E_t\theta_{t+1}. \tag{A.17}$$

Finally, the condition $B_{t+1}/P_{H,t} = \lambda$ can be written in linearized terms as

$$b_{t+1} = 0, \tag{A.18}$$

whereas the zero-default condition $1 - \Theta_t = 1$, by using that $-\theta_t = \log(1-\Theta_t)$ as defined above, becomes

$$\theta_t = 0. \tag{A.19}$$

A.3 Equations in Section 3

Here we derive the set of equations that are shown in Section 3. First, equations (A.16)-(A.17) correspond to equations (3.5)-(3.6) from the text. The policy equations (A.14)-(A.15) and (A.18)-(A.19) correspond to (3.7)-(3.8) in the text. Next, equation (3.4) is just the combination of equations (A.8) and (A.9). The three equations (3.1), (3.2) and (3.3) are obtained as follows. Insert risk sharing (A.7) into goods market clearing (A.10) to obtain equation (3.3). Rewrite the Euler equation (A.5) as

$$\begin{aligned} c_t &= E_t c_{t+1} - (r_t - E_t[(1-\omega)\pi_{H.t+1} + \omega\Delta e_{t+1}]) \\ &= E_t c_{t+1} - (r_t - E_t \pi_{H,t+1} - \frac{\omega}{\varpi} E_t(\Delta y_{t+1} - \Delta\mu_{t+1})), \end{aligned}$$

where we use (A.9) in the first line and (3.3) and (3.4) from the main text in the second line. Combine (A.7) and (3.3) to obtain

$$c_t = \frac{1-\omega}{\varpi}(y_t - \mu_t).$$

Use this expression to substitute for consumption in the Euler equation above to obtain

$$y_t = E_t y_{t+1} - \varpi(r_t - E_t \pi_{H,t+1}),$$

which is equation (3.1). Use equations (A.6), (A.7), (A.8), (A.9) and production technology (A.13) to rewrite marginal cost

$$mc_t^r = w_t^r - (p_{H,t} - p_t) = c_t + \varphi h_t - (p_{H,t} - p_t) = (\varpi^{-1} + \varphi) y_t.$$

Insert this into the Phillips curve to obtain equation (3.2) in the text.

A.4 Robustness: Model with incomplete markets

Here we discuss the model extension from Section 5 where international financial markets are incomplete. As explained in the text, we assume that private external debt, unlike public debt, is in foreign currency.

There are two changes relative to the baseline model. First, the household budget constraint is replaced by

$$\begin{aligned} \int_0^1 P_{H,t}(i) C_{H,t}(i) di + \int_0^1 P_{F,t}(i) C_{F,t}(i) di + R_t^{-1} D_{t+1} + R_t^{*-1} \mathcal{E}_t D_{t+1}^* + I_t^{-1} B_{t+1} \\ = W_t H_t + D_t + \mathcal{E}_t D_t^* + (1 - \Theta_t) B_t + \mathcal{Y}_t - P_t \tau_t. \end{aligned}$$

As in the baseline model, R_t denotes the interest rate on a bond in domestic currency. Foreign agents do not trade this bond in equilibrium, such that $D_{t+1} = 0$ at all times. In contrast, foreign agents trade bonds denominated in their own currency, at price R_t^{*-1}. Finally, domestic households hold risky government debt at price I_t^{-1}.

Second, as is well understood, incomplete assets markets induce non-stationarity (a unit root) to small open economy models—the steady state level of D_{t+1}^* is indeterminate. To avoid this property, we follow Schmitt-Grohé and Uribe (2003) and introduce an endogenous discount factor as ($\alpha > 0$ a small positive number)

$$\beta_{t+1} = \beta \left(1 + \alpha(\tilde{D}^*_{t+1} - \lambda^*)\right)^{-1} \beta_t, \quad \beta_0 = 1.$$

The discount factor depends on the country's (aggregate) net foreign asset position, which in equilibrium equals the net foreign asset position at the individual level (that is, $\tilde{D}^*_{t+1} = D^*_{t+1}$). By the arguments put forward in Schmitt-Grohé and Uribe (2003), this discount factor guarantees a net foreign asset position of λ^* in steady state.

Household maximization implies the following set of Euler equations

$$\begin{aligned}
1 &= \beta_{t+1} R_t E_t M_{t,t+1} \\
1 &= \beta_{t+1} R^*_t E_t M_{t,t+1} (\mathcal{E}_{t+1}/\mathcal{E}_t) \\
1 &= \beta_{t+1} I_t E_t M_{t,t+1} (1 - \Theta_{t+1}).
\end{aligned}$$

where $M_{t,t+1} := (C_{t+1}/C_t)^{-1}(P_t/P_{t+1})$. Using similar steps as in Section A.2 these can be approximated as

$$c_t = E_t c_{t+1} - (r_t - E_t \pi_{t+1} - \alpha \hat{d}^*_{t+1}) \tag{A.20}$$

$$r_t = E_t e_{t+1} - e_t \tag{A.21}$$

$$i_t = r_t + E_t \theta_{t+1}, \tag{A.22}$$

where in equation (A.21) we use that $R^*_t = \beta^{-1}$ remains in steady state throughout. Up to this point, the incomplete markets and the complete markets model coincide except for the endogenous discount factor in equation (A.20) (in contrast, equations (A.21) and (A.22) are also part of the complete markets model—see equations (4.4) and (3.6)).

The key difference between the two models arises because the risk sharing condition (A.7) and hence condition (3.3) are not part of the equilibrium. Instead, we keep track of net foreign assets via the aggregate resource constraint

$$P_t C_t + (R^*_t)^{-1} \mathcal{E}_t D^*_{t+1} = P_{H,t} Y_t + \mathcal{E}_t D^*_t,$$

where we have used the price indexes and consumption-demand functions from the main text to rewrite $\int_0^1 P_{H,t}(i) C_{H,t}(i) di + \int_0^1 P_{F,t}(i) C_{F,t}(i) di = P_t C_t$, where we have used the equilibrium expressions for profits $\mathcal{Y}_t = P_{H,t} Y_t - W_t H_t$ and replaced the government budget constraint $I_t^{-1} B_{t+1} = B_t(1 - \Theta_t) - P_t \tau_t$. Note that, as government debt and taxes drop out of the household budget constraint, Ricardian equivalence always obtains in this model. We divide both sides by P_t to re-write this as

$$C_t + (R_t^*)^{-1} Q_t D_{t+1}^* = \frac{P_{H,t}}{P_t} Y_t + Q_t D_t^*,$$

where we have inserted the real exchange rate $Q_t = \mathcal{E}_t / P_t$ (recall that $P_t^* = 1$ by our normalization). This expressions shows how a real depreciation (a rise in Q_t) harms consumption to the extent that net foreign assets are negative ($D_t^* < 0$)—the balance sheet effect of depreciation.

If net foreign assets are zero in steady state, the balance sheet effect drops out up to first order. Instead, we linearize around $D^*/P \equiv \lambda^* = -1.177$—which corresponds to 30% net foreign debt to GDP in steady state—to generate large balance sheet effects from depreciation.[25] From the last equation, this implies $C \neq Y$, such that the linearization is around a different steady state than in the complete markets model. To make results comparable, we keep the assumption of linearizing around a steady state of purchasing power parity: $Q = 1$, implying that also $P_H/P = 1$. The previous budget constraint then implies that $C = Y + (1-\beta)\lambda^*$, where we use that $R_t^* = 1/\beta$.[26] We therefore obtain

$$\beta(d_{t+1}^* + q_t \lambda^*) + C c_t = Y(y_t + p_{H,t} - p_t) + d_t^* + q_t \lambda^*, \tag{A.23}$$

where $Y = ((\gamma - 1)/\gamma)^{1+\varphi}$ is pinned down by the supply side of the model, and where C was given above. Finally, the demand side of the economy must also be adjusted, such that goods market clearing (A.10) needs to be replaced by

$$Y y_t = (1-\omega) C c_t - ((1-\omega)C + \omega C^*)\sigma(p_{H,t} - p_t) + C^* \omega \sigma q_t + \mu_t, \tag{A.24}$$

where C^* is pinned down by $Y = (1-\omega)C + \omega C^*$, and where μ_t is the demand shock as before. All remaining equations are unchanged from the baseline model. The incomplete markets model is as a set of variables $\{y_t, \pi_{H,t}, p_{H,t}, \pi_t, p_t, e_t, q_t, i_t, r_t, d_{t+1}^*, b_{t+1}, \theta_t, w_t, h_t, mc_t, c_t\}$ that satisfy the sixteen equations (A.6), (A.8)-(A.9), equation (A.24), equations (A.11)-(A.13), one of the two policy rules (A.14)-(A.15), equations (A.16)-(A.17), one of the two policy rules (A.18)-(A.19), equations (A.20)-(A.21) and (A.23), and the definitions for inflation $\pi_{H,t} = p_{H,t} - p_{H,t-1}$ and $\pi_t = p_t - p_{t-1}$, for given initial b_0 and d_0^*.

[25] To obtain this number, we compute $\lambda^* = -0.3 \times Y \times 4$, where $Y = ((\gamma-1)/\gamma)^{1+\varphi}$ is quarterly output in steady state.

[26] The reader may wonder how it is possible to *assume* that purchsing power parity holds in the steady state. From the demand function in steady state, $Y = (1-\omega)C + \omega C^*$, the underlying assumption is that foreign consumption C^* must adjust. We thus treat C^* as an additional parameter.

B Appendix: Propositions

B.1 Propositions 1-2

Here we provide the proof of Propositions 1-2. Consider the model in Section 3, but impose flexible prices $\xi = 0$. In this case the model collapses to

$$r_t = E_t \pi_{H,t+1} \tag{B.1}$$

$$e_t = p_{H,t} \tag{B.2}$$

$$\beta b_{t+1} = (1-\psi) b_t + \lambda(\beta i_t - \pi_{H,t} - \theta_t) \tag{B.3}$$

$$i_t = r_t + E_t \theta_{t+1} \tag{B.4}$$

as well as $y_t = q_t = 0$ and policy $r_t = \phi \pi_{H,t}$ or $e_t = 0$, and $b_{t+1} = 0$ or $\theta_t = 0$. We solve the model backwards by using the method of undetermined coefficients, thereafter we show that the solution derived is mean square stable whenever condition (4.2) holds.

In regime **Reform**, $e_t = 0$, such that from (B.2) $p_{H,t} = 0$ and therefore $\pi_{H,t} = 0$. Since $\theta_t = 0$ in this regime, and further regime change is ruled out, $r_t = i_t = 0$ from (B.1) and (B.4). Public debt hence evolves according to

$$\beta b_{t+1} = (1 - \psi_{\text{Reform}}) b_t,$$

and is mean-reverting by assumption ($\psi_{\text{Reform}} > 1 - \beta$).

Regime **Default** is identical to regime Reform, expect that fiscal policy is "active" such that debt has an explosive root ($\psi < 1-\beta$ by assumption). As a result, default θ_t must adjust such that $b_{t+1} = 0$ at all times, as argued in the main text. Hence the solution is $e_t = 0$, $p_{H,t} = 0$ and therefore $\pi_{H,t} = 0$ as before, as well as $\theta_t = ((1-\psi)/\lambda) b_t$ in the period upon entering the regime, as well as $b_{t+1} = 0$ and $\theta_{t+1} = 0$ in all periods thereafter.

There is no default in regime **Exit**, thus $i_t = r_t$ from equation (B.4). By contrast, generally $e_t = p_{H,t} \neq 0$ in this regime. The system (B.1)-(B.4) can be re-written as

$$\phi \pi_{H,t} = E_t \pi_{H,t+1}$$
$$\beta b_{t+1} = (1-\psi) b_t + \lambda(\beta\phi - 1)\pi_{H,t}.$$

It features one forward looking ($\pi_{H,t}$), one backward looking variable (b_{t+1}). As can be easily checked, in spite of $\psi < 1 - \beta$, the system exhibits bounded dynamics if the Taylor principle does not hold $\phi < 1$ (Leeper, 1991). A guess and verify approach yields

$$(\Delta e_t =)\ \pi_{H,t} = \frac{1-\psi-\beta\phi}{\lambda(1-\beta\phi)} b_t, \qquad b_{t+1} = \phi b_t.$$

In the main text, we show the special case of these equations when $\psi = 0$.

In the initial regime **Crisis**, $e_t = 0$ and hence $p_{H,t} = 0$ and $\pi_{H,t} = 0$ from equation (B.2). However, generally $r_t \neq 0$ because of expected changes in inflation and nominal depreciation (equations (B.1) and (B.2)), and $i_t \neq 0$ because of (in addition to the variation in r_t) expected outright default (equation (B.4)). Moreover, movements in the bond yield i_t feed back into b_{t+1} through equation (B.3).

By using the Markov chain we can write equation (B.3) as

$$i_t = \left[\varepsilon \frac{1-\psi-\beta\phi}{\lambda(1-\beta\phi)} + \delta \frac{1-\psi}{\lambda}\right] b_{t+1}, \tag{B.5}$$

where we have used the equilibrium default and nominal depreciation rates from regimes Default and Exit. Insert this into (B.3)

$$\beta b_{t+1} = (1-\psi) b_t + \lambda \beta i_t$$

and rearrange for b_{t+1} to obtain

$$b_{t+1} = (1-\psi)\frac{1}{\beta}\left(1 - \epsilon\left(\frac{1-\psi-\beta\phi}{1-\beta\phi}\right) - \delta(1-\psi)\right)^{-1} b_t =: (1-\psi)\Theta^b b_t$$

for public debt, and

$$i_t = (1-\psi)(\Theta^\theta + \Theta^r) b_t,$$
$$\text{where} \quad \Theta^\theta = \delta(1-\psi)\frac{\Theta^b}{\lambda} > 0 \quad \text{and} \quad \Theta^r = \epsilon\left(\frac{1-\psi-\beta\phi}{1-\beta\phi}\right)\frac{\Theta^b}{\lambda} > 0,$$

for the sovereign yield. In the main text, we show the special case of these two equations as $\psi = 0$. This completes the proof of Proposition 2.

Now turn to stability, which is Proposition 1. It is clear that the stability of the overall system hinges on the stability of its endogenous states, which are only b_t when prices are flexible. In this case, the general condition characterizing mean square stability, which is that all eigenvalues of

$$(\mathcal{P}' \otimes I_{n^2})\text{diag}(F_{\varsigma_1} \otimes F_{\varsigma_1}, ..., F_{\varsigma_h} \otimes F_{\varsigma_h})$$

must lie within the unit circle (n denoting the number of endogenous variables, h the number of regimes, F the solution matrices in the respective regimes, see the main text), reduces to the simple condition that all eigenvalues of

$$\begin{pmatrix} 1-f-\delta-\epsilon & 0 & 0 & 0 \\ f & 1 & 0 & 0 \\ \delta & 0 & 1 & 0 \\ \epsilon & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} \left(\frac{1}{\beta(1-\delta-\epsilon)}\right)^2 & 0 & 0 & 0 \\ 0 & \left(\frac{1-\psi_{\text{Reform}}}{\beta}\right)^2 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \phi^2 \end{pmatrix} \tag{B.6}$$

must lie within the unit circle. Because the target regimes are absorbing and the matrix on the right hand side is diagonal, a sufficient condition for this is that

$$(1-f-\delta-\epsilon)\left(\frac{1}{\beta(1-\epsilon-\delta)}\right)^2 < 1.$$

which is equation (4.2) in the main text. This completes the proof of Proposition 1.

B.2 Proposition 3

Here we derive the solution of the sticky price model shown in Section 4. To do so, we make a number of parametric assumptions. First and most importantly, we require that $f+\delta+\epsilon = 1$, that is, agents expect the first regime to persist with probability zero. Second, we set $\varphi = 0$, that is, we impose a linear disutility of labor. Third, we impose $\psi = 0$ such that taxes do not systematically respond to debt in the crisis regime, as well as $\kappa = 1-\beta$, i.e. that the slope of the Phillips curve relates in a particular way to the discount factor. The first assumption is strictly needed for a derivation of closed form results to be feasible. In contrast, the last three assumptions simplify the exposition.

We solve the model backwards using the method of undetermined coefficients. For convenience, we repeat the relevant system of equations

$$y_t = E_t y_{t+1} - \varpi(r_t - E_t \pi_{H,t+1}) \tag{B.7}$$

$$\pi_{H,t} = \beta E_t \pi_{H,t+1} + \kappa(\varphi + \varpi^{-1}) y_t, \tag{B.8}$$

$$(1-\omega) y_t = \varpi q_t, \tag{B.9}$$

$$q_t = (1-\omega)(e_t - p_{H,t}) \tag{B.10}$$

$$\beta b_{t+1} = (1-\psi) b_t + \lambda(\beta i_t - \pi_{H,t} - \theta_t), \tag{B.11}$$

$$i_t = r_t + E_t \theta_{t+1} \tag{B.12}$$

along with the definition for inflation $\pi_{H,t} = p_{H,t} - p_{H,t-1}$. Furthermore, differing across regimes are the conduct of monetary policy

$$e_t = 0 \qquad \text{or} \qquad r_t = \phi \pi_{H,t} \tag{B.13}$$

and the rate of equilibrium default

$$\theta_t = 0 \qquad \text{or} \qquad b_{t+1} = 0. \tag{B.14}$$

Starting with regime **Reform**, it holds that $e_t = \theta_t = 0$ and that $\psi_{\text{Reform}} > 1-\beta$. We also derive that $y_t = -\varpi p_{H,t}$ from (B.9)-(B.10). Inserting this in (B.8) allows us to derive a second order difference equation in the price level

$$\beta p_{H,t+1} = (1+\beta+\kappa\varpi(\varphi+\varpi^{-1}))p_{H,t} - p_{H,t-1}.$$

Guessing that $p_{H,t} = G_{pp}p_{H,t-1}$ for some unknown coefficient G_{pp} we obtain the restriction $1 = G_{pp}(1+(1-G_{pp})\beta+\kappa\varpi(\varphi+\varpi^{-1}))$. This is a quadratic equation in G_{pp} with one root inside the unit circle. To obtain this root we rewrite this as

$$\frac{(1-\beta G_{pp})(1-G_{pp})}{G_{pp}} = \kappa\varpi(\varphi+\varpi^{-1}).$$

Recognizing that $\kappa \equiv (1-\beta\xi)(1-\xi)/\xi$ reveals that, once we impose our assumption $\varphi = 0$, the solution is $G_{pp} = \xi < 1$, where ξ is the price-stickiness parameter. We now determine the equilibrium behavior of interest rates and public debt. First, combining (B.7), (B.9)-(B.10) yields $r_t = \Delta e_{t+1}$, such that $r_t = 0$ in this regime. Second, it follows that $i_t = 0$ from (B.12) as there is no possibility of default. The equilibrium behavior of debt can now be derived from (B.11)

$$b_{t+1} = \frac{1-\psi_{\text{Reform}}}{\beta}b_t + \frac{\lambda(1-\xi)}{\beta}p_{H,t-1},$$

where we have inserted the solution for $p_{H,t}$. This is a stable difference equation, because of our assumption $\psi_{\text{Reform}} > 1-\beta$ above.

The solution for the price level is the same in regime **Default**, given that $e_t = 0$ holds in this regime, too. As a result, it also holds that $r_t = 0$. Instead, θ_t is generally non-zero. Because $b_{t+1} = 0$ at all times in this regime, it must be that

$$0 = (1-\psi)b_t + \lambda(\beta\theta_{t+1} + (1-\xi)p_{H,t-1} - \theta_t),$$

where we have used that $i_t = \theta_{t+1}$ under $r_t = 0$, see equation (B.12). This is a first order difference equation in θ_t, for given states b_t and $p_{H,t-1}$. To solve it, we guess that $\theta_t = G_{\theta b}b_t + G_{\theta p}p_{H,t-1}$ for coefficients $G_{\theta b}$ and $G_{\theta p}$ to be determined. Note that, at time $t+1$, the guess reduces to $\theta_{t+1} = G_{\theta p}\xi p_{H,t-1}$, where we have used that $b_{t+1} = 0$ and that $p_{H,t} = \xi p_{H,t-1}$. Inserting this in the previous equation yields

$$0 = (1-\psi)b_t + \lambda(\beta G_{\theta p}\xi p_{H,t-1} + (1-\xi)p_{H,t-1} - (G_{\theta b}b_t + G_{\theta p}p_{H,t-1})),$$

which reveals that $G_{\theta b} = (1-\psi)/\lambda$ and that $G_{\theta p} = (1-\xi)/(1-\beta\xi)$.

In regime **Exit** we use that $r_t = \phi\pi_{H,t}$ and that $\theta_t = 0$. We solve the two by two system

$$\begin{aligned} y_t &= y_{t+1} - \varpi(\phi\pi_{H,t} - \pi_{H,t+1}) \\ \pi_{H,t} &= \beta\pi_{H,t+1} + \kappa\varpi^{-1}y_t, \end{aligned}$$

along with the evolution of debt

$$b_{t+1} = \frac{1-\psi}{\beta}b_t + \frac{\lambda(\beta\phi-1)}{\beta}\pi_{H,t}.$$

Guess that $\pi_{H,t} = G_{\pi b}b_t$ and $y_t = G_{yb}b_t$ for coefficients $G_{\pi b}$ and G_{yb} to be determined. We obtain the two restrictions

$$\begin{aligned} &G_{yb}[1-(1-\psi)/\beta - (\lambda(\beta\phi-1)/\beta)G_{\pi b}] + G_{\pi b}\varpi[\phi - (1-\psi)/\beta - (\lambda(\beta\phi-1)/\beta)G_{\pi b}] = 0 \\ &G_{\pi b}[1-(1-\psi) - \lambda(\beta\phi-1)G_{\pi b}] - \kappa\varpi^{-1}G_{yb} = 0. \end{aligned}$$

Solving the second equation for $G_{\pi b}\varpi[-(1-\psi)/\beta - (\lambda(\beta\phi-1)/\beta)G_{\pi b}] = (\kappa/\beta)G_{yb} - (\varpi/\beta)G_{\pi b}$ and replacing in the first equation yields

$$\begin{aligned} &G_{yb}[1-(1-\psi)/\beta - (\lambda(\beta\phi-1)/\beta)G_{\pi b}] + G_{\pi b}\varpi(\phi - 1/\beta) + (\kappa/\beta)G_{yb} = 0 \\ \Leftrightarrow\ &G_{yb}[\kappa - (1-\beta-\psi) - \lambda(\beta\phi-1)G_{\pi b}] + G_{\pi b}\varpi(\beta\phi-1) = 0. \end{aligned}$$

Because we assume that $\kappa = 1-\beta-\psi$ (which is implied by our assumptions $\kappa = 1-\beta$ and $\psi = 0$), this equation reduces to

$$G_{\pi b}(\beta\phi-1)[\varpi - \lambda G_{yb}] = 0$$

which reveals that $G_{yb} = \varpi/\lambda$. Using this information we can use the second restriction above to obtain a quadratic equation for $G_{\pi b}$ as

$$G_{\pi b}^2 + \frac{\psi}{\lambda(1-\beta\phi)}G_{\pi b} - \frac{\kappa}{\lambda^2(1-\beta\phi)}.$$

The two roots of this equation are

$$G_{\pi b} = -\frac{\psi}{2\lambda(1-\beta\phi)} \pm \sqrt{\frac{\psi^2}{4\lambda^2(1-\beta\phi)^2} + \frac{\kappa}{\lambda^2(1-\beta\phi)}}$$

Under our assumption $\psi = 0$ the single positive root is $G_{\pi b} = \sqrt{\kappa/(1-\beta\phi)}/\lambda$. Hence we have verified that equilibrium output and inflation evolve as $\pi_{H,t} = (\sqrt{\kappa/(1-\beta\phi)}/\lambda)b_t$ and $y_t = (\varpi/\lambda)b_t$. Inserting this into the equation for debt above

$$\begin{aligned} b_{t+1} &= (1/\beta)b_t + (1/\beta)\lambda(\beta\phi-1)(\sqrt{\kappa/(1-\beta\phi)}/\lambda)b_t \\ &= (1/\beta)(1-(1-\beta\phi)\sqrt{\kappa/(1-\beta\phi)})b_t. \end{aligned}$$

Because of $\kappa = 1 - \beta$, it holds that $1 - \sqrt{\kappa} < \beta$. Furthermore, necessarily $1 - \beta\phi < 1$. From this follows that the coefficient on debt is smaller than one, such that debt is indeed mean reverting after exit.

Finally, in the initial regime **Crisis** we use our assumption on the transition probabilities. Because $e_t = 0$ in this regime we again write the difference equation

$$\begin{aligned} \beta E_t p_{H,t+1} &= (1+\beta+\kappa)p_{H,t} - p_{H,t-1} \\ \beta[(f+\delta)p_{H,t+1}|\text{Reform} + \epsilon p_{H,t+1}|\text{Exit}] &= (1+\beta+\kappa)p_{H,t} - p_{H,t-1}, \end{aligned}$$

where we have evaluated the expectations operator using that $f + \delta + \epsilon = 1$, used that $p_{H,t+1}$ is the same in both Reform and Default (see the earlier derivation), and where we have imposed our assumption $\varphi = 0$. Inserting the solutions $p_{H,t+1}|\text{Reform} = \xi p_{H,t}$ and $p_{H,t+1}|\text{Exit} = p_{H,t} + (\sqrt{\kappa/(1-\beta\phi)}/\lambda)b_{t+1}$ we rewrite this as

$$\beta[(f+\delta)\xi p_{H,t} + \epsilon(p_{H,t} + (\sqrt{\kappa/(1-\beta\phi)}/\lambda)b_{t+1})] = (1+\beta+\kappa)p_{H,t} - p_{H,t-1}. \qquad (*)$$

To evaluate this further, we require the equilibrium behavior of b_{t+1}. Public debt is given by

$$\beta b_{t+1} = b_t + \lambda(\beta i_t - (p_{H,t} - p_{H,t-1})),$$

where we have used our assumption $\psi = 0$ and the fact that $\theta_t = 0$. The evolution of i_t is obtained from (B.12). Using that $r_t = E_t\Delta e_{t+1}$ we have

$$\begin{aligned} i_t &= r_t + E_t\theta_{t+1} = E_t(\Delta e_{t+1} + \theta_{t+1}) \\ &= \epsilon e_{t+1}|\text{Exit} + \delta\theta_{t+1}|\text{Default} \\ &= \epsilon[(1/\varpi)y_{t+1}|\text{Exit} + p_{H,t+1}|\text{Exit}] + \delta\theta_{t+1}|\text{Default}. \end{aligned}$$

Here we have used that $e_t = 0$ in the initial regime and combined equations (B.9)-(B.10) to replace e_{t+1}. Inserting our solutions $p_{H,t+1}|\text{Exit} = p_{H,t} + (\sqrt{\kappa/(1-\beta\phi)}/\lambda)b_{t+1}$, $y_{t+1}|\text{Exit} = (\varpi/\lambda)b_{t+1}$ as well as $\theta_{t+1}|\text{Default} = (1/\lambda)b_{t+1}$ we rewrite this further as

$$i_t = \epsilon\{[(1/\lambda) + (\sqrt{\kappa/(1-\beta\phi)}/\lambda)]b_{t+1} + p_{H,t}\} + \delta(1/\lambda)b_{t+1}$$

such that public debt can be written as

$$\begin{aligned} & \beta b_{t+1} = b_t + \beta[\epsilon\{[1 + \sqrt{\kappa/(1-\beta\phi)}]b_{t+1} + \lambda p_{H,t}\} + \delta b_{t+1}] - \lambda(p_{H,t} - p_{H,t-1}) \\ \Leftrightarrow\ & (\beta[1 - \epsilon(1 + \sqrt{\kappa/(1-\beta\phi)}) - \delta])b_{t+1} = b_t - \lambda(1-\beta\epsilon)p_{H,t} + \lambda p_{H,t-1}. \qquad (**) \end{aligned}$$

Combining equations (*) and (**) yields the solution for $p_{H,t}$ and b_{t+1} in the initial regime. Here we merely state the solution for the price level

$$([1-\epsilon(1+\sqrt{\kappa/(1-\beta\phi)})-\delta][\xi^{-1}-\beta\epsilon(1-\xi)]+\epsilon(1-\beta\epsilon)\sqrt{\kappa/(1-\beta\phi)})p_{H,t} \\ =\epsilon(\sqrt{\kappa/(1-\beta\phi)}/\lambda)b_t+(1-\epsilon-\delta)p_{H,t-1},$$

where we have used that $\xi(1+\beta(1-\xi)+\kappa)=1$. We simplify this a bit further to obtain

$$((1-\epsilon-\delta)(1-\beta\epsilon\xi(1-\xi))-\epsilon\sqrt{\kappa/(1-\beta\phi)}(1-\xi(1-\beta\epsilon\xi))p_{H,t} \\ =\epsilon\xi(\sqrt{\kappa/(1-\beta\phi)}/\lambda)b_t+(1-\epsilon-\delta)\xi p_{H,t-1}.$$

Note how this nests the case of $\epsilon=0$: in this case, the price level reverts at the speed of the Calvo parameter $p_{H,t}=\xi p_{H,t-1}$ (as in regimes Reform and Default). The solutions for the real exchange rate and for output follow directly from this equation, from the fact that $e_t=0$ in the initial regime, hence $q_t=-(1-\omega)p_{H,t}$, then from the risk sharing condition $(1-\omega)y_t=\varpi q_t$,, which is equation (B.9). This completes the proof of Proposition 3.

C Appendix: Additional robustness

In this Appendix, we present two additional robustness checks of our quantitative analysis, which complements the results from Section 5.4 from the main text.

In a first experiment, we continue to assume that financial markets are incomplete (as in robustness exercise 2 in the main text), and in addition relax the assumption that there are no resource costs of default. Such costs feature prominently in quantitative sovereign default models (e.g. Arellano, 2008). We therefore assume that there is a persistent resource loss in the domestic economy that is proportional to the size of the haircut. Specifically, we assume an impact-loss of 10 percent of output due to the default. We assume this output loss to be autocorrelated with a persistence parameter of 0.9. The (black) dotted lines with pluses in Figure 6 show the result: the country suffers from an additional output contraction, although this effect is moderate. More striking is that the real exchange rate depreciates faster after default and public debt and deficits are larger in the scenario with default costs.

In a second experiment, we allow for the possibility that foreign interest rates decline in the wake of the Greek sovereign debt crisis. This is meant to capture a possible response of monetary policy at the union level. In this robustness, rather than treating foreign interest rates r^* as a constant, we model them as a stochastic process: we assume that r_t^* follows a random walk. We then treat the process $\{r_t^*\}$ as an additional observable, as we match the ECB's main refinancing rate during our sample period. We find that yields, debt to GDP and

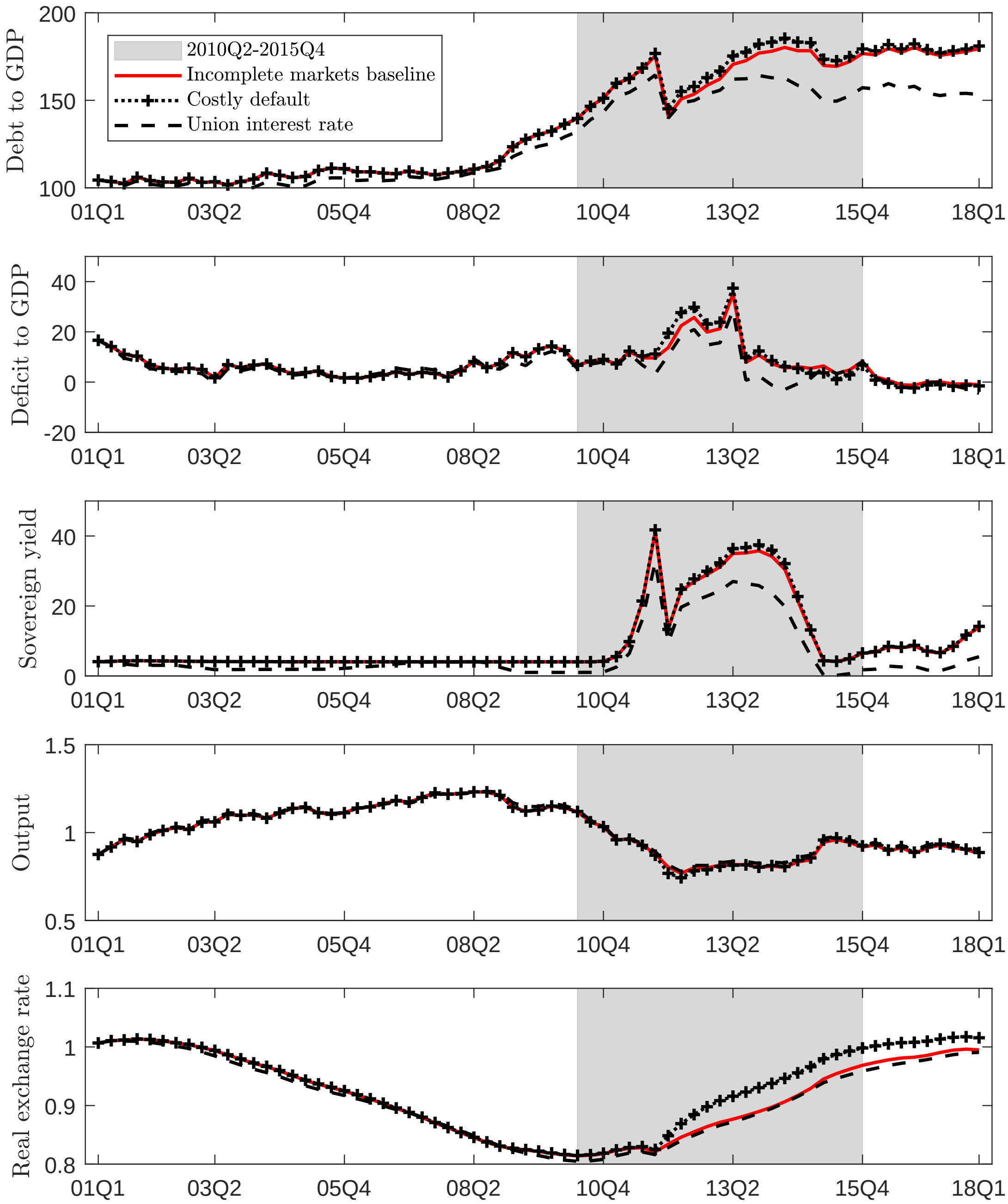

Figure 6: Macroeconomic performance in Greece, baseline under incomplete markets (red solid line) and robustness. Robustness exercise 1: costly default (black dotted line with pluses). Robustness exercise 2: response of union-wide interest rate (black dashed line).

deficits to GDP are somewhat lower during the crisis phase, and that the output contraction is less severe (see the black dashed lines in Figure 6).